OPTIMAL OPERATION
OF
POWER SYSTEMS

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ABSTRACT

This thesis is primarily concerned with the solution of the long-term scheduling problem for mixed hydro-thermal systems such as that of the former New Zealand Electricity Department (NZED - now the Electricity Division of the Ministry of Energy).

Chapters 2 to 6 describe our model for the solution of the deterministic version of this problem. This model, an adaptation of the Electricité de France (EDF) models outlined in Appendix A, involves the use of "energy prices" (Lagrange multipliers) to decompose the problem into manageable sub-problems. Our model differs from that of the EDF in the following ways. Firstly, instead of dealing with aggregate national loads, it is a multi-load model in which losses and restrictions in the transmission system are explicitly modelled. Secondly, we have adopted a more flexible scheme to ensure that short-term requirements are met. In particular we have developed a new approach to the scheduling of river chains in the short-term. Thirdly, we have generalised their approach so as to handle more realistic (non-convex) cost (and loss) functions. Finally, the nature of the NZED system (and of our model) has required the adoption of a more sophisticated approach to the adjustment of the "prices". Chapter 7 describes the application of this approach to the NZED system.

Chapter 8 uses some recent results on the optimal recourse problem to generalise this model into a stochastic framework. Chapter 9 uses this abstract stochastic

model to analyse some earlier approaches and to suggest a new method for the solution of realistic stochastic scheduling problems.

Finally, Chapter 10 shows how the prices developed in the solution of the scheduling problem may be used in other contexts. In particular, a simple adaptation allows the "optimal tariff problem" to be solved exactly.

CHAPTER 1

INTRODUCTION

1.1 THE POWER SYSTEM

1.1.1 Introduction

Electricity plays a central role in all modern industrial societies. Over the years electric utilities have built up large complex systems to produce electrical energy and distribute it to consumers.

Figure (1-1) illustrates the arrangement of an idealised (mixed hydro-thermal) system. Energy enters the system in a variety of forms. One important source of energy is the chemical potential energy locked up in fuels such as oil or coal. This energy is converted into electrical energy in conventional thermal power stations. Another important source is the gravitational potential energy of water which has been lifted to some elevation above sea level by the mechanisms of evaporation and precipitation. This potential may be converted to electrical energy in hydro-electric (hydro) plants. This form of potential energy is highly desirable in that it can easily be stored for future use in natural or artificial reservoirs. Other sources of energy include nuclear fission, geothermal steam, tidal variations and, potentially, solar radiation and wind. The electrical energy, once produced, is distributed through a network of high voltage alternating current (AC) or direct current (DC), transmission lines, broken down to lower voltages, then further distributed to

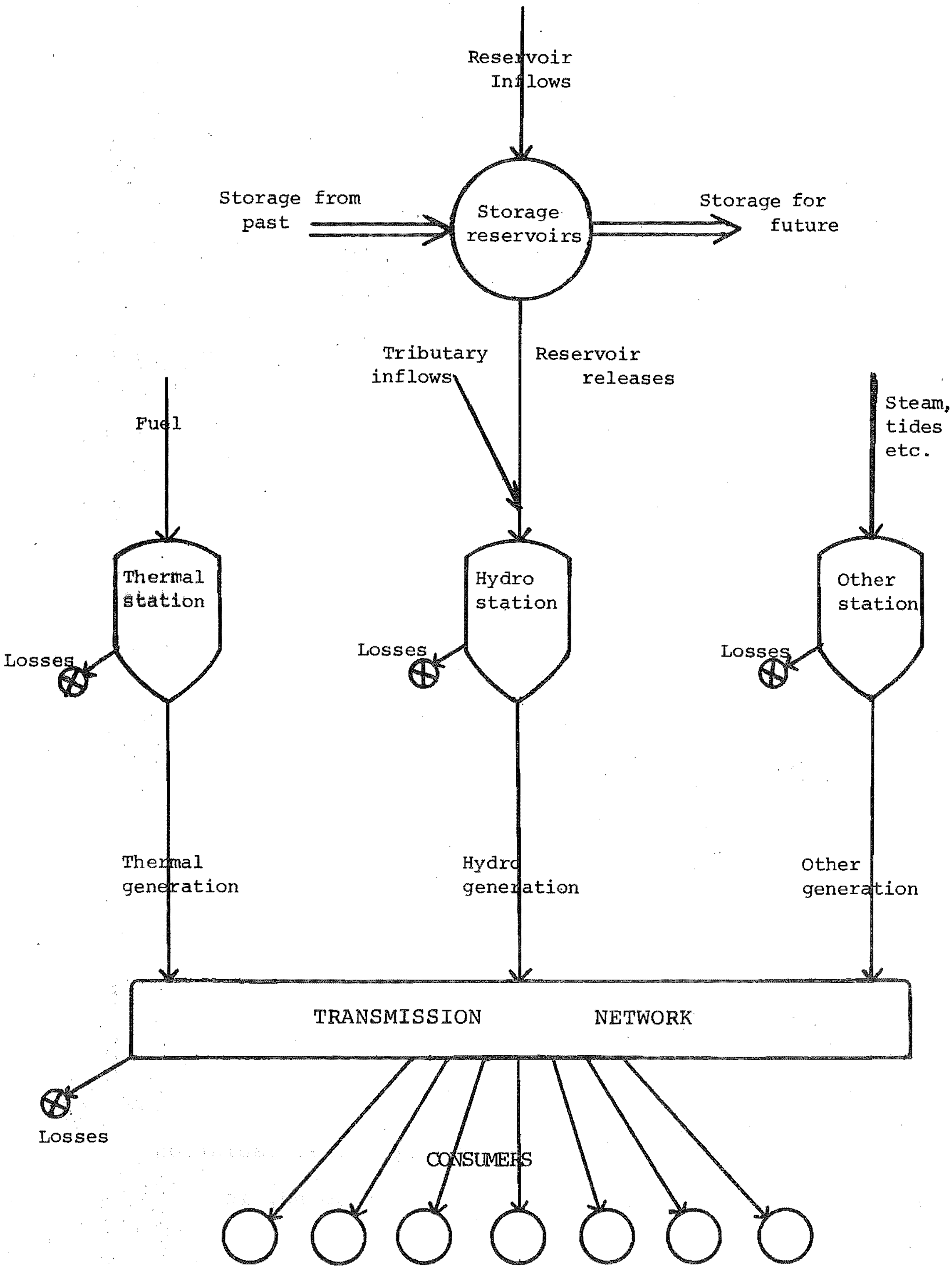


FIGURE (1-1): Idealised power system.

the individual consumers. Some energy is lost at each stage due to the inherent inefficiency of the equipment used while the remainder passes out of the system at the point of consumption. Costs are incurred by the purchase and maintenance of equipment, the administration and operation of the system and the purchase of fuel.

This study is primarily concerned with the system operated by the (former) New Zealand Electricity Department (NZED) (now the Electricity Division of the newly formed Ministry of Energy). The next sub-section is devoted to a description of this particular system.

1.1.2 The New Zealand Electricity System

In New Zealand the NZED is responsible for most (about 98%) of the electricity produced. The balance is produced by small plants operated by various local authorities or industries and sold to the national system. Only one of these plants (Waipori, near Dunedin) is at all significant. A simplified version of the system is shown in Figure (1-2). Figure (1-3) shows details of the more complex hydro-electric schemes while Figure (1-4) shows the relative importance of the various generation and load areas. One significant feature of this system is the direct current (DC) link between the North and South Islands. This limited capacity link serves to transfer energy produced by hydro plants in the South Island to the more populous North Island. We have developed a multi-load model partly in order to model this link.

We further note the sparse nature of the whole

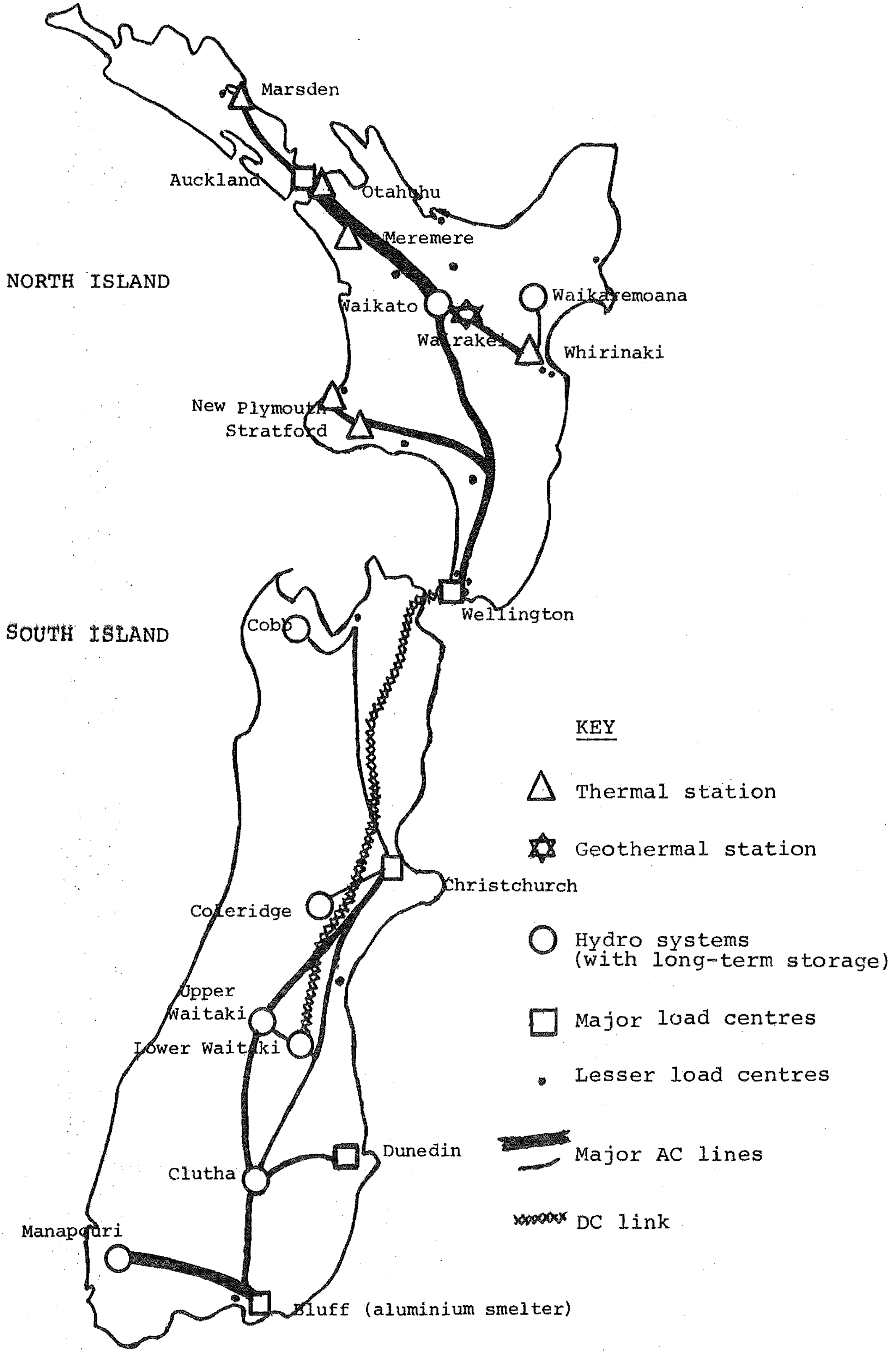


FIGURE (1-2): NZED system (as at 1/1/79).

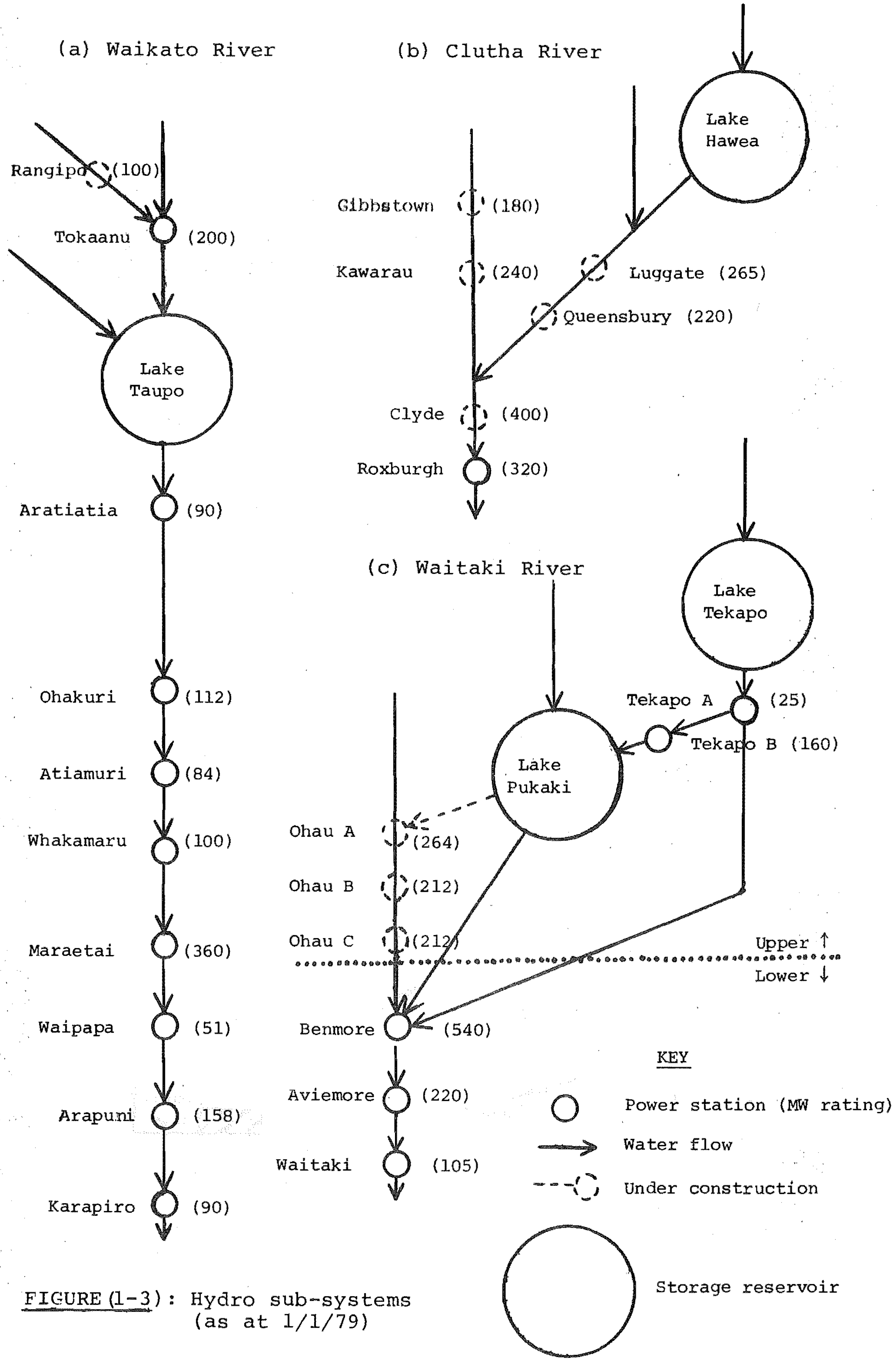


FIGURE (1-3): Hydro sub-systems
(as at 1/1/79)

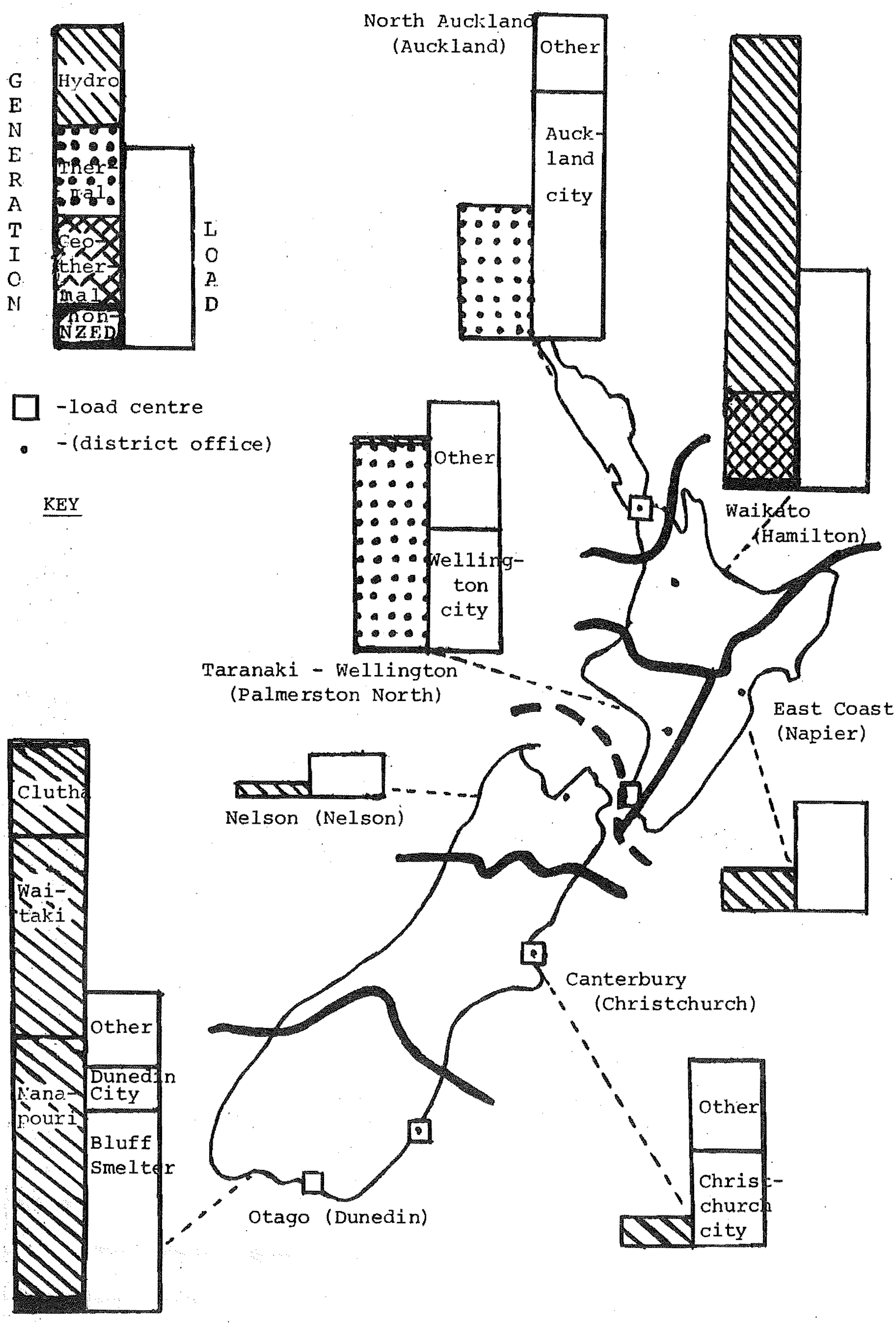


FIGURE (1-4): Regional energy balances for NZED system (in year ended 31/3/77. Sources [43] and [46].)

network and the distance of most major generating plants from major load centres. Thus losses (averaging about 6% over the whole system) and capacity restrictions impose significant limitations on the spatial distribution of generation. This represents a further incentive for the development of a multi-load model.

Note also that the system contains no nuclear or pumped storage hydro plant so that these will not be considered in our study. There is however one geothermal (steam) plant which is easily incorporated.

Tables (1-1) and (1-2), giving the capacity and output of each individual station, complete our description of the system. In Section 1.4 we outline the previous optimisation studies carried out on the system while in Chapter 7 we give some results from the application of our model.

1.2 POWER SYSTEMS OPTIMISATION PROBLEMS

1.2.1 Introduction

Naturally, given the expense involved in operating the system and its importance to the nation, a great deal of effort has been expended to ensure that each component is as efficient and reliable as is reasonably possible. In recent years the "energy crisis" has focussed attention on the problem of ensuring that the best use is made of the resources and equipment available. This general problem area involves a great diversity of optimisation problems reflecting the scale and complexity of electrical supply systems. Thus, at one end of the

"River System"	Station	Capacity (Megawatt rating)	Output: Gigawatt-hours (in year ended 31/3/77)	
Waikato (+ Tongariro)	Tokaanu	200	625	
	Aratiatia	90	363	
	Ohakuri	112	418	
	Atiamuri	84	312	
	Whakamaru	100	539	
	Maraetai	360	889	
	Waipapa	51	276	
	Arapuni	158	884	
	Karapiro	90	573	
			Waikato Total	4,879
Waikaremoana	Kaitawa, Piripaua, Tuai	124	594	
Other NZED	Mangahao, Matahina	92	398	
Non-NZED	-	-	170	
			Hydro Total	6,041
Thermal	Marsden	240	747	
	Meremere	210	1,104	
	New Plymouth	600	1,867	
	Otahuhu	180	39	
	Stratford	208	976	
	Whirinaki	108	3	
	Non NZED	-	19	
			Thermal Total	4,755
Geo-thermal	Wairakei	192	1,233	
			North Island Total	12,029

TABLE (1-1): North Island generation.

"River System"	Station	Capacity (Megawatt rating)	Output: Gigawatt-hours (in year ended 31/3/77)	
Cobb	Cobb	32	209	
Coleridge	Coleridge	35	173	
Waitaki	Tekapo (A)	25	113	
	Benmore	540	1,738	
	Aviemore	220	723	
	Waitaki	105	379	
			Waitaki Total	2,953
Clutha	Roxburgh	320	1,455	
Manapouri	Manapouri	700	3,717	
Other NZED	Arnold, Highbank, Monowai	34	120	
(Dunedin City Council)	Waipori	91	154	
Other non-NZED	-	-	104	
			South Island Total	8,885
			New Zealand Total	20,914

TABLE (1-2): South Island generation

(Sources, for both tables, [43] and [46]).

scale, one may attempt to determine the optimal place of electrical energy in the world economy over the next century, while, at the other end, one may be concerned with the instantaneous control of a generator or the most efficient way to boil an egg. Our principal concern in this study will be with the "long-term scheduling problem". In order to solve this problem accurately we also consider the "short-term scheduling problem". Further, our solution procedure yields information which can be used to solve the "optimal tariff problem" and also to make a significant contribution to the "optimal development problem". In the next few sub-sections we outline the nature of these problems.

1.2.2 The Long-Term Scheduling Problem

In the long-term scheduling problem the planning horizon is of the order of one year and this is usually divided into weekly or monthly intervals. The objective in this problem is to use the available equipment to meet an exogenously determined load (or demand) pattern in such a way as to minimise system operating costs, particularly fuel costs. The decision variables, then, are the generation levels of the various plants and the transmission levels through the various lines of the network. Owing to the size of the problem one can only deal with aggregate quantities for each interval and, possibly, plant. For example we may wish to determine the optimal total peak and off-peak generation from each plant in each interval. In this problem we must deal with constraints due to plant characteristics, upper and lower reservoir storage limits, river flow limits and limitations on transmission capacity. Non-linearities are introduced by the performance characteristics of generating plant and transmission lines.

If there were no storage capacity in the system the long-term optimisation problem would become trivial since there would then be no iteraction between the decisions made in successive intervals. Thus we would face in each interval a short-term problem, independent of any consideration of the future. However the presence of long-term storage reservoirs introduces the possibility of storing potential energy for future conversion to electricity. Then the long-term problem consists of finding the optimal balance between production from the various plants in each of the intervals of the planning horizon. This large and complex problem is further complicated by the fact that we do not know what inflows will occur in the future. Whether or not this is taken into account in the optimisation model this means that the model must be regularly re-optimised in order to provide solutions appropriate to the situations encountered as time progresses. (Unless, of course, the model involves the optimisation of future utilisation for all possible sequences). So, typically, a long-term scheduling model would be used by a utility each week (say) to determine the amount of water to be released from each reservoir in that week. In order to do this, however, the model must be able to determine the optimal future utilisation of the water if it were stored for future use. Thus it may arrive at an optimal balance between present and future utilisation. Further, in order to ensure realistic solutions, we must have some model of short-term management.

1.2.3 The Short-Term Scheduling Problem

Having decided how much water should be released from each reservoir in a particular interval the utility must then decide a detailed schedule for generation (and transmission) within the interval. The objective here is to maintain secure supply at minimum cost. We refer to this as the short-term scheduling problem. Typically it will involve determining the optimal generation level at each plant for each hour of a day or week. (With transmission levels to match). Here again the demand (load) pattern and available equipment are considered fixed. Difficulties are introduced by start-up costs in the thermal sector, flow delays in the hydro-sector and the need to provide "spinning reserve" to cover possible break-downs. Further difficulties arise if the detailed behaviour of the transmission system is modelled. Again, this problem must be re-optimised frequently to account for variations from the expected demand patterns, tributary inflows or availability of equipment.

1.2.4 The Optimal Tariff Problem

In the scheduling problems already discussed the decision-maker is faced with an exogenously determined (predicted) demand pattern which must be met with the (given) available equipment. In the optimal tariff problem, on the other hand, the utility may set tariffs so as to induce the consumers to modify their demand pattern. The objective of this problem may be to set tariffs so as to maximise the utility's profit, to lower costs, to reduce demand or achieve some other policy objective. Here the operating schedules may be considered constant or they may be decision variables along with the tariffs.

1.2.5 The Optimal Development Problem

Most electric utilities are, in the long run, faced with a steadily rising demand for electricity. In order to meet this demand an on-going program of system development is required. Thus the utility is continually faced with the problem of deciding between alternative expansion plans. We refer to this as the optimal development problem. Here the planning horizon may be many years long and a fairly aggregated model is required. The decision-maker may be able to exercise some control over demand but in any case there is considerable uncertainty about his predictions. There is similar uncertainty about future fuel costs, but on the other hand the long-term (aggregate) behaviour of the hydro inflows is basically constant. The decision variables for this problem are the characteristics of new plant (constrained by technical considerations) and (possibly) the operating pattern by which the system would be managed. The objective is to minimise combined capital and operating costs while ensuring that sufficient plant is available to meet demands as they occur. The principal difficulties in this problem result from uncertainty about future demand levels coupled with long lead times for the installation of new plant.

1.3 POSSIBLE APPROACHES TO THE LONG-TERM SCHEDULING PROBLEM

This problem concerns any utility whose system includes long-term reservoirs. Consequently a number of approaches to this problem have been attempted by various groups. In [17], we have already evaluated the approaches taken by Pacific Gas and Electric (PG & E), the Swedish State Power Board (SSPB) and Electricité de France (EDF). Because our own model is an extension of that developed by the EDF, we have included some relevant portions of [17] as appendix A in this study. Another survey, [55], classifies a number of techniques according to various criteria without undertaking any detailed analysis. Here we shall briefly indicate some at least of the approaches which may be taken to scheduling systems such as that of the NZED.

The long-term scheduling problem in its most general form is a stochastic non-linear problem of huge size. The multiplicity of variables and constraints results from the large number of plants,lines, loads and river systems each with their own "efficiency" characteristics and limitations, coupled with the need to supply power continuously as required, at each node of the system, under any probable set of inflows, demand levels or plant availability. Each approach to this problem simplifies it in various ways so as to enable its solution. For instance, in all those reported studies which could be applied to the NZED system, future demand levels have been considered known, as has equipment availability.

All of these models necessarily involve the use of a relatively small number of discrete time intervals rather than a continuous representation. Thus in order to ensure that the detailed demand pattern within each interval can be met the model should incorporate some model of short-term management. Otherwise, while the release specified by the long-term model may provide sufficient total energy within an interval, the combined effect of storage, release and flow restrictions, along with flow delays, may not allow the energy to be produced when it is required. Further, even though there may be a feasible short-term production schedule, the thermal costs involved may be much higher than those predicted by a simple long-term model. Consequently the solution from any model not allowing for this is likely to be rather "over-optimistic" in its predictions.

The construction of a fully stochastic model is made extremely difficult on two counts. The computational requirements are likely to be excessive and in any case the data may not be available. The former problem has led to a number of investigators adopting purely determistic models. Two exceptions to this are the SSPB and EDF models. Both of these approximate the theoretical (and unobtainable) probability distribution by simulating some form of optimal management over all available historically observed inflow sequences. This aspect of these models is discussed and evaluated in Section 9.2.3.

The techniques which have so far been proposed to deal with the long-term problem fall into three categories; dynamic programming, linear or non-linear programming (without decomposition) and (non-linear) decomposition methods. We give here only a cursory description of the models and their application. Each is designed for a specific system and has numerous features making it particularly useful for that system, but not necessarily for ours. We make no attempt to evaluate their usefulness except inasmuch as they can be applied to our system.

Dynamic programming was first suggested as a solution technique for this type of problem by Little [32]. A modified form of stochastic dynamic programming has been applied in the Swedish model ([58]) discussed in Section 4 of [17]. This model handles non-linear costs by piece-wise approximation and attempts to ensure that the load is met in each load segment of each week of the planning horizon. Its main drawback is the degree of aggregation involved. Not only are all loads aggregated into one, but so are all reservoirs. This model was used principally for development planning and for setting energy prices. The validity of this approach depends on high correlations between inflows in different reservoirs and a fairly compact high capacity transmission network. These conditions are not really met in our system. While in theory this model could be generalised to allow several reservoirs the "curse of dimensionality" leads to excessive computational requirements. A (multi-reservoir) stochastic dynamic programming model for the NZED system is referred to in the next section.

Linear programming was applied by Pacific Gas and Electric ([38] and [39]) in a model discussed in Section 3 of [17]. This model approximated some non-linear functions by piece-wise linear functions. Sixteen reservoirs were considered explicitly in the model but all loads were aggregated. The model used monthly intervals and attempted no explicit accounting for short-term requirements, merely using "average" conversion factors etc. It was suggested that a number of schedules could be prepared, each corresponding to some inflow sequence, and the results combined for scheduling purposes. This model was apparently used to some extent for scheduling purposes. A linear network flow model of the NZED system is referred to in the next section.

Non-linear programming methods have been applied directly in a number of studies, three of which are discussed here. The major drawback with this class of methods is the incorporation of the stochastic aspects. None of the reported methods has, as yet, attempted to model this aspect. The solution of large scale non-linear programs is not generally easy to achieve within reasonable computation times. These models have, however, been successfully applied to realistically sized problems, scheduling weekly hydro releases over one or two years. They each deal with the major long-term storage reservoirs individually but take no account of the detailed short-term management of river chains. Each deals solely with the hydro-electric sector, summarising the remainder of the system by the value given to hydro production in the objective function. Loads are aggregated into one and average losses subtracted, line capacity restrictions being ignored except for some "bottle-necks" in the Hydro-Quebec model.

The Bonneville Power Administration developed a model of the Pacific North-West System ([22] and [26]). This model used a modified conjugate gradient algorithm to solve problems involving as many as thirty-eight long-term reservoirs. The objective used was to minimise "energy deficits", and to spread them as uniformly as possible through the year. This objective was modified by the use of penalty functions to represent many of the operating constraints. The model was used to determine how much energy could be extracted from the hydro system under specified adverse stream-flow conditions, rather than to directly schedule output so as to meet demand. Thus, in particular, no account was taken of the necessity of meeting short-term peak power requirements.

Hydro-Quebec have developed a model of their system ([25]) using a reduced gradient approach. This system involved nine "equivalent mid-term reservoirs" (where their "mid-term" reservoirs would be considered "long-term" in our system). Their objective was to minimise the cost of burning fuel or buying energy to make up the loads not met by hydro. No account was taken of short-term peak power requirements, but some transmission "bottle-necks" were modelled. Run times were "surprisingly good" and this model is intended for use as a scheduling tool.

The Tennessee Valley Authority have developed a model of their system ([54]), simplified to consist of six long-term reservoirs within one river valley. This model applies a specialised reduced gradient technique to a network model. The objective function is rather complex, being designed to represent the benefits from the displacement of thermal power by hydro power. Again no explicit modelling is currently undertaken of short-term management although some account of this is included in the objective function. The transmission network is not (explicitly) modelled. Computation times have been encouraging and it is intended that this model be extended to model the whole Tennessee Valley system (19 reservoirs) and to incorporate stochastic inflow data. Currently the model is used for planning purposes although, when it is extended, it is intended to become a scheduling tool.

Finally we consider the (non-linear) decomposition technique developed by the Electricité de France and discussed in Section 5 of [17] (cf. Appendix A). The central idea of this model is the use of a system of "energy prices" (or "dual variables" or "Lagrange multipliers" or "shadow prices") to co-ordinate output from the different units. The model involves a hierarchy of programs. The global program, P3, determines a set of trial prices for energy delivered in each load segment of each time interval. There is a local program, P2, which, in the light of these prices, optimises the long-term schedule of each long-term reservoir in turn, using a "trajectory method". Another program, P4, sometimes considered as part of P3, determines an optimal long-term thermal schedule on the same basis. The global program then compares total generation with total demand, adjusting the prices so as to decrease the difference. This process continues until supply and demand are matched in each segment of each interval. During each iteration the local programs make repeated use of tables prepared by a river scheduling program, P1.

In this model each long-term reservoir is modelled explicitly. Also the limitations imposed by realistic short-term river management are given more weight than in any other reported model. Peak load requirements are treated realistically although all loads are aggregated into one. Two methods have been proposed for the incorporation of stochastic inflow data. These schemes are discussed in more detail in Section 9.2.3. This model is in the process of being applied to the rather large EDF system. A fuller description of the model may be found in Appendix A.

The purpose of this study has been to develop a tool suitable for realistic long-term scheduling in the NZED system. The acceptability of such a model will depend on its modelling of certain key factors. One of these is the stochastic nature of the inflows. Large-scale linear or non-linear programs cannot really model this. For this reason attention was focussed, in [34], on the SSPB and EDF methods, although a large linear programming/network-flow model has been developed separately ([5]). Given the nature of the NZED system the single reservoir approach of the SSPB model seemed inadequate. (Although, as an interim measure, a single reservoir adaptation of the EDF method ([6]) should prove useful). We decided to base our approach on that of the EDF because:

(a) it has a natural, intuitively appealing, economic interpretation;

(b) it models the thermal system explicitly;

(c) it ensures sufficient peak power;

(d) it models each long-term reservoir explicitly;

(e) it models each river chain, and so makes realistic allowances for short-term requirements;

(f) it can be adapted to account for stochastic inflows, (although some LP or NLP approaches could be similarly adapted, the resulting programs would be too large for solution by present codes);

(g) it can be adapted to model the transmission network.

1.4 EARLIER WORK IN NEW ZEALAND

Scheduling problems in the NZED have been considered in a number of studies either prior to, or concurrent with, the present investigation. We will not analyse any of this work in detail. This section briefly outlines the nature and scope of the investigations undertaken and the use to which they have been put.

The short-term scheduling problem has received the most attention. Turner ([62]) developed a dynamic programming model for the short-term scheduling of a chain of hydro reservoirs. This model was applied to derive operating rules for the Waitaki river system involving, at that time, three stations in a single chain. This approach was developed further by Green ([23] and [24]), but eventually abandoned, partly because it could not easily schedule river chains with several stations (in particular the eight stations on the Waikato).

This method was superseded by a heuristic approach originally developed by Bull ([10]). The objective of this heuristic is to use the hydro system to supply peak power, flattening the residual load curve so as to allow economical operation of the thermal system. Accordingly the algorithm commences by scheduling production from the top station in the chain so as to reduce the peaks as far as possible. It then proceeds to schedule each of the stations down the chain, assuming the releases scheduled from the upstream stations, so as to reduce the peaks still further. If the schedule produced by this method is unsatisfactory (in particular if it involves spill) it is modified until it is satisfactory. This heuristic produces feasible solutions which, while not necessarily optimal, allow reasonably good short-term operation. It is currently used in the NZED.

The above methods deal only with the short-term scheduling of individual river systems. Turner ([61]), on the other hand, developed a short-term scheduling model for an inter-connected system with a simplified hydro sector. This model used the "co-ordination equation" approach, maximising efficiency by equating the marginal costs of delivered power from all sources. Here "water values" (γ) were adjusted so as to ensure that specified release targets were met, while incremental costs (λ) were manipulated in order to satisfy demand. Development work on this model was halted before it became fully operational. Currently a simpler model, utilising the previously described heuristic, is used to produce "reasonable" short-term schedules ([42]).

Further studies on optimising transmission patterns were undertaken by Boshier ([2] and [3]). He applied a non-linear programming method to the "voltage scheduling problem", a sub-optimisation of the short-term scheduling problem. He utilised a "created response" (or "penalty function") method to produce an unconstrained problem which was solved using a standard IBM "Fletcher-Powell minimisation" package. The method worked well on small problems and certain large ones but could not be applied more generally.

Long-term scheduling has received comparatively little attention until quite recently. Currently ([37] and [63]) a model in which all storages are aggregated into a single equivalent reservoir is used.

A minimum storage "guideline" (cf. Basic Rule Curve in [9]) is developed. Starting from any point on this guideline secure supply can be ensured (with 95% certainty) for the remainder of the year by base-loading all thermal plant. A set of such guidelines is produced each corresponding to some lower, and hence cheaper, level of base-loaded thermal plant.

Then, if during the year total storage falls below one of these guidelines, the corresponding thermal stations are base-loaded. This approach is easily seen to be optimal if, and only if, the adverse inflow sequence on which the guidelines are based (5% Design Dry Year) actually occurs. Thus this method is designed to ensure system security rather than to minimise costs.

As with any aggregated equivalent reservoir model the aggregate releases must be apportioned among the individual reservoirs. For this purpose a program has been developed by Boshier ([4]). His approach is to operate the lakes so that they each have an equal probability of eventually becoming over full and so being forced to spill water.

The possibility, and desirability, of developing techniques for optimising long-term scheduling was recognised in the late sixties ([37]). A number of techniques were considered ([34]) with attention being focussed on the methods developed by the Swedish State Power Board and Electricité de France. The former method has been outlined in the previous section and is described in some detail in [17]. The latter method, the earlier of the two EDF models described in Appendix A (also in [17]), was adapted for New Zealand conditions by Lusk ([33]). However research on this model was abandoned before any useable tool was developed.

McKerchar, on the other hand, developed a dynamic programming technique to optimise the long-term management of a chain of reservoirs on one river ([35] and [36]). He used synthetic hydrology to generate a large set of "inflows" then used deterministic dynamic programming to determine optimal decisions for each one, using regression analysis to derive a single resonable decision for the immediate problem. This study was primarily concerned with hydrological considerations and could not, in any case, be easily generalised to deal with more than one river chain. Thus, although the program was tested on the Waitaki River system, no attempt was ever made to apply it to the national system.

More recently a number of models including the one reported in this thesis have been developed in parallel. While this has involved a considerable duplication of research effort it should in the long run result in the selection of a management tool whose optimality can be checked by other methods, and which is well suited to the NZED.

Firstly, Boshier ([6]) has developed a "single-reservoir trajectory model". This model works with an aggregate national reservoir, but rather than developing guidelines, attempts to find an optimal total release for the initial period. (Naturally the individual reservoir releases must again be derived using some scheme such as that of [4]). The optimal initial release is found by simulating trajectories for each year of the available historical inflow data. In order to do this some hypothesis must be made as to the future management of the hydro system. The effect of a number of alternative hypotheses is currently being investigated. It is assumed that short-term management will follow certain traditional patterns. The major limitation of this model is its highly aggregated nature. The extent to which this affects the optimality of the solution can only be judged by comparison with a more detailed model such as that developed here. We are not yet in a position to undertake any meaningful comparison.

Secondly, a linear multi-reservoir model has been developed by Boshier and Lermit ([5]). Here the system is represented by a network consisting of a sub-network similar to that of Figure (1-1) for each time interval, the intervals being connected by the arcs representing storage in the various reservoirs. This model allows for several load classes and incorporates the DC link. It has since ([14]) been generalised to model some of the non-linearities by piece-wise linear functions. The network flow solution procedure of [5] has been abandoned in favour of a standard linear programming package. Thus the model can cope (approximately) with all features of the long-term problem except the stochastic nature of the inflows. It has, however, been developed to solve the "optimal tariff problem"(utilising the "shadow prices" from the LP) and no attempt has been made to use it for scheduling purposes.

Finally, Daellenbach ([16]) has developed a multi-reservoir stochastic dynamic programming model. The purpose of this model was to aid in assessing the gains which could be realised by using various stochastic models. The major drawback of this model is that, even restricting attention to major reservoirs, the dimensionality of the problem is too high to allow reasonable computation times. Thus, while the model should prove useful for its intended purpose of stochastic model evaluation, it is not likely to be adopted for realistic scheduling.

So there is currently no optimal long-term scheduling program in use by the NZED. In view of the importance of the system to the national economy, both in terms of its contribution and its cost, the introduction of some such program is clearly desirable. It is intended that initially the single-reservoir trajectory model will be run experimentally, in parallel with the current management methods. Hopefully, as management are convinced of the model's reliability, its recommendations will be given greater weight in day to day decision making. During this period, our more detailed model is to be developed to the stage of becoming a useful tool for real, week by week, system operation. In the next section we outline the scope of our study.

1.5 OUTLINE OF THIS STUDY

The purpose of our study has been to develop a realistic technique which can be applied to determine optimal release, generation and transmission schedules, week by week, for the NZED system. The development, evaluation and implementation of such a model is a massive task which will not be complete for some time yet. Our intention here is to present a theoretical model along with some preliminary results from its application to the system. The remainder of this section outlines the content of this thesis, the structure of which is summarised by Figure(1- 5).

Firstly, in Chapter 1 we have outlined the nature of the system and some of the problems involved in operating it efficiently. We have briefly outlined some approaches to the long-term scheduling problem. A survey paper ([17]), previously published, considers three of these in more detail. We have adopted the basic approach taken by the EDF and so include a description of this as Appendix A. Previous studies on the NZED system have also been outlined.

In Chapter 2 we describe our basic deterministic model. The central concept of this model is that of decentralisation by prices. We show that, provided the functions involved display the appropriate convexity, the optimal solution to the long-term scheduling problem may be found by treating each component of the system as if it were to be managed by an independent profit maximising manager. The central authority is then given the task of ensuring that the right amount of energy is delivered in the right place at the right time. It can do this by setting appropriate "energy prices". Any particular set of prices will induce predictable (maximum profit) production and transmission schedules for each independently managed component. The problem faced by the central authority consists then of adjusting energy prices until the induced schedules are satisfactory from a national point of view. This price adjustment problem is known as the dual problem (DC or DA), while the optimal response of the system to any set of prices can be found by solving the Lagrangian problem (PC′ or PA′). This latter problem can, of course, be decomposed into a multitude of smaller problems, each corresponding to the problem faced by the manager of some component. This approach, similar to that of the EDF, is merely another form of the classical economic concept of using prices to equate supply and demand in a competitive economy. (Note that we do not propose operating the system in this manner. We model it in this way in order to derive optimal policies which are then communicated to the operators of each component).

Our approach differs from that of the EDF in two major respects.

Firstly, we have generalised their model so as to allow some representation of the losses and limitations involved in the transmission system. This generalisation was motivated by the presence in the NZED system of the DC link between the North and South Islands and also by the generally sparse nature of the transmission network. The EDF model is a single-load model, treating all energy produced as a single commodity delivered to a central point to meet a national load (plus fixed losses). Our model, on the other hand, is a multi-load model in which each load centre or generation plant may be treated separately. It is easy to see that an optimal solution to a single-load model may not be optimal (because of the losses involved) or even feasible (because of the limited capacity of certain lines) when applied to the real system. These problems are very real in the NZED system. Thus our model, taking into account both losses and constraints in the network, represents a significant step towards realistic modelling of this system.

Note that we do not concern ourselves with the type of detailed load-flow calculations commonly encountered in short-term optimisation. Rather we are concerned with the approximate losses likely to be incurred by major inter-regional energy transfers under "average" conditions. The inclusion of this representation of the transmission system naturally complicates the solution procedure. The extra computation involved is, however, surprisingly minor. We have an extra set of sub-problems, the exchange problems (PE), which are easily solved via calculus. Apart from this the dual problem (DC or DA) has to adjust more variables, which, in our method, does not present any significant difficulties. This multi-load model, without the complications discussed below, has previously been presented in [47].

The other major area in which our model differs from that of the EDF is in the treatment of short-term management. In their model (see Appendix A) the dual program (P3) is supposed to adjust national energy prices (λ), for each load segment of each period, so as to ensure sufficient generation in each such segment. In order to model the short-term management of hydro stations, a "price duration curve" is derived from these prices. This curve is then used by a heuristic program (P1) to model the short-term management of each river chain. The results of this procedure are stored in tabular form and used as input into the long-term hydro scheduling program (P2). We have generalised this by allowing for the derivation from the "aggregate" (λ) prices of a "detailed" price curve (μ). This price curve, unlike a price duration curve, allows us to model exactly the dynamics of short-term management: flow delays and restrictions, storage restrictions and the like in the hydro sector, start-up costs, delays and restrictions in the thermal sector. (See [49]). Consequently, in Chapter 5, we are able to develop an optimal short-term scheduling procedure (PASH) for a river chain (given the μ prices), the results of which can be tabulated as a function of the λ prices for use in the long-term hydro scheduling problem (PALH). Naturally, given our modelling of the transmission network, we have a different set of (λ) prices for each region and allow that the detailed (μ) prices may vary spatially as well as temporally.

We introduce this generalisation by first proving some decomposition results about a complete model (PC) in which we require a detailed optimisation over the whole planning horizon. We approximate this by an aggregate model (PA) corresponding to the global EDF model(or to that of [47]). In our aggregate model we require, however, that the aggregate quantities determined by this model be managed in accordance with the short-term sub-models. We assume that we can derive the detailed prices, $\mu(\lambda)$, from the aggregate prices, λ, so that they are sufficiently close to the true detailed price (μ for the complete dual DC) that, at the optimal λ, the corresponding detailed schedules as determined by the sub-models on the basis of $\mu(\lambda)$, are satisfactorily close to the true optimal schedules. Here, rather than the arbitrary assumptions made in P1, we allow a flexible price derivation scheme to be based on feed-back from the application of the model. This approach allows us not only to model short-term management more accurately, but because of its flexibility, to match the supply and demand patterns far more exactly.

This increased accuracy is achieved at no expense at all in the aggregate problem (which must be re-run frequently). The extra computational burden is incurred entirely in the preparation,via the short-term sub-models,of the tables summarising system response.

Chapters 3 and 4 deal with the solution of the thermal (PAT) and exchange (PAE) sub-models resulting from the decomposition of PA. There we relax the assumptions of Chapter 2 to allow the cost and loss functions to be non-convex. This does not interfere with the solution of the problem because the response functions can be modified so as to maintain the concavity of the dual objective function. These problems may be solved as required or they may be used to build up tables describing the response of each component.

Chapter 5 deals with the solution of the hydro sub-model. This problem is approximated by a long-term problem (PALH) and a short-term problem (PASH). The short-term problem, corresponding to P1, is solved by a true optimisation procedure utilising the detailed prices. It is used to build up tables summarising the output (and its value) from each hydro system, when optimally managed in the short term, as a function of the aggregate prices (and tributary inflows). The "value of release" functions so determined are used in the long-term problem which determines the optimal long-term schedule using the "trajectory method" developed by EDF.

Chapter 6 deals with two topics, the derivation of detailed prices from aggregate prices and the adjustment of aggregate prices in the dual problem (DA). In this latter problem we have adopted a rather more sophisticated approach than that of the EDF model. This is required partly because of our modelling of the transmission network and partly because the small size of the NZED system results in very "erratic" system response surfaces. This general approach should, however, produce even better results when applied to a larger, and hence less "lumpy", system.

Chapter 7 presents some preliminary results from the application of our model to the NZED system. These results are really only illustrative examples since the model has not, as yet, been implemented in its totality, using real system data. This chapter completes our description of the deterministic long-term scheduling model.

Chapter 8 summarises some recent results from the theory of optimal recourse problems and then generalises our model into this stochastic framework. This stochastic long-term scheduling model can be decomposed analagously to PC, using prices for energy delivered to each region, in each load segment, of each period, under each of the observed historical inflow sequences. The resultant thermal and exchange sub-models are identical to the deterministic sub-models of Chapters 3 and 4. The hydro sub-model, however, is much more complex than its deterministic equivalent. Some general principles relating to its solution are developed.

Chapter 9 considers the practical solution of this stochastic hydro sub-problem. A number of approaches, particularly those of the EDF, are evaluated and a new method proposed. The chapter concludes with a consideration of the "stochastic dual problem" involved in adjusting the prices involved in the stochastic model. Note that in these two chapters we have ignored any consideration of the short-term problem. Thus this model, a direct extension to that of [47], can be considered as the stochastic version of the complete deterministic model (PC), with rather long "instants", or of the deterministic aggregated model (PA), with no restriction on the detailed utilisation of aggregate quantities. In this respect the stochastic problem could be treated in exactly the same manner as was the deterministic problem. The resultant complexity of notation would, however, seriously detract from the intelligibility of the formulation.

Chapter 10 suggests ways in which the prices developed in the solution of the long-term scheduling problem could be used in the optimal development problem and the optimal tariff problem. In order to solve this latter problem we extend the basic model by allowing the consumers to vary their demand in response to the "energy prices". The resultant model decomposes yielding thermal, exchange, hydro and demand sub-problems. The solution of these latter is not difficult, the relevant data being readily available. Here again we simplify the notation by dealing with the simple aggregate model, ignoring short-term considerations. A brief outline of the approach of this model may be found in [50].

Further to the above, it is obvious that the short-term sub-models for the thermal, exchange and hydro sub-problems, developed in Chapters 3 - 5, could be incorporated into an optimal short-term scheduling algorithm. Some tentative suggestions as to the development of such an algorithm may be found in [49].

So we may summarise the structure of this thesis as in Figure (1- 5). Chapter 1 introduces the long-term scheduling problem, while Chapter 2 develops our (deterministic) model and decomposes it into a dual problem and a number of sub-models. Chapters 3 - 5 deal with the solution of the sub-models while Chapter 6 covers the solution of the dual. Chapter 7 completes our consideration of the deterministic long-term scheduling problem with some results from the NZED system. Chapters 8 and 9 extend this model into a stochastic framework. Chapter 10 considers the economic implication of the dual variables.

Also, the following are relevant.

Firstly, [17] extends Chapter 1, giving a detailed survey of three alternative approaches. Then [47] gives a simplified version, in which we do not require realistic short-term management, of the basic model in Chapter 2 (and 6). In [48] we present a heuristic discussion of our results as they apply to long-term scheduling using a single-reservoir model. The model is presented in simple economic terms in [50]. Further, [49] discusses the incorporation of relevant portions from Chapters 2 to 5 into a short-term scheduling procedure. Finally, [51] discusses the inclusion of pumped storage into our model.

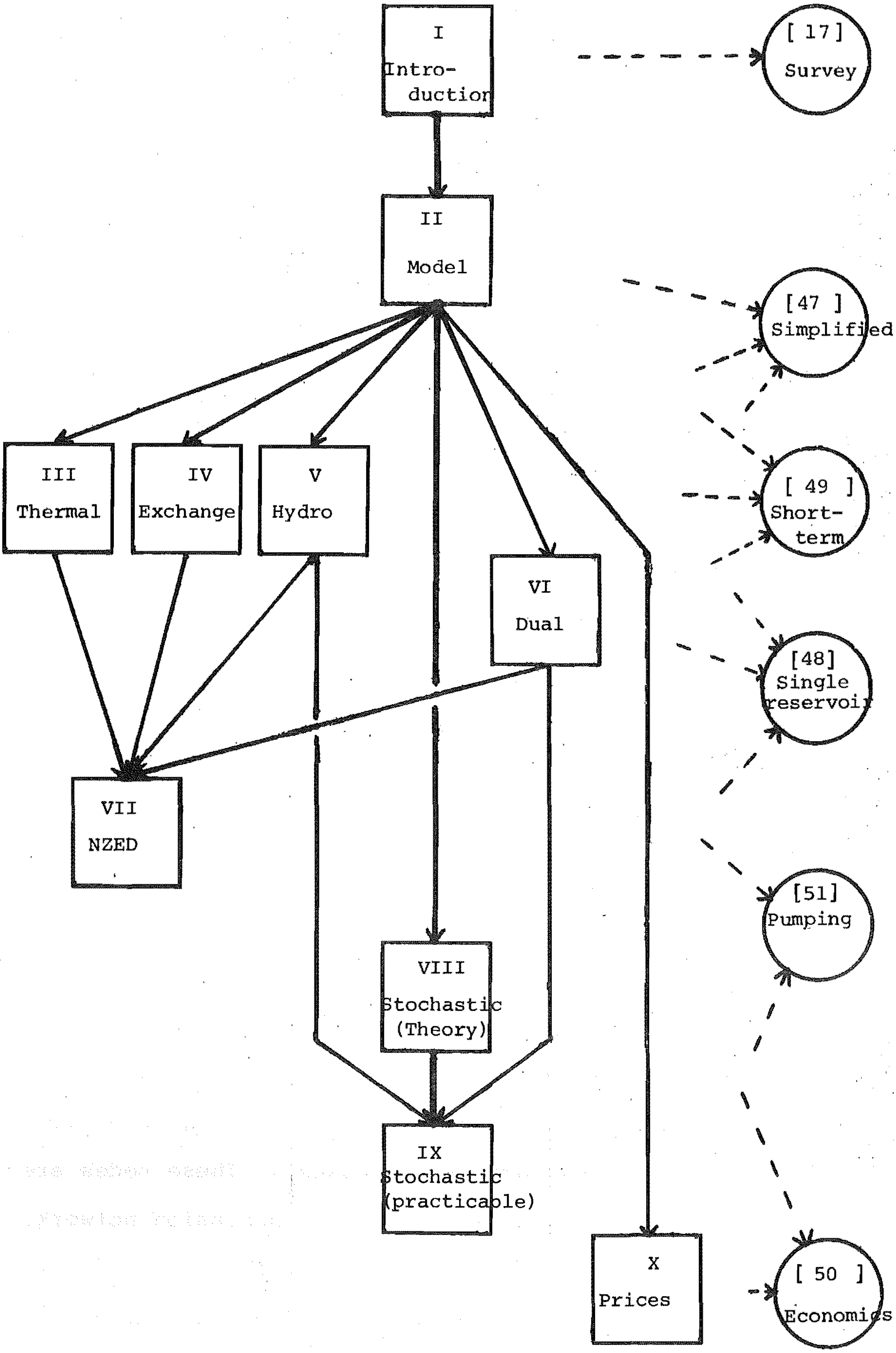


FIGURE (1-5): Structure of thesis.

CHAPTER 2

A COMPLETE MODEL

2.1 INTRODUCTION

This model is a generalisation of the deterministic model presented in [47]. The purpose of this generalisation is to provide a conceptual framework for both long-term (as in [47]) and short-term scheduling. The direct solution of the complete model would in general be computationally infeasible. However we introduce an "aggregated model" which can be solved and which approximates the complete model. We restrict our attention here to the deterministic case. A stochastic generalisation of the model of [47] is considered in Chapter 8.

In Section 2.2 we formulate the complete model, while in Section 2.3 we discuss its theoretical solution. In Section 2.4 we outline an approach to obtaining approximate solutions.

2.2 THE MODEL

We will characterise our planning horizon by a finite, but possibly very large, number of "instants", indexed by $r = 1,\ldots R$. (e.g., the instants may be hours and the horizon one year). We have a national power system consisting of many nodes, indexed by $i \varepsilon I$, at which power is produced or consumed by various processes. These nodes are inter-connected by a national grid, or transmission network, enabling the transfer of energy between them.

Firstly, we have demand nodes (load centres) which, at each instant r, have an exogenously determined demand level, D_i^r. We assume that our electric utility has been assigned the task of meeting the consumers' demand at minimum cost. A model in which the utility is allowed to influence demand by facing the consumer with the costs of energy production (or some function thereof) is considered in Chapter 10.

Secondly, we have "fixed generation" nodes at which energy may be produced by geothermal or nuclear power plants (which because of their operating characteristics are generally base-loaded if available) or at which contracted energy may be delivered from other utilities. At instant r such a node, i, produces a fixed amount of energy, G_{iF}^r.

Thirdly, we have thermal generation nodes producing energy from any conventional type of thermal plant. At instant r node i will produce output g_{iT}^r (to be determined). The running cost of a thermal plant is given by the function $C_i^r(g_{iT}^r)$ (where C_i may vary over the year owing to changes in fuel costs, equipment availability etc). Here (cf. [49]) we ignore start-up costs, maintenance costs and the like. We also assume that the cost function C(g) is convex and increasing, but see Chapter 3 for a relaxation of this. Thus for each successive increment of energy we must pay no less than for its predecessor. We assume that there are bounds on g_{iT}^r expressed by:

$$\underline{G}_{iT}^r \leqslant g_{iT}^r \leqslant \bar{G}_{iT}^r \tag{C-1}$$

For convenience we consider these two types of generation node (fixed and thermal) as one type, which we will refer to as thermal. We treat each fixed generation node as a thermal node with the fixed generation, G_{iF}^{r}, as the minimum and maximum generation.

i.e. let: $\underline{G}_{iT}^{r} = G_{iF}^{r} = \bar{G}_{iT}^{r}$ for all $r=1,\ldots R$ (C-1′)

We index these (thermal) nodes by $b\varepsilon\mathcal{B}\subset\mathcal{I}$.

So as to ensure feasible solutions to the ensuing mathematical program we will also introduce fictitious "shortage" thermal nodes with very high cost functions. If the theoretical solution derived would require such a station to produce energy for any instant then a shortage of energy is indicated for this instant. The very high cost associated is the "shortage cost". A typical thermal cost curve is shown in Figure (2-1).

Finally, we have "hydro" nodes consisting of all kinds of hydro-electric generation, storage or pumping facilities. These nodes will be grouped into valleys. We index these valleys by $h\varepsilon\mathcal{H}$ and use $i\varepsilon h$ to mean that reservoir i is in valley h. Each "valley" consists of a natural river system or a group of such systems, inter-connected by canals etc., in a power scheme. Each valley (h) is subject to an uncontrollable vector of inflows, F_h^r, and has storage vector s_h^r at instant r.

The amount of water released by each station in the valley is denoted by a vector q_h^r and the generation from each is given by the function: $g_{iH}^{r}(q_i^r, s_h^r, F_h^r)$. We define the total generation from valley h to be:

$$g_{hH}^{r} = \sum_{i\varepsilon h} g_{iH}^{r}(q_i^r, s_h^r, F_h^r) \qquad \text{(C-2)}$$

Here, for any reservoir i, the storage level at instant r, s_i^r, is determined by the initial storage level and the inflows and releases up to instant r. Hence the releases, q_h^r, are the only independent variables. For our purposes we will require g to be a concave function of q. Thus we assume that g_i^r is concave in q_i^r and convex in s_i^r. This first condition is easily guaranteed by the shape of the generation functions. Figure (2-2) shows a typical curve (ignoring head variations). The second condition can not be guaranteed. However, head variation is not a major feature of the NZED system. Of the long-term reservoirs, with which we are primarily concerned, only one small station (Tekapo A with 25 MW) is subject to this effect. Also we follow the TVA (p14 in [54]) and Hydro Quebec (p23 in [25]) in our expectation that the comparatively minor concavity introduced by the head effect be overwhelmed by the convexity of $g_i^r(q_i^r)$.

For each valley the release vector, q_h^r, (and hence the storage vector, s_h^r) must conform to certain physical and operational requirements. These are detailed in Chapter 5. Here we shall summarise these constraints by requiring:

$$q_h^r \varepsilon Q_h^r \qquad \text{for all } r=1,\dots R \qquad \text{(C-3)}$$

We assume that these sets are convex.

All of these nodes are interconnected by a transmission system allowing exchanges of energy. Let e_{ij}^r be the exchange between nodes i and j for instant r. For ease of notation we will restrict e^r_{ij} to be positive and let e^r_{ji} denote an energy transfer from j to i for instant r. Obviously there are limits to the amount which can be transferred:

$$\underline{E}^r_{ij} \leqslant e^r_{ij} \leqslant \bar{E}^r_{ij} \qquad \text{for all } i,j \in I \quad r=1,\ldots R \qquad \text{(C-4)}$$

(Where, in general, $\underline{E}^r_{ij} = 0$).

Note that many nodes are not directly connected by transmission lines, in which case: $\bar{E}^r_{ij} = 0$, for all $r=1,\ldots R$. Also, we do not allow a node to transfer energy to itself. Thus: $\bar{E}^r_{ii} = 0$, for all $i \in I$, $r=1,\ldots R$.

These transfers are subject to losses, $L^r_{ij}\ (e^r_{ij})$ (which may depend on the equipment available at a particular instant). Because of these losses the amount of energy received by node j from node i is less than that despatched from i for j. We denote this amount by f^r_{ij}. Thus:

$$f^r_{ij}(e^r_{ij}) = e^r_{ij} - L^r_{ij}\ (e^r_{ij}) \quad \text{for all } i,j \in I \quad r=1,\ldots R \qquad \text{(C-5)}$$

We assume here that the loss function, L(e), is convex (line losses are in fact generally quadratic, but see Chapter 4 for a fuller discussion).

Note that it is not our intention to introduce full scale load-flow considerations into our optimisation. Our approach is primarily motivated by a significant aspect of the NZED system - the existence of a limited capacity DC link between the two major islands. More generally, line losses (and capacity restrictions) can have

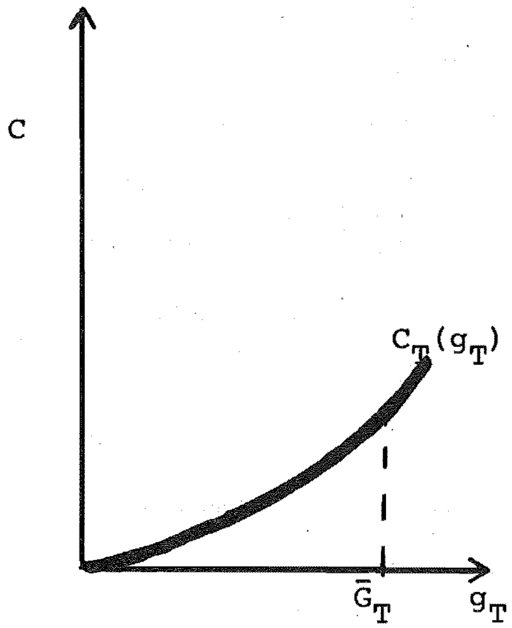


FIGURE (2-1):
Thermal cost function

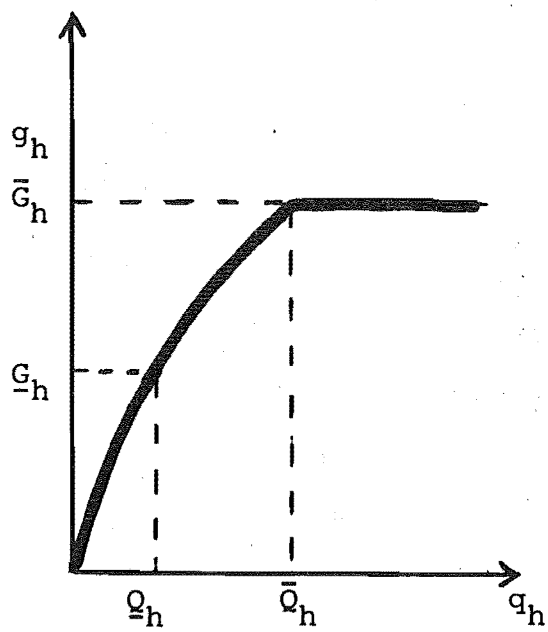


FIGURE (2-2):
Hydro output function

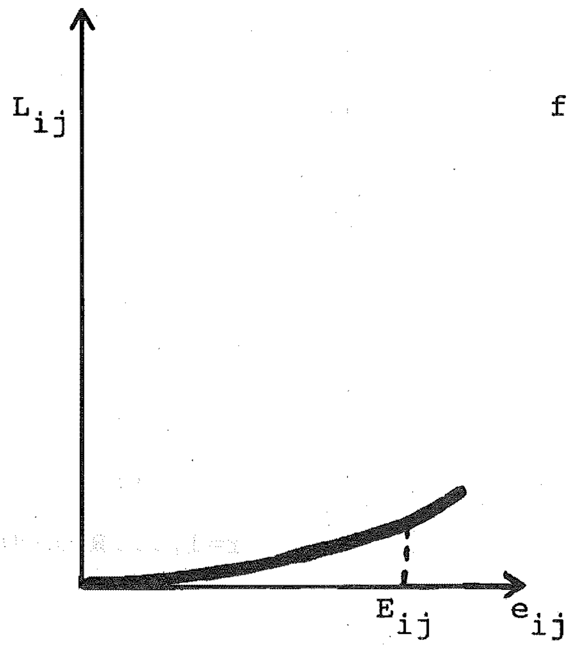


FIGURE (2-3):
Tranmission loss function

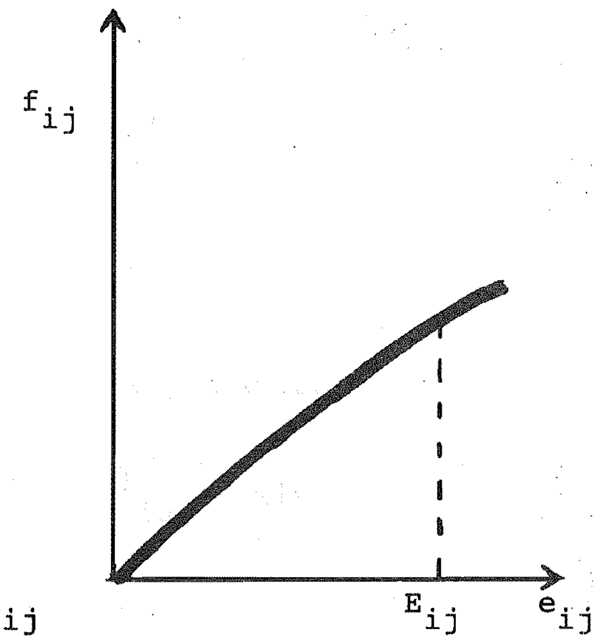


FIGURE (2-4):
Energy received function

significant impact on the optimal generation pattern required to meet a particular load pattern. So it seems appropriate that our model should be able to account for this, even if the "loss functions" can not be determined exactly. We ignore any running costs for the transmission system. Typical curves for L(e) and f(e) are shown in Figures (2-3) and (2-4).

This completes our description of the physical aspects of the system. The network in Figure (2-5) summarises the flow of energy in the system during a single instant. In much of what follows we shall not differentiate between the different kinds of nodes. We assume that for all r=1,...R:

$$g^r_{iT} = 0 \quad \text{if } i \notin \mathcal{B} \qquad \text{(C-6)}$$

$$g^r_{iH} = 0 \quad \text{if } i \notin h, \text{ for some } h \in \mathcal{H} \qquad \text{(C-7)}$$

$$D^r_i = 0 \quad \text{if } i \in h, \text{ for some } h \in \mathcal{H} \text{ or } i \in \mathcal{B} \qquad \text{(C-8)}$$

Our aim is to ensure that all demands are met while minimizing the total fuel cost of thermal generation. Thus we have a further constraint:

$$g^r_{iT} + g^r_{iH} + \sum_{j \in \mathcal{I}} (f^r_{ji} - e^r_{ji}) - D^r_i \geqslant 0 \quad \text{for all } i \in \mathcal{I}, \ r=1,\dots R \qquad \text{(C-9)}$$

Our objective function is:

$$\sum_{r=1}^{R} \sum_{i \in \mathcal{I}} C^r_i \, (g^r_{iT}) \qquad \text{(C-10)}$$

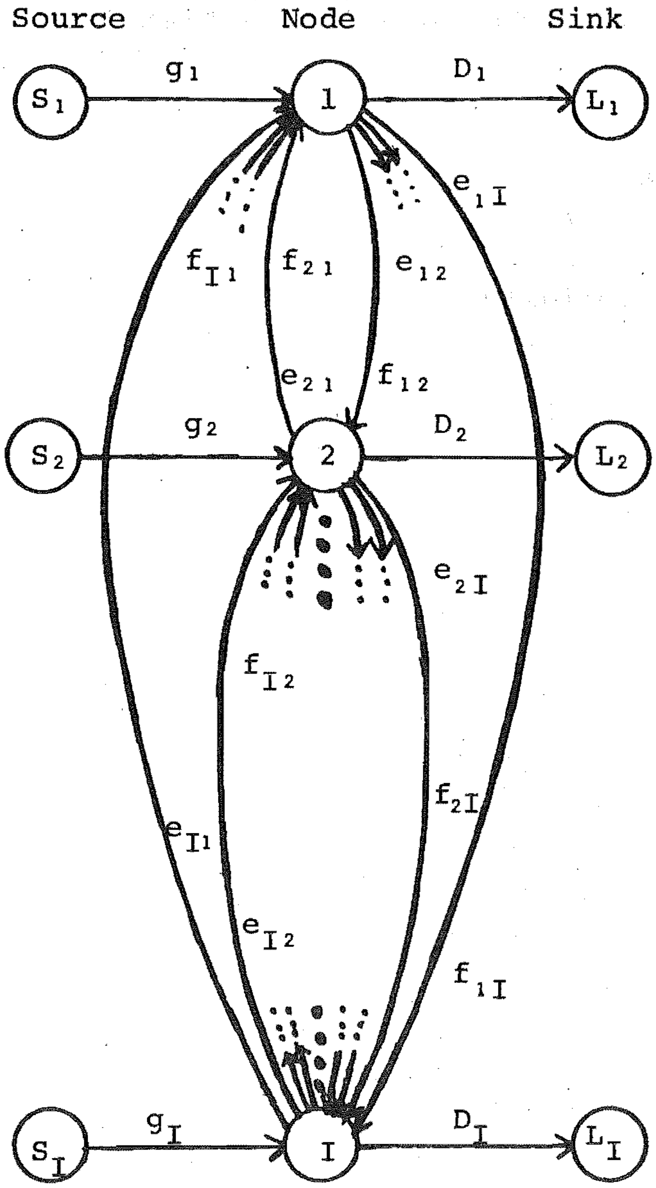


FIGURE (2-5): Transmission network

2.3 SOLUTION OF THE COMPLETE PROBLEM

2.3.1 The Primal Problem

Our problem now is to find vectors $\tilde{q}_T, \tilde{q}$, and $\tilde{e}$ so as to minimise the total cost of energy production while satisfying the constraints. Thus we have problem PC (renumbering the equations):

$$\text{(PC) Find } \underset{(g_T,q,e)}{\text{MIN}} \sum_{r=1}^{R} \sum_{i\varepsilon I} C_i^r \, (g_{iT}^r) \qquad \text{(C-11)}$$

Such that the following hold:

$$g_{iT}^r + g_{iH}^r + \sum_{j\varepsilon I} (f_{ji}^r - e_{ij}^r) \geq D_i^r \quad \text{for all } i\varepsilon I, \quad r=1,\ldots R \qquad \text{(C-12)}$$

$$\underline{E}_{ij}^r \leq e_{ij}^r \leq \bar{E}_{ij}^r \quad \text{for all } i,j\varepsilon I, \quad r=1,\ldots R \qquad \text{(C-13)}$$

$$\underline{G}_{iT}^r \leq g_{iT}^r \leq \bar{G}_{iT}^r \quad \text{for all } i\varepsilon\, B, \quad r=1,\ldots R \qquad \text{(C-14)}$$

$$q_h \, \varepsilon\, Q_h \quad \text{for all} \quad h\varepsilon H \qquad \text{(C-15)}$$

We let Z denote the set of g,q and e satisying (C-13) - (C-15).

2.3.2 The Dual Problem

Form the Lagrangian:

$$\mathcal{L}_C(z,\mu) = \sum_{r=1}^{R} \sum_{i\varepsilon I} \Big[C_i^r(g_{iT}^r)$$

$$- \mu_i^r(g_{iT}^r + g_{iH}^r + \sum_{j\varepsilon I} (f_{ji}^r - e_{ij}^r) - D_i^r) \Big] \qquad \text{(C-16)}$$

$\mathcal{L}_C(z,\mu)$ will be continuous in z if C (g_T), L(e) and $g_H(q)$ are continuous in their respective arguments. In practice this may not be the case. However we shall show, in Chapters 3 to 5, that these functions may be modified so as to be both convex and continuous. Thus we may safely assume that $\mathcal{L}_C(z,\mu)$ is continuous in z and so the 'dual objective function' may be defined over the whole positive orthant (see [30] Section 8.4).

Thus the dual objective function may be defined as:

$$P_C(\mu) = \underset{(z\varepsilon Z)}{\text{MIN}} \ \mathcal{L}_C(z,\mu) \quad \text{for all } \mu \geqslant 0 \qquad \text{(C-17)}$$

Let $z^*(\mu) = (g_T^*(\mu), q^*(\mu), e^*(\mu))$ be the corresponding point in Z. The problem of determining $z^*(\mu)$, and hence $P_C(\mu)$, is referred to as the Lagrangian problem, PC$'$.

Now we can set up the dual problem (DC):

$$\text{Find} \quad \underset{\mu}{\text{MAX}} \ P_C(\mu) \qquad \text{(C-17}'\text{)}$$

Such that: $\mu_i^r \geqslant 0$ for all $i \varepsilon I$

$$r=1,\ldots R \tag{C-18}$$

Given the assumptions which we have made about the form of the relevant functions we may apply Theorem 8.7.1 of [30] (an extension of Karlin's Theorem 7.1.1 in [28]). This theorem assures us that, in our case:

If z^* solves the primal problem (PC) then there is a vector of multipliers $\mu^* \geqslant 0$, which solves the dual (DC), with:

$$P_C(\mu^*) = \sum_{r=1}^{R} \sum_{i\varepsilon I} C_i^r(g_{iT}^{r*}(\mu^*)) \tag{C-19}$$

Also, by Corollary 2 to Theorem 8.4.1 in [30],

If:

(i) $\tilde{z}$ is a feasible solution for PC

(ii) $\tilde{\mu}$ is a feasible solution for DC

(iii) (C-19) holds.

Then:

(a) $\tilde{z}$ is the optimal solution to the primal problem

(b) $\tilde{\mu}$ is the optimal solution to the dual problem.

In our problem (C-19) will hold when supply exactly matches demand so that:

$$\mathcal{L}_C(z^*(\mu),\mu) = \sum_{r=1}^{R} \sum_{i\varepsilon I} C_i^r(g_{iT}^{r*}(\mu)) + 0 \tag{C-16'}$$

In this case z^* will also be primal feasible and, for any reasonable power system, μ will be dual feasible (i.e. $\mu \geqslant 0$).

Further, by Theorem 8.4.2 of [30], the dual objective function, $P_C(\mu)$, is concave. So we can apply the following general algorithm, a flow chart for which appears in Figure (2-6), to solve the problem DC.

(1) Initialise μ.

(2) Solve the Lagrangian problem to determine $P_C(\mu)$, $z^*(\mu)$.

(3) IF: $$\left| g^r_{iT}+g^r_{iH} + \sum_{j\varepsilon I} (f^r_{ji} - e^r_{ij}) - D^r_i \right| < \varepsilon \qquad \text{(C-19'')}$$

(where ε is some pre-determined tolerance)

THEN STOP,

ELSE GO TO (4).

(4) Adjust μ

4.1 Determine a feasible search direction $\delta(\mu)$

4.2 Determine a step size θ such that:

$$P_C\ (\mu + \theta\delta(\mu)) \geqslant P_C(\mu) \qquad \text{(C-20)}$$

4.3 let $\mu = \mu + \theta\delta(\mu)$ (C-21)

(5) GO TO (2).

When this algorithm stops in Step 3 we will have found the optimal solution, not only to the dual problem (DC), but to the original primal problem (PC).

We may give a very natural economic interpretation to this algorithm. The multiplier μ^r_i plays the role of the "price" for energy delivered to node i in instant r. First, our algorithm announces a tentative set of energy prices. The solution to the Lagrangian problem PC′ gives the schedule which a profit maximising manager would follow faced with the announced prices. The dual algorithm then adjusts the prices so as to better match supply and demand.

This process is reiterated until supply and demand are approximately equal.

The advantage of this scheme is that the Lagrangian problem, PC$'$, may be decomposed into a number of sub-problems. Each of these corresponds to the problem which a profit maximising manager of an individual system component (station or line) would face, given the prices μ. Thus our model is essentially equivalent to the perfect competition model of an economy, with the dual algorithm playing the role of the "invisible hand" co-ordinating the decisions of individual managers. In the next three sections we concentrate on Step 2, which involves the optimisation of the Lagrangian problem for a particular set of multipliers. Step 3, the adjustment of the μ prices, is discussed in Chapter 6.

2.3.4 Decomposition

Step 2 of the dual algorithm involves the evaluation of $\mathcal{P}_C(\mu)$, the value of the response of the supply system to the current suggested prices (evaluated at those prices).

Now:

$$\mathcal{P}_C(\mu) = \min_{z \in Z} \mathcal{L}_C(z,\mu) \qquad \text{(C-17)}$$

So we must solve the following problem, (Lagrangian):

$$\text{(PC}'\text{)} \quad \text{Find} \ \min_{z} \mathcal{L}_C(z,\mu) \qquad \text{(C-17}'\text{)}$$

Such that (C-13) - (C-15) hold.

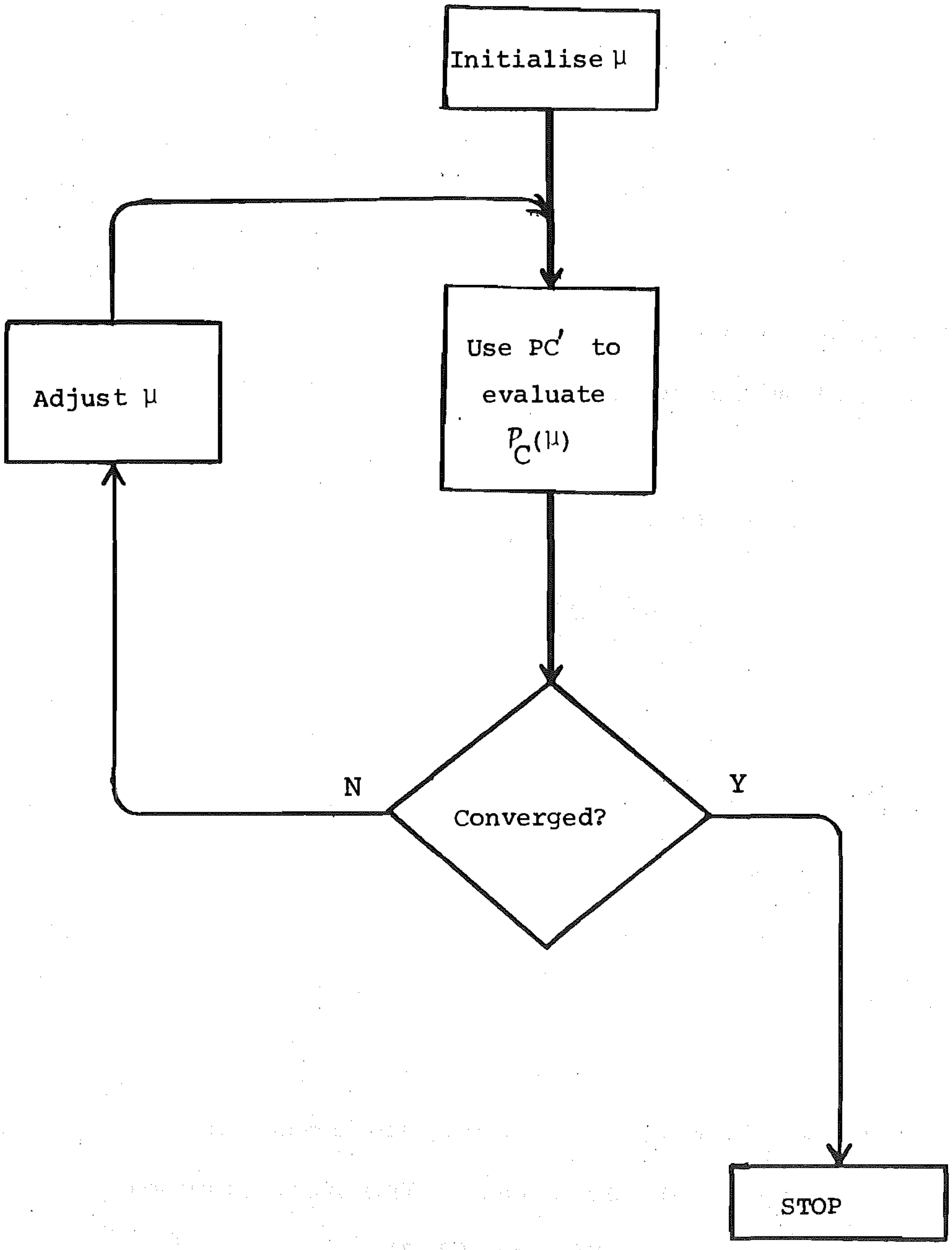


FIGURE (2-6): Flow chart for DC

Where:

$$\mathcal{L}_C(z,\mu) = \sum_{r=1}^{R} \sum_{i \varepsilon I} \Big[C_i^r(g_{iT}^r)$$

$$- \mu_i^r(g_{iH}^r + g_{iT}^r + \sum_{j \varepsilon I} (f_{ji}^r - e_{ij}^r) - D_i^r) \Big] \quad \text{(C-16)}$$

Thus we could imagine this as the problem confronting a manager in charge of the supply system. Faced with the prices μ for his output, he wishes to maximise his profits. Equivalently, he minimises total net costs, that is fuel costs $(C_i^r(g_{iT}^r))$ minus value of production (sold at the prices μ).

Rearranging we get:

$$\mathcal{L}_C(z,\mu) = \sum_{i \varepsilon I} \sum_{r=1}^{R} \Big[C_i^r(g_{iT}^r) - \mu_i^r g_{iT}^r - \mu_i^r g_{iH}^r \Big] \quad \text{(C-22)}$$

$$- \sum_{r=1}^{R} \sum_{i \varepsilon I} \Big[\mu_i^r (\sum_{j \varepsilon I} (f_{ji}^r - e_{ij}^r)) \Big] \quad \text{(C-23)}$$

(Ignoring the term $\mu_i^r D_i^r$ which is fixed for any iteration of the dual algorithm).

There are now no constraints in (C-13) to (C-15) linking (C-22) and (C-23), so we can optimise these terms separately, getting problems PG and PE. These problems are decomposed further in the next two sections. The hierarchy of decomposition is shown in Figure (2-7).

2.3.5 The Generation Problem

Problem PG, the generation problem, may be stated as:

$$\text{Find } \underset{g,g}{\text{MIN}} \sum_{r=1}^{R} \sum_{i \varepsilon I} \Big[C_i^r(g_{iT}^r) - \mu_i^r(g_{iT}^r + g_{iH}^r) \Big] \quad \text{(C-22')}$$

Such that (C-14) - (C-15) hold.

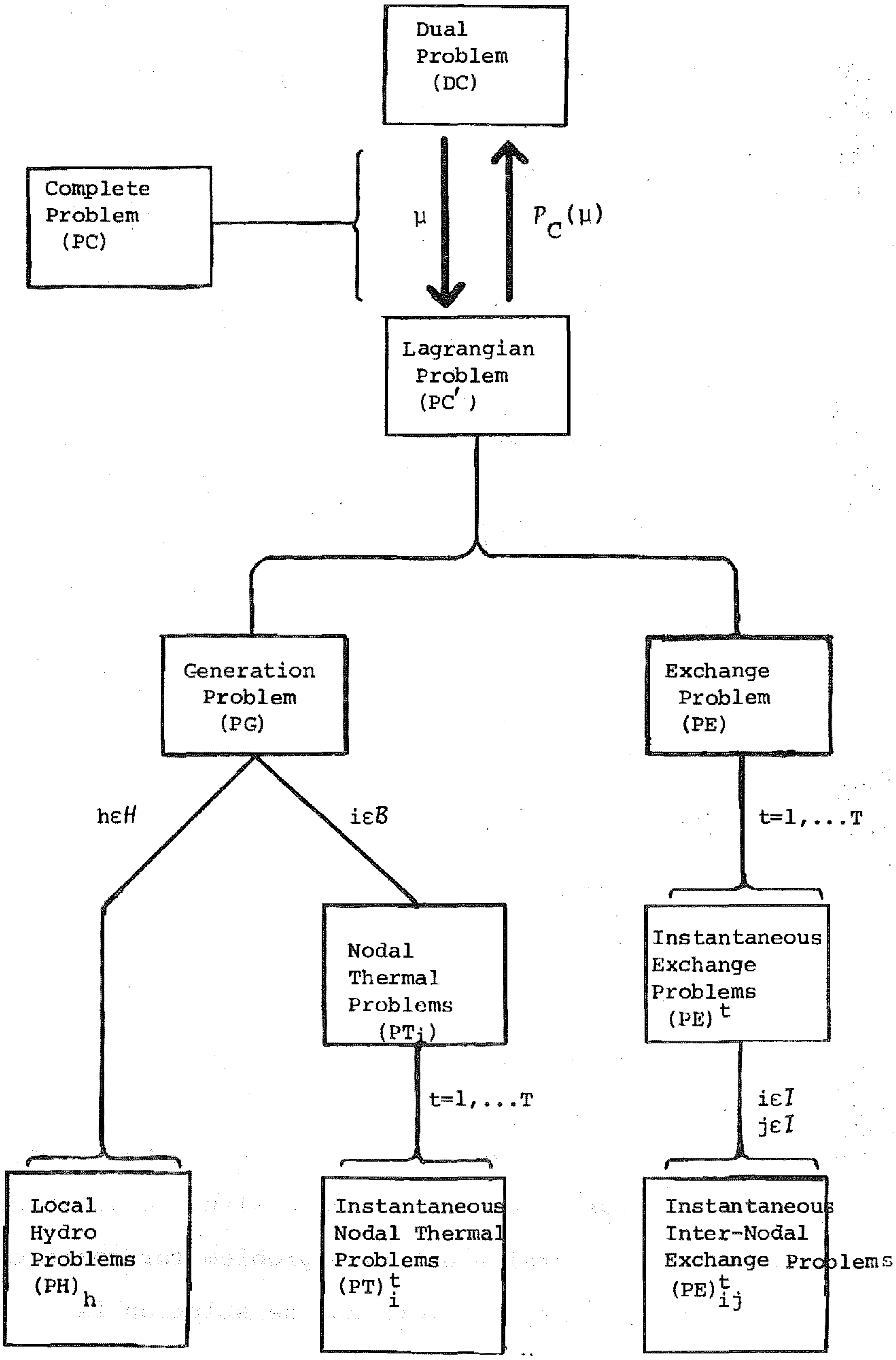


FIGURE (2-7) Decomposition of PC.

Here, in problem PG, we attempt to maximise the "profits" (minimise the net costs) from generation at the (μ) prices given.

This problem decomposes, according to the nature of the various nodes, into many local problems. We have one such problem for each thermal node - the nodal thermal problem, PT_i $(i \varepsilon \mathcal{B})$. We also have one for each hydro valley - the local hydro problem, PH_h $(h \varepsilon \mathcal{H})$.

The nodal thermal problem, PT_i, may be stated as:

$$\text{Find} \quad \underset{g_{iT}}{\text{MIN}} \sum_{r=1}^{R} C_i^r(g_{iT}^r) - \mu_i^r g_{iT}^r \qquad \text{(C-22')}$$

$$\text{Such that:} \quad \underline{G}_{iT}^r \leqslant g_{iT}^r \leqslant \bar{G}_{iT}^r \quad \text{for all } r=1,\ldots R \qquad \text{(C-14)}_i$$

This can obviously be further decomposed into R instantaneous nodal thermal problems, PT_i^r:

$$\text{Find} \quad \underset{g_{iT}^r}{\text{MIN}} \left[C_i^r\ (g_{iT}^r) - \mu_i^r g_{iT}^r\right] \qquad \text{(C-22')}_i^r$$

Such that:

$$\underline{G}_{iT}^r \leqslant g_{iT}^r \leqslant \bar{G}_{iT}^r \qquad \text{(C-14)}_i^r$$

The solution of these problems may be found quite easily via calculus. Chapter 3 deals with the solution of (the aggregated version of) this problem for realistic thermal cost functions. Provided the solution is unique we can use the thermal sub-model of that chapter to determine the optimal thermal generation as a function of the price (μ). So we have:

$$g_{iT}^{r*}(\mu) = g_{iT}^{r*}(\mu_i^r) \qquad \text{(C-24)}$$

The local hydro problem, PH_h, may be stated as:

$$\text{Find} \ \underset{q_h}{\text{MAX}} \sum_{r=1}^{R} \mu_h^r g_{hH}^r (q_h^r) \qquad \text{(C-22}')_h$$

Such that:

$$q_h \ \varepsilon \mathcal{Q}_h \qquad \text{(C-15)}$$

This problem may be solved by the techniques developed by the EDF (see Appendix A or [17]). Chapter 5 deals with the approach which we have taken. So we can find:

$$g_{hH}^{r*}(\mu) = g_{hH}^{r*} \ (\mu_h) \qquad \text{(C-25)}$$

<u>2.3.6 The Exchange Problem</u>

Problem PE (Exchange Problem) may be stated as:

$$\text{Find} \quad \underset{e}{\text{MAX}} \sum_{r=1}^{R} \sum_{i \varepsilon I} \left[\mu_i^r \left(\sum_{j \varepsilon I} (f_{ji}^r - e_{ij}^r) \right) \right] \qquad \text{(C-23}')$$

Such that:

$$\underline{E}_{ij}^r \leqslant e_{ij}^r \leqslant \bar{E}_{ij}^r \qquad \text{for all } i,j \varepsilon I, \ r=1,\ldots R$$

$$\text{(C-13)}$$

In this problem we attempt to maximise the benefits from inter-nodal exchange.

This clearly breaks into R instantaneous exchange problems, PE^r:

$$\text{Find} \quad \underset{e^r}{\text{MAX}} \sum_{i \varepsilon I} \left[\mu_i^r \left(\sum_{j \varepsilon I} (f_{ji}^r - e_{ij}^r) \right) \right] \qquad \text{(C-23)}^r$$

Such that $(C\text{-}13)^r$ holds.

Substituting (C-5) into (C-23) we get:

$$(C\text{-}23)^r = \sum_{i\varepsilon I}\left[\mu_i^r \sum_{j\varepsilon I} [e_{ji}^r - L_{ji}^r(e_{ji}^r)] - \mu_i^r(\sum_{j\varepsilon I} e_{ij}^r)\right]$$

$$= \sum_{i\varepsilon I}\sum_{j\varepsilon I}\left[e_{ji}^r(\mu_i^r - \mu_j^r) - \mu_i^r(L_{ji}^r(e_{ji}^r))\right] \qquad (C\text{-}26)$$

Now it can be seen from a consideration of the objective function as expressed by (C-26), and of the constraints $(C\text{-}13)^r$, that each "instantaneous" exchange problem can be broken into instantaneous inter-nodal exchange problems, PE_{ij}^r, of the form:

$$\text{Find } \underset{e_{ij}^r}{\text{MAX}} \left[e_{ij}^r(\mu_j^r - \mu_i^r) - \mu_j^r(L_{ij}^r(e_{ij}^r))\right] \qquad (C\text{-}26')_{ij}^r$$

Such that:

$$\underline{E}_{ij}^r \leqslant e_{ij}^r \leqslant \bar{E}_{ij}^r \qquad (C\text{-}12)_{ij}^r$$

This problem may easily be solved via calculus. Chapter 4 deals with the solution of (the aggregated version of) this problem. So we have that:

$$e_{ij}^{r*}(\mu) = e_{ij}^{r*}(\mu_i^r, \mu_j^r) \qquad (C\text{-}27)$$

2.4 AGGREGATED MODELS

2.4.1 Introduction

If we were to produce a complete model of the NZED system consisting of about 30 hydro stations, 6 thermal stations, 20 load centres (cities > 21,000 pop^n) and interconnecting transmission lines, using hours for "instants" and a planning horizon of one year (8736 hours), we would have to solve a dual problem in about 500,000 variables. Each iteration of this would require the solution of a Lagrangian problem with countless variables for hourly production, release, storage and transmission. Thus the direct solution of the complete model is beyond the computational capabilities of present-day computers. Our concern here is to develop an approximate model which can be solved.

The multiplicity of dual variables results from the large number of demand constraints, (C-12), one for each node for each instant. In order to ameliorate this difficulty we will relax the requirement that our model explicitly ensures that demand at each node is exactly met at each instant. To facilitate this we aggregate "nodes" into "regions" (indexed by $n=1,\ldots N$), and "instants" into "periods" (indexed by $t=1,\ldots T$). We will use the shorthand notations: $r\varepsilon t$, to mean instant r is in period t, $i\varepsilon n$, to mean node i is in region n. Further, we divide each period into "load segments" (indexed by $k=1,\ldots K$). The instants in these load segments need not be contiguous. A typical load segment would be "the twenty instants of highest demand within the week". We say: $r\varepsilon k$, to mean that instant r is in segment k.

Then we reduce the number of dual variables (μ) which need to be adjusted by introducing an "aggregate dual problem", DA, which adjusts a much smaller set of "price parameters", λ. From these parameters we derive detailed prices, $\mu(\lambda)$, and then solve the Lagrangian problem using these prices. The (λ) price parameters appear as multipliers on certain aggregate demand constraints approximating (C-12). We will also refer to λ as "aggregate prices". In the aggregate dual problem we adjust the λ prices until the aggregate constraints are met. The optimality of the solution depends then on the following assumption:

(AA) That for each node, $i \varepsilon n$, for any period t, the optimal vector of multipliers, $\tilde{\mu}_i^t$, can be constructed with adequate accuracy from the optimal vector of aggregate multipliers, $\tilde{\lambda}_n^t$, in the sense that optimal generation and transmission schedules based on the approximate price vectors, $\tilde{\mu}$, so constructed, come acceptibility close to satisfying conditions (C-12).

In other words, having decided that for the purposes of long-term scheduling a particular degree of accuracy in matching supply and demand in future "instants" is required, we must be careful to select aggregate demand constraints, (C-12$'$), so as to ensure that accuracy. This selection is to be determined by experience.

Thus we can say:

$$\mu_i^r \approx \mu_i^r(\lambda_n^t) \qquad \text{for all } r\varepsilon t, \; i\varepsilon n \tag{A-1}$$

Here we allow that the construction of the detailed price curve (μ_i^t) may well depend on the character of the node (i) and the time of the year (t). For instance, the shape of the load curve for a residential area in

Winter will be completely different to that for an industrial area in Summer. In Section 6.1 we outline the process by which we intend to derive suitable μ prices from the λ prices.

In the next two sections we deal with the aggregate problem and its solution. In Chapters 3 to 5 we discuss the solution of the resultant sub-problems.

2.4.2 The Model

Firstly we will define:

Demand for load segment k of period t in region n:

$$D_n^{tk} = \sum_{i\varepsilon n} \sum_{r\varepsilon t\cap k} D_i^r \quad \text{for all } n=1,\ldots n \quad t=1,\ldots T \quad k=1,\ldots K \tag{A-2}$$

Thermal generation in k of t in n:

$$g_{nT}^{tk} = \sum_{i\varepsilon n} \sum_{r\varepsilon t\cap k} g_{iT}^r \quad \text{for all } n=1,\ldots N \quad t=1,\ldots T \quad k=1,\ldots K \tag{A-3}$$

Hydro generation in k of t in n:

$$g_{nH}^{tk} = \sum_{h\varepsilon n} \sum_{r\varepsilon t\cap k} g_{hH}^r \quad \text{for all } n=1,\ldots N \quad t=1,\ldots T \quad k=1,\ldots K \tag{A-4}$$

Enchange of energy between region n and m in k of t:

$$e_{nm}^{tk} = \sum_{i\varepsilon n} \sum_{j\varepsilon m} \sum_{r\varepsilon t\cap k} e_{ij}^r \quad \text{for all } n,m=1,\ldots N \quad m\neq n \quad t=1,\ldots T \quad k=1,\ldots K \tag{A-5}$$

Energy loss within region n in k of t:

$$L_{nn}^{tk} = \sum_{i\varepsilon n} \sum_{j\varepsilon n} \sum_{r\varepsilon t\cap k} L_{ij}^r(e_{ij}^r) \quad \text{for all } n=1,\ldots N \quad t=1,\ldots T \quad k=1,\ldots K \tag{A-6}$$

In our aggregate model we approximate the constraints (C-12) by the following:

$$g_{nT}^{tk}+g_{nH}^{tk}-L_{nn}^{tk} + \sum_{m=1}^{N}(f_{mn}^{tk} - e_{nm}^{tk}) \geqslant D_n^{tk} \qquad \text{(A-7) (=(C-12'))}$$

for all $n=1,\ldots N$
$t=1,\ldots T$
$k=1,\ldots K$

And:

$$|g_{iT}^{r} + g_{iH}^{r} + \sum_{j\varepsilon I}(f_{ij}^{r} - e_{ij}^{r}) - D_i^{r}| \leqslant \Delta$$

for all $i\varepsilon I, r=1,\ldots R$

(A-8) (= (C-12''))

Where Δ is an "acceptable" level of error as in (AA).

Now we introduce as our aggregate primal problem, PA:

$$\text{Find} \quad \underset{z}{\text{MIN}} \sum_{r=1}^{R}\sum_{i\varepsilon I} C_i^{r}(g_{iT}^{r}) \qquad \text{(A-9) (=(C-11))}$$

Such that:

$$g_{nT}^{tk}+g_{nH}^{tk}-L_{nn}^{tk}+ \sum_{m=1}^{N}(f_{mn}^{tk} - e_{nm}^{tk}) \geqslant D_n^{tk}$$

for all $n=1,\ldots N$
$t=1,\ldots T$
$k=1,\ldots K$ (A-7)

$$|g_{iT}^{r}+g_{iH}^{r}+ \sum_{j\varepsilon I}(f_{ij}^{r}-e_{ij}^{r}) - D_i^{r}| \leqslant \Delta$$

for all $i\varepsilon I, r=1,\ldots R$

(A-8)

$$\underline{E}_{ij}^{r} \leqslant e_{ij}^{r} \leqslant \bar{E}_{ij}^{r}$$

for all $i,j\varepsilon I, r=1,\ldots R$

(A-10) (= (C-13))

$$\underline{G}_{iT}^{r} \leqslant g_{iT}^{r} \leqslant G_{iT}^{r}$$

for all $i\varepsilon B, r=1,\ldots R$

(A-11) (= (C-14))

$$q_h \ \varepsilon Q_h$$

for all $h\varepsilon H$

(A-12) (= (C-15))

Here we can associate multipliers, λ, with the aggregate constraints, (A-7), forming the Lagrangian:

$$\mathcal{L}_A(z,\lambda) = \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} \Bigg[\Bigg(\sum_{r\varepsilon t\cap k} \sum_{i\varepsilon n} C_i^r(g_{iT}^r) \Bigg) - \lambda_n^{tk}\Bigg(g_{nT}^{tk} + g_{nH}^{tk} - L_{nn}^{tk} + \sum_{m=1}^{N} (f_{mn}^{tk} - e_{nm}^{tk}) - D_n^{tk} \Bigg) \Bigg] \qquad \text{(A-13)}$$

λ_n^{tk} can be thought of as the "average" price for energy delivered to region n in load segment k of period t.

Now, just as for the complete problem, we can define:

$$P_A(\lambda) = \underset{\{z \mid (A-8)-(A-12)\ \text{hold}\}}{\text{MIN}} \mathcal{L}_A(z,\lambda) \qquad \text{(A-14)}$$

Then introduce the aggregate dual problem, DA:

$$\text{Find} \quad \underset{\lambda}{\text{MAX}}\ P_A(\lambda) \qquad \text{(A-15)}$$

$$\text{Such that:} \quad \lambda_n^t \geqslant 0 \qquad \text{for all } n=1,\ldots N,\ t=1,\ldots T \qquad \text{(A-16)}$$

The aggregate dual objective function, $P_A(\lambda)$, is concave just as was the complete dual objective function, $P_C(\lambda)$. Now we could solve DA by an iterative technique, adjusting the λ prices so as to meet the constraints. At each iteration of this algorithm we would face the aggregate Lagrangian problem, PA′:

$$\text{Find} \quad \underset{z}{\text{MIN}} \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} \left[\left(\sum_{r \varepsilon t \cap k} \sum_{i \varepsilon n} C_i^r(g_{iT}^r) \right) \right.$$

$$\left. - \lambda_n^{tk} \left(g_{nT}^{tk} + g_{nH}^{tk} - L_{nn}^{tk} + \sum_{m=1}^{N} (f_{mn}^{tk} - e_{nm}^{tk}) - D_n^{tk} \right) \right] \qquad \text{(A-10}'\text{)}$$

Such that (A-8)-(A-12) hold.

Now this Lagrangian problem is not separable because of the constraints (A-8).

However, given assumption (AA), we can ensure that the constraints (A-8) are met by solving, instead, the following modified aggregate primal problem, PA$''$:

$$\text{Find} \quad \underset{z}{\text{MIN}} \sum_{n=1}^{N} \sum_{t=1}^{T} \left[\sum_{r \varepsilon t} \sum_{i \varepsilon n} \left[C_i^r(g_{iT}^r) \right. \right.$$

$$\left. \left. - \mu_i^r(\lambda_n^t) \left(g_{iT}^r + g_{iH}^r + \sum_{j \varepsilon I} (f_{ji}^r - e_{ij}^r) - D_i^r \right) \right] \right] \qquad \text{(A-17)}$$

Such that (A-9) - (A-12) hold.

Now this problem is identical in form to the complete Lagrangian problem, PC$'$. We propose the following algorithm for its solution. (cf. Section 2.3.2).

(1) Initialise λ.

(2): 2.1 Derive $\mu(\lambda)$.

2.2 Solve the modified aggregate Lagrangian problem, PA$''$, to determine $z^*(\mu(\lambda))$.

2.3 Aggregate the responses to get:

$g_{nT}^{tk*}(\lambda), g_{nH}^{tk*}(\lambda), e_{nm}^{tk*}(\lambda), L_{nn}^{tk*}(\lambda)$

for $n,m=1,\ldots N$

$t=1,\ldots T$

$k=1,\ldots K$.

(3) If: $\left| g_{nT}^{tk*}(\lambda)+g_{nH}^{tk*}(\lambda)-L_{nn}^{tk*}(\lambda)+\sum_{m=1}\left(f_{mn}^{tk*}(\lambda)-e_{nm}^{tk*}(\lambda)\right)-D_{n}^{tk}\right| < \varepsilon$

for all $n=1,\ldots N$
$t=1,\ldots T$
$k=1,\ldots K$

THEN STOP (A-7′)

ELSE GO TO (4).

(4) Adjust λ:

4.1 Determine a feasible search direction, $\delta(\lambda)$.

4.2 Determine a step size, θ, such that:

$$P_A(\lambda + \theta\delta(\lambda)) \geqslant P_A(\lambda) \quad \text{(A-18)}$$

4.3 Let $\lambda = \lambda + \theta\delta(\lambda)$. (A-19)

4.4 GO TO (2).

When this algorithm terminates in Step 3 we will have the solution to the aggregate dual problem DA, with assumption (AA) assuring us that, at this optimum, the constraints (A-8) hold. So then we have the approximate solution to the aggregate primal problem (PA) just as we had for the complete primal in Section 2.3.2. Thus we have the solution, to within the tolerances guaranteed by (AA), to the original complete problem.

In Chapter 6 we deal with the adjustment of the λ prices (Step (4)), the derivation of the μ prices and the aggregation of the responses (Steps (2.1),(2.3)). In the next section we consider Step (2.2) - the solution of the modified aggregate Lagrangian problem, PA″.

The inter-relationship of these various problems is demonstrated in Figure (2-8).

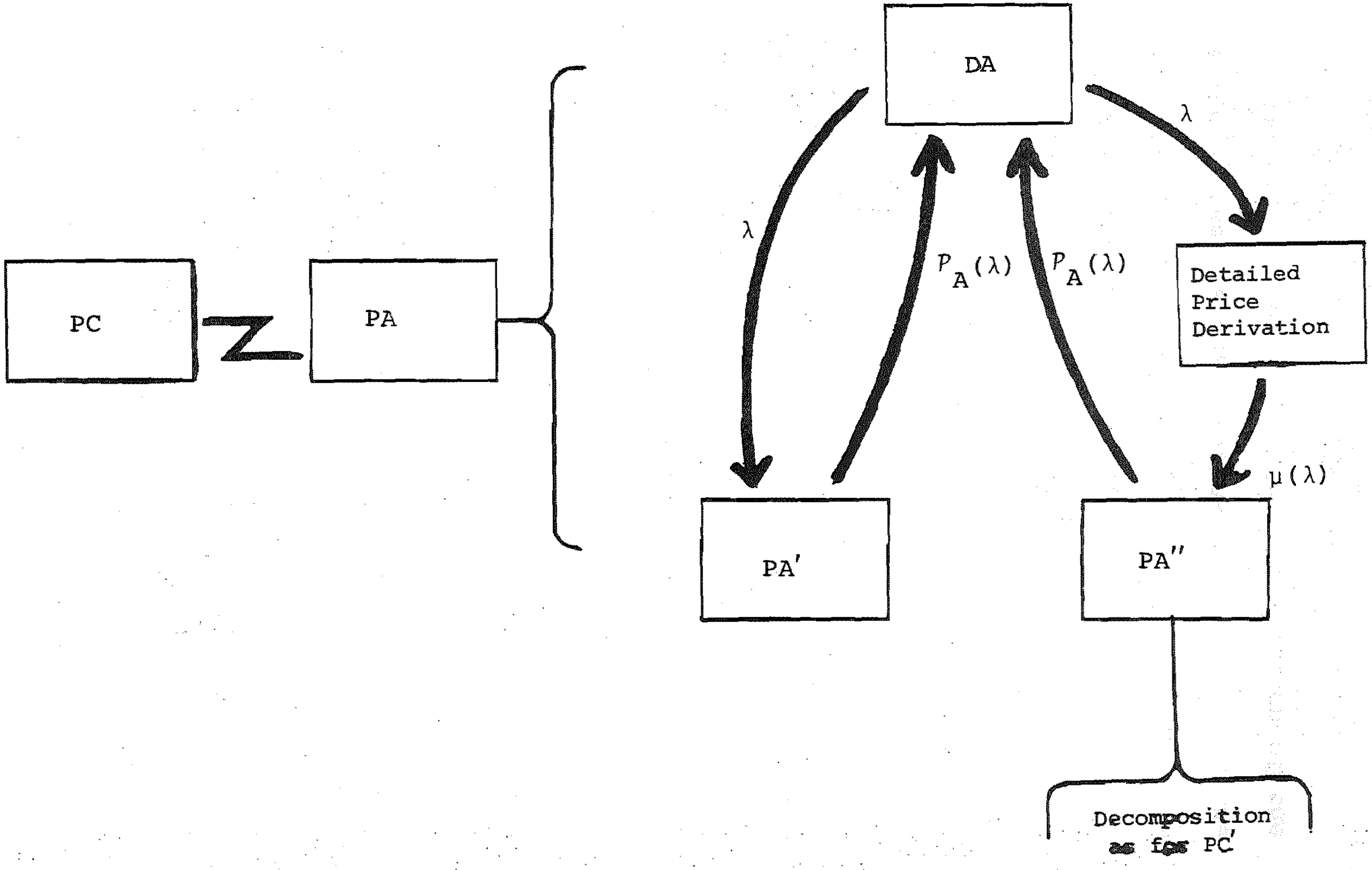


FIGURE (2-8): PA vs PC

2.4.3 Decomposition

The problem PA″, faced at Step 2.2, is separable in exactly the same way as was the complete Lagrangian problem, PC′. Thus we can use the results of Section 2.3.3 to decompose this problem to the level we desire.

Firstly, we have one-period inter-regional exchange problems, (PAE^{t}_{nm}):

$$\text{Find } \max_{e^{t}_{nm}} \sum_{r\varepsilon t} \sum_{i\varepsilon n} \sum_{j\varepsilon m} \left[\mu^{r}_{j}(\lambda^{t}_{m}) f^{r}_{ij} - \mu^{r}_{i}(\lambda^{t}_{n}) e^{r}_{ij}\right] \qquad (A\text{-}17)^{t}_{nm}$$

Such that:

$$\underline{E}^{r}_{ij} \leqslant e^{r}_{ij} \leqslant \bar{E}^{r}_{ij} \quad \text{for all } i\varepsilon n, j\varepsilon m, r\varepsilon t. \qquad (A\text{-}10)$$

These problems can easily be solved by the technique of Chapter 4. We can simplify the solution of the overall problem by applying the exchange sub-model to evaluate the aggregate optimal exchange between the two regions for a number of different combinations of prices λ_n and λ_m, then storing the results in a look-up table. Then, in the aggregate dual, we can determine the optimal solution to the exchange sub-problem merely by referencing these tables. (In forming such tables we incorporate the derivation of the μ prices from the λ prices into the sub-model). In Chapter 4 we also allow non-convex loss functions. Provided that this does not cause non-concavities in the "profit" function (and hence in $P_A(\lambda)$) it will not cause problems in the aggregate dual problem.

Secondly, we have aggregate regional problems, (PA''_n):

$$\text{Find} \quad \underset{(g_n, e_n)}{\text{MIN}} \sum_{t=1}^{T} \sum_{r\varepsilon t} \sum_{i\varepsilon n} \Bigg[\Big(C_i^r(g_{iT}^r) - \mu_i^r(\lambda_n^t) g_{iT}^r\Big) - \mu_i^r(\lambda_n^t) g_{iH}^r - \mu_i^r(\lambda_n^t) \sum_{j\varepsilon n} (f_{ji}^r - e_{ij}^r)\Bigg] \qquad (A-17)_n$$

Such that $(A-11)_n$, $(A-12)_n$ hold.

These problems decompose, just as did the original complete problem, into nodal thermal, local hydro and inter-nodal exchange problems. The thermal and exchange problems, being temporally separable, decompose into a collection of one-period problems. So we have:

One-period nodal thermal problems, (PAT_i^t):

$$\text{Find} \quad \underset{g_{iT}^t}{\text{MIN}} \sum_{r\varepsilon t} \Big(C_i^r(g_{iT}^r) - \mu_i^r(\lambda_n^t) g_{iT}^r\Big) \qquad (A-17)_{iT}^t$$

Such that:

$$\underline{G}_{iT}^r \leqslant g_{iT}^r \leqslant \bar{G}_{iT}^r \qquad \text{for all } r\varepsilon t \qquad (A-11)_{iT}^t$$

One-period inter-nodal exchange problems, (PAE_{ij}^t).

$$\text{Find} \quad \underset{e_{ij}^t}{\text{MAX}} \sum_{r\varepsilon t} \Big(\mu_j^r(\lambda_n^t) f_{ij}^r - \mu_i^r(\lambda_n^t) e_{ij}^r\Big) \qquad (A-17)_{ij}^t$$

Such that:

$$\underline{E}_{ij}^r \leqslant e_{ij}^r \leqslant \bar{E}_{ij}^r \qquad \text{for all } r\varepsilon t \qquad (A-10)_{ij}^t$$

Local hydro problems, (PAH_h):

$$\text{Find} \max_{q_h} \sum_{t=1}^{T} \left(\sum_{r \varepsilon t} \sum_{i \varepsilon h} \mu_i^r(\lambda_n^t)\, g_{iH}^r(q_i^r, s_i^r) \right). \qquad (A-17)_h$$

Such that:

$$q_h \ \varepsilon \mathcal{Q}_h$$

In Chapter 3 we deal with the solution of the one-period nodal thermal problems. This is quite easily achieved via calculus. Just as for the inter-regional exchange problems we can, for each λ_n vector, derive μ_n, solve the sub-problem and store the aggregate results in tabular form. Then in the aggregate dual problem we can use these tables to determine optimal aggregate output from the thermal system. We will also allow rather more general, non-convex and possibly inseparable, cost functions. Again these will not cause any problems provided that the "profit" from thermal generation remains a concave increasing function of λ.

In Chapter 5 we deal with the solution of a local hydro problem. This is by far the most complex of the sub-problems. We will divide this problem into a long-term problem and a collection of short-term problems. The long-term problem must be resolved at each iteration of the aggregate dual. It utilises tables summarising the results from the short-term hydro model.

In Chapter 4 we deal with the solution of inter-regional exchange problems (as above). However we do not intend to use these techniques to solve the intra-regional exchange problems $(PAE_{ij})_{i,j\varepsilon n}$. Rather, we intend to aggregate the production in each region using "traditional" internal transmission patterns. This point is discussed in Section 6.2.3.

Thus we may solve the problem PA$''$ at each iteration of the aggregate dual problem by looking up tables to determine the optimal aggregate response from the thermal and exchange systems and solving the long-term hydro problem. The techniques required for the solution of these sub-problems are found in the next three chapters.

2.5 CONCLUSION

We have outlined a scheme whereby the complete, computationally intractable, model may be approximated by a far more manageable aggregate model. This model relies on two things for its accuracy: the accuracy of the sub-models and the accuracy with which the μ prices may be derived from the λ prices. The former topic is taken up in Chapters 3 to 5.

The accuracy with which we can derive the detailed prices from the aggregate prices depends on the flexibility allowed by the price parameter system. Obviously, if K or T are large enough, we can make our parameter system so flexible as to be able to achieve any desired degree of accuracy. However we wish to trade off flexibility for computational tractability.

For example, suppose that we had one period for each week and let K=1. Then we would have a model very similar to that of [47] except for the very important additional constraint that the detailed demand pattern within each week and region must be (approximately) met. In this model λ_n^t is the representative price for energy delivered to region n in period t. The aggregate dual problem for this model will have only 52 dual variables (λ_n^t) to adjust for each region. However, unless special provision is made, this scheme will attempt to conform production at all times of the year to a single load distribution. For this reason, the more flexible multi-segment model could be more appropriate.

However, provision can be made for regional and load variations. This can be achieved by appropriate variations in the way in which the detailed, μ, prices are derived from the λ parameters. In order to determine an appropriate scheme we will need to know how the pattern of optimal μ prices varies. The multi-segment model of Section 2.4.3 can be used to gain the necessary experience.

The scheme proposed in this section has obvious similarities with that proposed by the EDF (see Appendix A). The EDF model has two independent price parameters corresponding to the "normal" and "off-peak" segments. From these they derive "price duration curves" appropriate to various times of the year and week. These curves are used as input into heuristic short-term scheduling programs (e.g., P1). The chief advantages of our scheme are:

(i) Its accuracy - rather than price duration curves we have used price curves, giving the pattern of prices as they vary with time. This enables us to model dynamic phenomena in our sub-models. In the thermal sub-model we can take account of start-up costs, etc. (See [49]). In the short-term hydro sub-model we can employ an optimising procedure, taking into account time delays and storage limits for downstream reservoirs in each valley.

(ii) Its consistency - we have developed a consistent framework in which an aggregate model can be seen as an approximation to the true detailed model rather than a collection of ad-hoc assumptions.

(iii) Its flexibility - we have assumed no fixed relationships among the λ prices nor any specific form for the μ price curve as a function of the λ price parameters.

Further, we note that there is no need for the periods to be of equal length. If it is possible for us to determine, say, a weekly price curve for a period and region on the basis of just one "aggregate price", it may equally well be possible to determine a monthly price curve on such a basis with sufficient accuracy for our purposes. This may be particularly appropriate for periods in the more distant future, say a year away. Our forecasts as to the situation prevailing at that time are necessarily vague and we have a corresponding degree of uncertainty about the appropriate price structures. Thus a less accurate representation could be quite acceptable. We could most easily introduce variable length periods by amalgamating several "periods" into one "super-period" (so that we can use the amalgamated output from our standard sub-models).

As is pointed out in [17], this could allow a considerable reduction in the number of dual variables required and hence in computational effort needed to adjust them. However preliminary experience indicates that computation time will not be a major problem. The only way in which this possibility can reasonably be evaluated is, again, by experience with a more detailed model. If the price curves derived from this model do exhibit a sufficiently consistent pattern this alternative can easily be incorporated into the general framework.

Finally we consider the relationship between our present model and that put forward earlier in [47]. That earlier work presented a multi-load generalisation of the EDF model in which no account was taken of short-term requirements. Thus it merely ensured that sufficient energy was available in each region and period without ensuring that the system would be able to allocate energy delivery so as to meet peak requirements without increasing costs. The extra complexity of the present model has been introduced to formalise these extra (realistic) requirements. Mathematically, the model of [47] is identical to the complete model, PC, the only difference being in the number (and hence length) of the time intervals involved. That model is also equivalent to a single-segment aggregate model, PAI (say), without the extra detailed restrictions summarised by (A-8). For such a model we would have:

$$\mu_i^r = \lambda_i^t \qquad \text{for all } r \varepsilon t \qquad \text{(A-20)}$$

implying a constant level of production throughout the period. A similar multi-segment model (PAK), in which we allowed:

$$\mu_i^r = \lambda_i^{tk} \qquad \text{for all } r \varepsilon t \cap k \qquad \text{(A-21)}$$

could ensure approximate satisfaction of the detailed constraints. Such a model could be stated much more simply as a generalisation of the model in [47].

However our more general approach, which covers such models, allows for greater flexibility. Surprisingly, despite its more formidable statement, the resultant aggregate problem is no harder to solve than PAK, the extra detail being confined solely to the preparation of input tables.

CHAPTER 3

THE NODAL THERMAL PROBLEM

3.1 INTRODUCTION

Our concern here is with the solution to the nodal thermal sub-problems of our aggregated model. We deal with a single thermal node, $i\varepsilon n$, and period t. Thus, given a vector of prices $(\mu_i^r)^{r\varepsilon t}$, derived from the price parameters λ_n^t, we wish to determine the generation schedule for node i which maximises the "net return" at those prices. In fact we can further decompose our problem to the level of the individual sets within the station. Unless stated otherwise, we shall treat each station as an individual set for the remainder of this chapter. We shall drop the subscripts n, i and "T".

Our problem may formally be stated as:

$$\text{(PAT) Find} \quad \underset{g}{\text{MIN}} \quad \sum_{r\varepsilon t} C^r(g^r) - \mu^r g^r \qquad \text{(T-0)}$$

Such that:

$$\underline{G}^r \leqslant g^r \leqslant \bar{G}^r \quad \text{for all } r\varepsilon t \qquad \text{(T-1)}$$

Because we have ignored set-up and shut-down costs the objective and constraint set are temporally separable. So we can separate problem PAT into instantaneous problems, $(\text{PAT}^r)^{r\varepsilon t}$ Where there is no possibility of ambiguity we drop the superscript r. The ease with which these instantaneous problems can be solved depends on the form of the cost function. In the next two sections we deal with the two cases illustrated by simple realistic examples.

3.2 CONVEX COSTS

Firstly, if $C(g)$ is convex then we can solve PAT^r by merely equating the derivative of the objective (w.r.t g) to zero. That is we find $\tilde{g}$ such that:

$$\left.\frac{\partial C(g)}{\partial g}\right|_{\tilde{g}} = \mu \tag{T-2}$$

Then set:

$$g^* = \text{MIN}\{\text{MAX}\{\tilde{g},\underline{G}\},\bar{G}\} \tag{T-3}$$

In the quadratic case, where:

$$C(g) = \alpha g^2+\beta g \tag{T-4}$$

$$(T\text{-}2) \Rightarrow 2\alpha\tilde{g}+ \beta=\mu$$

$$\Rightarrow \tilde{g}(\mu) = \frac{\mu-\beta}{2\alpha} \tag{T-5}$$

The cost and marginal cost curves for this case are shown in Figures (3-1) and (3-2). Figure (3-3) shows the resultant "generation response" curve. In economic terms we can imagine a manager for the thermal station maximising his profit by equating marginal revenue (μ) with marginal cost. If μ is below his minimum marginal cost (β) he does not generate. Above that point his generation increases linearly (due to the quadratic cost function) until, at price $\bar{\mu}$, it reaches $\bar{G}$.

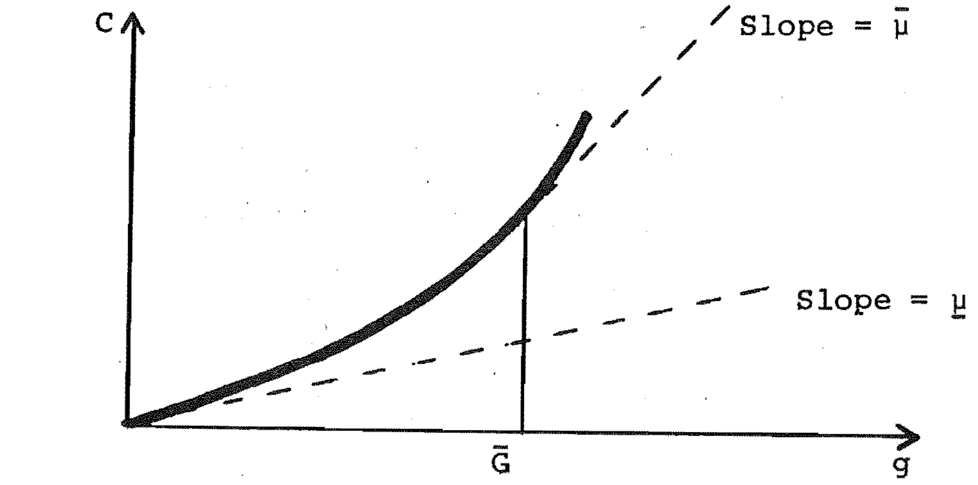


FIGURE (3-1): Thermal cost function (convex quadratic case).

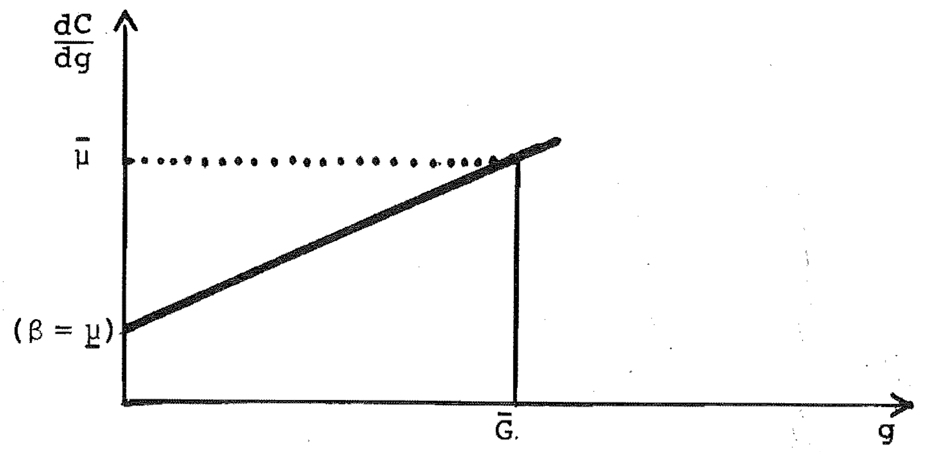


FIGURE (3-2): Marginal cost curve.

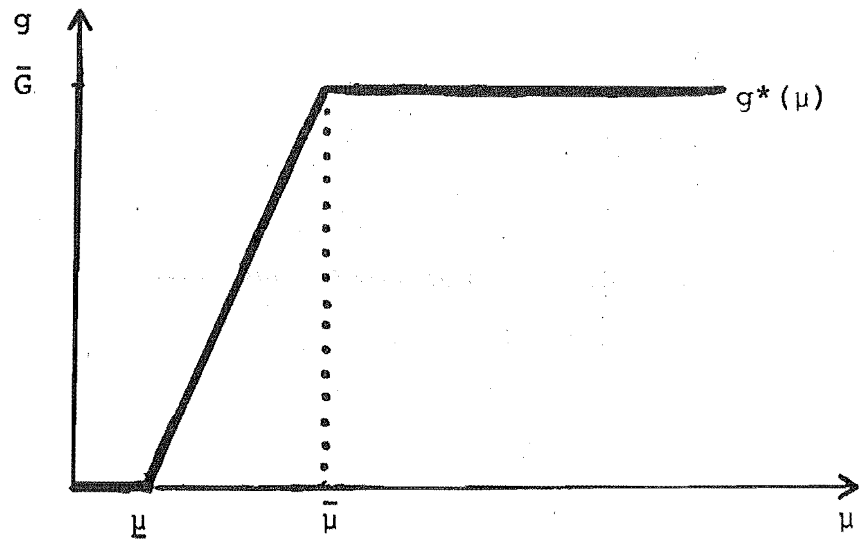


FIGURE (3-3): Thermal response curve.

3.3 NON-CONVEX COSTS

Usually a thermal station or set which is running does in fact have an approximately convex cost curve which, in general, can be adequately represented by a quadratic. However the quadratic usually has a significant constant term and thus the option of turning the thermal station off will be attractive. As is shown in Figure (3-4) this could be modelled by a non-convex (in fact discontinuous) cost curve. (An alternative model could include zero-one integer variables introducing a new difficulty).

Now, if our thermal station can be turned off without incurring any penalty, we will never run the station if the value of the energy produced (μg) is lower than the cost of producing it ($C(g)$). If $\hat{g}$ is the level of generation giving minimum average cost for the units generated then there should be no generation at all from the station while:

$$\mu g < C(\hat{g}). \qquad \text{(T-6)}$$

This point $\hat{g}$ corresponds in economic terms with the shut-down point and, in engineering terms, with the peak efficiency of the station. If we let:

$$AC(g) = \frac{C(g)}{g} \text{ - the average cost function,} \qquad \text{(T-7)}$$

then $\hat{g}$ can be found by setting:

$$\left.\frac{\partial AC(g)}{\partial g}\right|_{\hat{g}} = 0 \qquad \text{(T-8)}$$

Or, equivalently:

$$AC(g) = \frac{\partial C(g)}{\partial g}\bigg|_{\hat{g}} \tag{T-9}$$

i.e. marginal cost is equal to average cost.

$$\left[\text{Since (T-8)} \Rightarrow g\frac{\partial C(g)}{\partial g}\bigg|_{\hat{g}} - C(\hat{g}) = 0\right.$$

$$\left.\Rightarrow \frac{\partial C(g)}{\partial g}\bigg|_{\hat{g}} = \frac{C(\hat{g})}{g} = AC(\hat{g})\right]$$

$$\text{Further (T-9)} \Rightarrow g\frac{\partial C(g)}{\partial g}\bigg|_{\hat{g}} = C(\hat{g}) \tag{T-10}$$

So $\hat{g}$ is the generation level at which a line drawn from the origin is tangent to $C(g)$.

Figure (3-4) demonstrates condition (T-10) for the quadratic case. Figure (3-5) shows the corresponding marginal and average cost curves, demonstrating conditions (T-9) and (T-10).

For this quadratic case $\hat{g}$ is easily found (e.g. from (T-5)) to be:

$$\hat{g} = \sqrt{\frac{\gamma}{\alpha}} \quad . \tag{T-11}$$

If the station is running then, as in Section 3.2, optimal operation involves generating so that the marginal cost of generation is equal to the marginal value of generation (i.e. the price μ). This is again equivalent, in economic terms, to equating marginal revenue (= price, in our case) with marginal cost so as to maximise "profit". Thus the response of a thermal station can be summarised by a curve such as that in Figure (3-6). Here, as the "price" μ increases, there is no response from the station until $\mu = \hat{\mu}$, the marginal cost of generation at the station's most efficient generation level, $\hat{g}$. Then the optimal generation level jumps to $\hat{g}$. After this point the optimal generation level increases gradually until the price reaches $\bar{\mu}$, the marginal cost of generation at the station's maximum output, $\bar{G}$. After this point generation remains constant at $\bar{G}$.

If we were to modify our non-convex cost curve, replacing the segment from 0 to $\hat{g}$ by the tangent line of slope $\hat{\mu}$, we would in fact obtain a response curve identical to that of Figure (3-6). We shall refer to this modified curve as $C'(g)$. In fact the generation range $[0, \hat{g})$ constitutes a "duality gap" in the sense of [30], Section 8.4]. As is pointed out there, the Lagrangian approach cannot be successful if the optimal solution lies in such a "gap". By modifying the curve we have removed the gap, the tangent line providing the necessary supporting hyperplane.

This modification to the curve is sufficient to guarantee optimal solutions, but it does not give us unique solutions. We can guarantee these however by "patching" the response curve, removing the discontinuity by inserting a very steep portion joining $(\hat{\mu}^-, 0)$ and $(\hat{\mu}^+, \hat{g})$. This is equivalent to putting a slight curvature on the initial segment of C' so as to make it strictly convex.

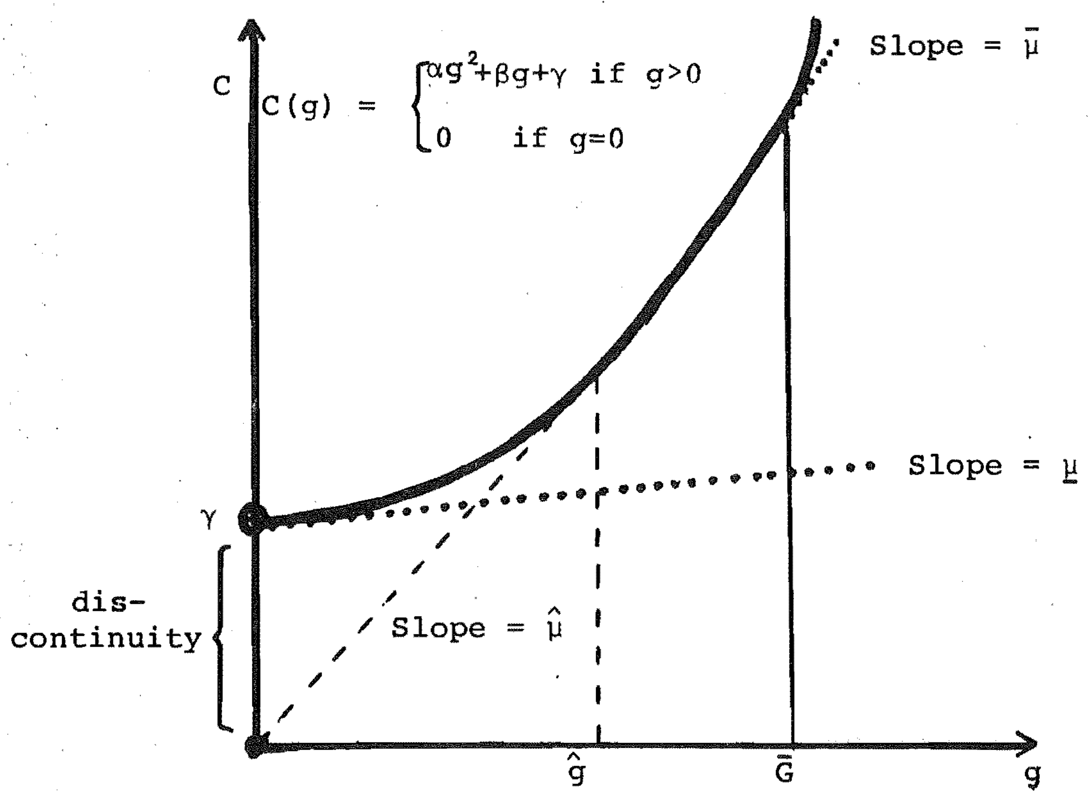


FIGURE (3-4): Thermal cost function (non-convex quadratic case).

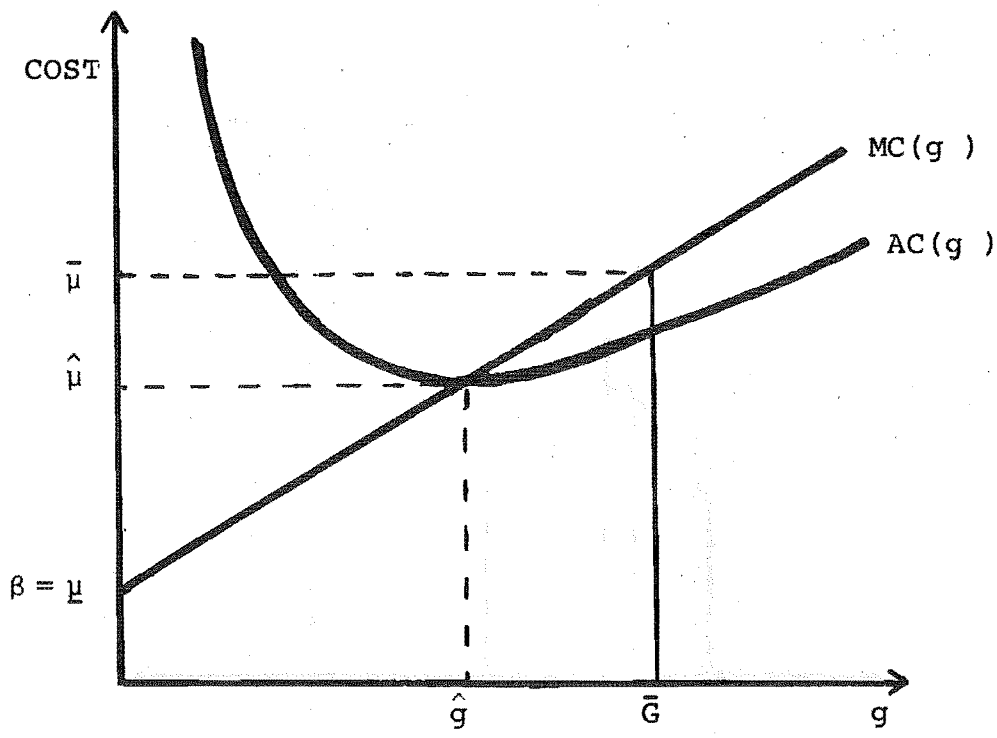


FIGURE (3-5): Marginal and average cost curves.

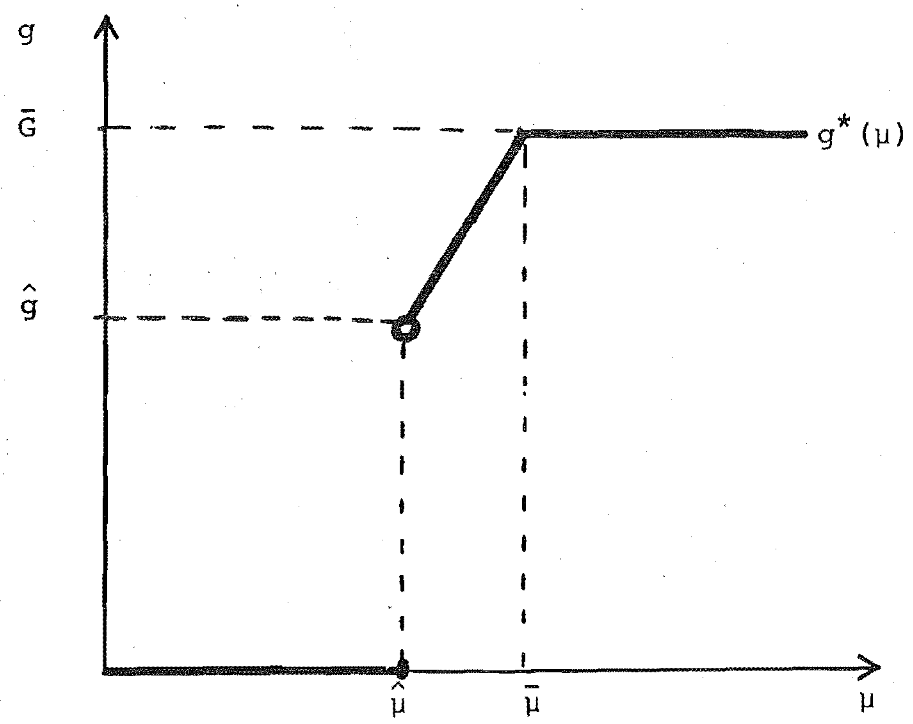


FIGURE (3-6): Thermal response curve.

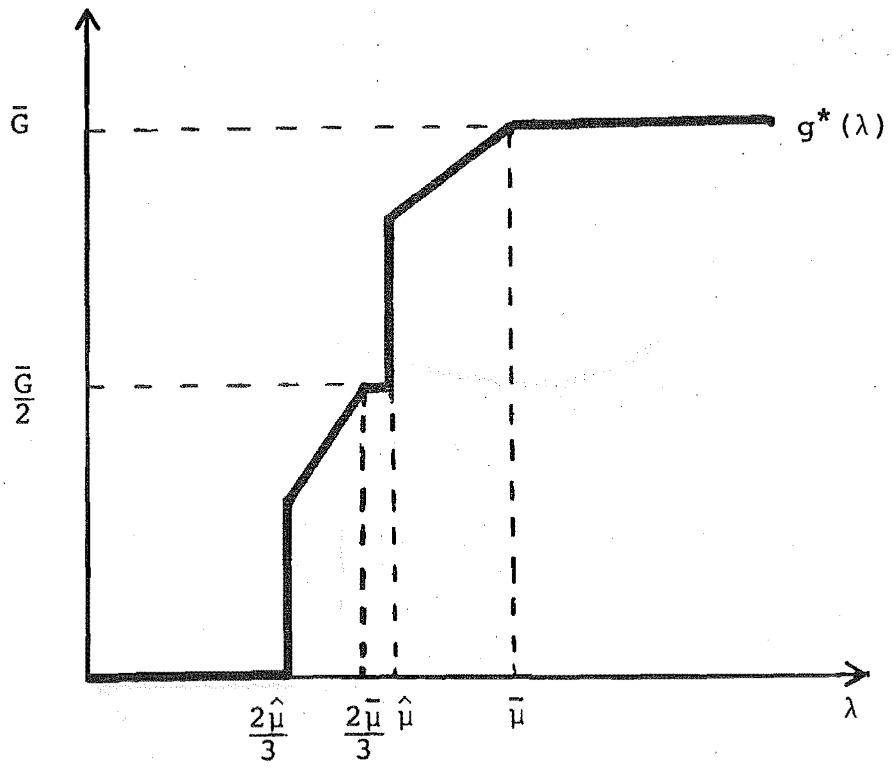


FIGURE (3-7): Aggregate response curve.

These modifications allow us to solve the mathematical model, but to what do they correspond in practice? If $0 < g^* < \hat{g}$ we can generate g^* at marginal cost $\hat{\mu}$ within an instant by generating at the level of maximum efficiency, $\hat{g}$, for part of the instant and shutting the plant down for the remainder. If this is impossible for some reason then, if we were implementing the solution immediately and this problem occurred for an "instant" in the near future, we should re-run the short-term optimisation twice - once with the station forced to be on, once off. Then we would implement the better solution. If the instant involved was further in the future it would seem appropriate, in view of the uncertainties involved, to ignore the problem.

3.4 CONCLUSIONS

We have seen that, provided we ignore set-up costs, the solution to the thermal sub-problem of our aggregated model is quite straightforward. However set-up costs may be a significant factor in some circumstances. These can be dealt with by a short-term scheduling algorithm such as that of [41] which is outlined and extended in [49].

We have dealt with each "node" as if it consisted of only one set. We can aggregate the sets in a station just as we will aggregate the stations in a region. (See Section 6.2.3).

Note that, for the quadratic case, the instantaneous response curve, being piece-wise linear, can be summarized by its corners and so stored very compactly.

For the purposes of our global algorithm we do not, in fact, need to know the "instantaneous" output from each node, but merely some aggregate characteristics. We restrict our attention here to aggregation over the time period - aggregation over the region being dealt with in Section 6.2.3.

For any particular aggregate price vector (λ) we are interested in one aspect only of the response of this thermal node - its contribution to the satisfaction of the chosen aggregate constraints (e.g. total peak and off-peak energy produced) when run optimally with respect to that price vector. Since we can derive the vector μ_i^t from λ_n^t this is easily determined - and need only be determined once, giving us a standard generation response curve, $g_i^{t*}(\lambda_n^t)$, which can be stored in tabular form. However outages or a significantly different load curve would require an alternative generation response curve. The derivation of a very simple response curve in this way is shown in Figure (3-7). Here we have one price from which are derived two prices (μ^1, μ^2) which correspond to two periods of the day of equal length, and our concern is only with total generation.

It can be seen that the response curve will become increasingly smooth as the price vector μ becomes more detailed.

Finally we note that, in practice, a set may be so constructed that its maximum generation is limited to a maximum level ($\bar{G}$) lower than its, theoretical, most efficient operating level ($\hat{g}$). This situation is typical of much plant designed for base-load operation. In this case the optimal economic generation level is clearly $\bar{G}$ and the three cornered generation response curve (see Figure (3-6)) reduces to a single step function. In this situation the marginal cost of generation is no longer relevant since it is lower than the average cost. Thus, if the price μ is lower than the average cost at maximum generation ($AC(\bar{G})$), then we should not run the set at all. If, on the other hand, we have $\mu > AC(\bar{G})$ we should run the set at $\bar{G}$. Thus $\hat{\mu}$, the price at which generation from the set becomes an economic proposition, may be found by:

$$\hat{\mu} = \alpha\bar{G} + \beta + \frac{\gamma}{\bar{G}} \tag{T-12}$$

We may aggregate the sets in each station and also the generation in the various instants just as for the generation response curves discussed previously.

CHAPTER 4

THE INTER-REGIONAL EXCHANGE SUB-PROBLEM

4.1 INTRODUCTION

We are concerned here with the solution of the inter-regional exchange problem of our aggregated model. At each iteration of this model we will have specified the price parameters, λ, and will be able to derive $\mu(\lambda)$. We are concerned with the exchange of energy between two regions, n and m, for which we have price vectors $(\mu_i^r)_{i\varepsilon n}^{r\varepsilon t}$ and $(\mu_j^r)_{j\varepsilon m}^{r\varepsilon t}$. We wish to determine the optimal transmission schedule between the two regions (n,m) so as to maximise the benefits from exchange at these prices. We will here concern ourselves entirely with the inter-nodal exchange problem (i.e. taking n,m to be nodes) since the inter-regional exchange is simply a sum of inter-nodal exchanges. The solution to this problem is, in fact, very similar to that of the thermal problem outlined in Chapter 3.

First, let us restate the problem from Section 2.3.5:

$$(PAE_{nm}^t)\ \text{Find} \quad \underset{e_{nm}^t}{\text{MAX}} \sum_{r\varepsilon t} \left[e_{nm}^r(\mu_m^r - \mu_n^r) - \mu_m^r(L_{nm}^r(e_{nm}^r)) \right] \qquad (E\text{-}0)$$

Such that:

$$\underline{E}_{nm}^r \leqslant e_{nm}^r \leqslant \bar{E}_{nm}^r \qquad (E\text{-}1)$$

Of course we also have the closely related problem PAE_{mn}^t, whose solution, since it depends on the same parameters, we will deal with simultaneously.

As for the thermal problem, the solution to this problem depends on the form of the functions - in this case losses rather than costs. Here, just as in that chapter, we are assuming that there are no penalties involved in changing the transmission level and thus the problem breaks down into a series of "instantaneous problems". We will henceforth drop the superscript r. Again we have two cases - either the losses are strictly convex, or not. We consider both in the following sections.

Recall that our primary motivation for introducing the exchange problem is the presence in New Zealand of a limited capacity DC link between the two main islands. For this type of link the methods outlined in this chapter are entirely appropriate. Most AC lines are subject to square law (i.e. pure quadratic) losses. However, in an AC network, the determination of exact flows of active and reactive power with their corresponding losses is a major undertaking. Our simple form for the loss function cannot reflect this complexity. It is intended rather to reflect the approximate losses likely to be incurred. Studies (e.g. [29]) indicate that such approximate loss functions can be computed with a reasonable degree of accuracy. More exact forms for the loss functions could be accommodated. They would be likely to destroy the spatial separability of the exchange problem and so complicate both its solution and the nature of the response surfaces derived.

4.2 CONVEX LOSSES

We deal here with the case where the loss function is convex (and increasing) in e as shown in Figure (4-1). For example, L(e) could be a pure quadratic. Figure (4-2) shows the resultant "energy received" function, f(e).

Since the loss function is convex the objective (E-0) is concave, and so the exchange problems easily solved by differentiating the objective, finding $\tilde{e}^r_{nm}$ such that:

$$\left.\frac{dL_{nm}(e_{nm})}{de_{nm}}\right|_{\tilde{e}_{nm}} = \frac{\mu_m - \mu_n}{\mu_m} \tag{E-2}$$

and setting:

$$e^*_{nm} = \text{MIN}\left\{\text{MAX}\left\{e_{nm}, \bar{E}_{nm}\right\}, \underline{E}_{nm}\right\} \tag{E-3}$$

For example, in the pure quadratic case we have:

$$L_{nm}(e_{nm}) = \alpha_{nm} e^2_{nm} \tag{E-4}$$

$$(E\text{-}3) \Rightarrow \quad 2\alpha_{nm}\tilde{e}_{nm} = \frac{\mu_m - \mu_n}{\mu_m} \tag{E-5}$$

$$\Rightarrow \quad \tilde{e}_{nm} = \frac{\mu_m - \mu_n}{2\alpha_{nm}\mu_m} = \frac{1}{2\alpha_{nm}}\left(1 - \frac{\mu_n}{\mu_m}\right) \tag{E-6}$$

Thus $\tilde{e}_{nm}(\mu_n, \mu_m)$ is linear in $\left(\frac{\mu_n}{\mu_m}\right)$ and hence $\tilde{f}_{nm}(\mu_n, \mu_m)$ is quadratic in this.

In economic terms we could imagine a "manager" buying energy at the price μ_n in region n and selling it at the price μ_m in region m. He maximises his profits by equating marginal revenue with marginal cost, that is:

$$\mu_m\left(1 - \left.\frac{dL_{nm}}{de_{nm}}\right|_{\tilde{e}_{nm}}\right) = \mu_n \tag{E-7) cf. (E-3}$$

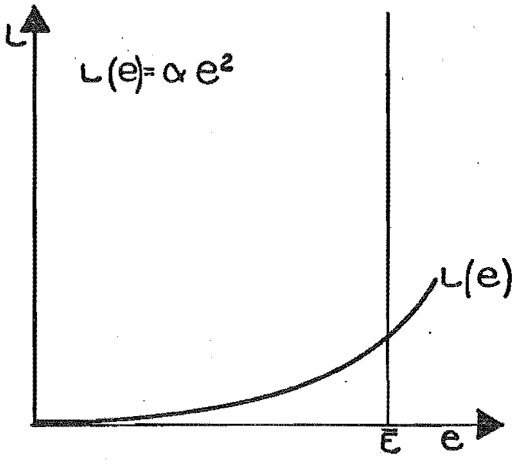


FIG (4·1) :
LOSS FUNCTION.

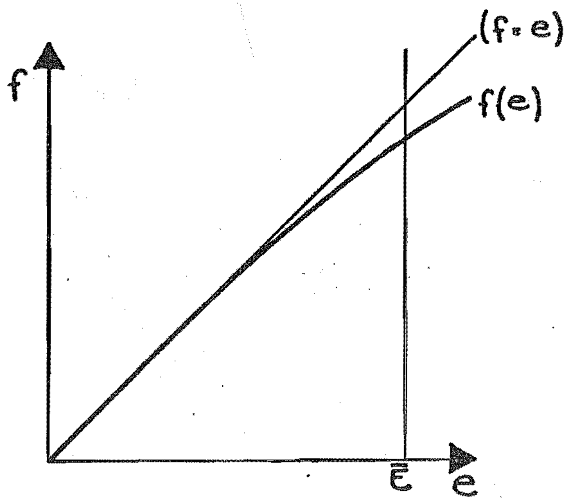


FIG (4·2) :
ENERGY RECEIVED FUNCTION.

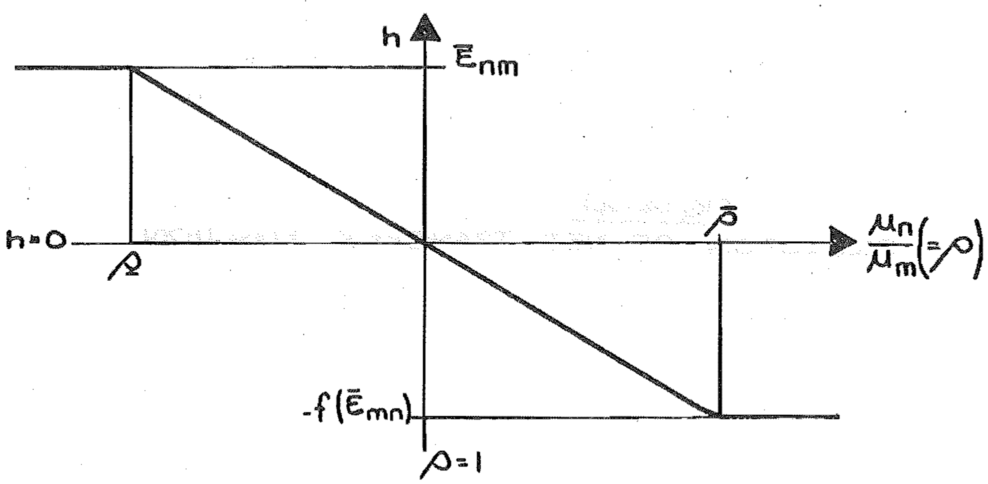


FIG (4·3) :
NET TRANSFER.

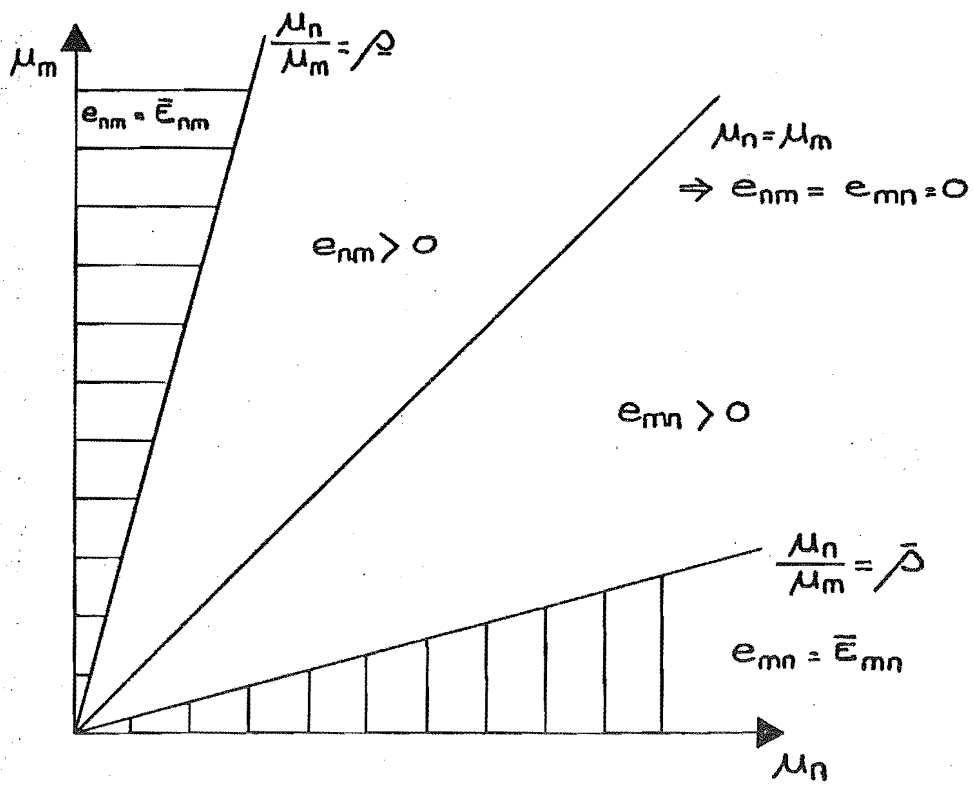


FIG (4-4):
PROJECTION OF NET TRANSFER FUNCTION.

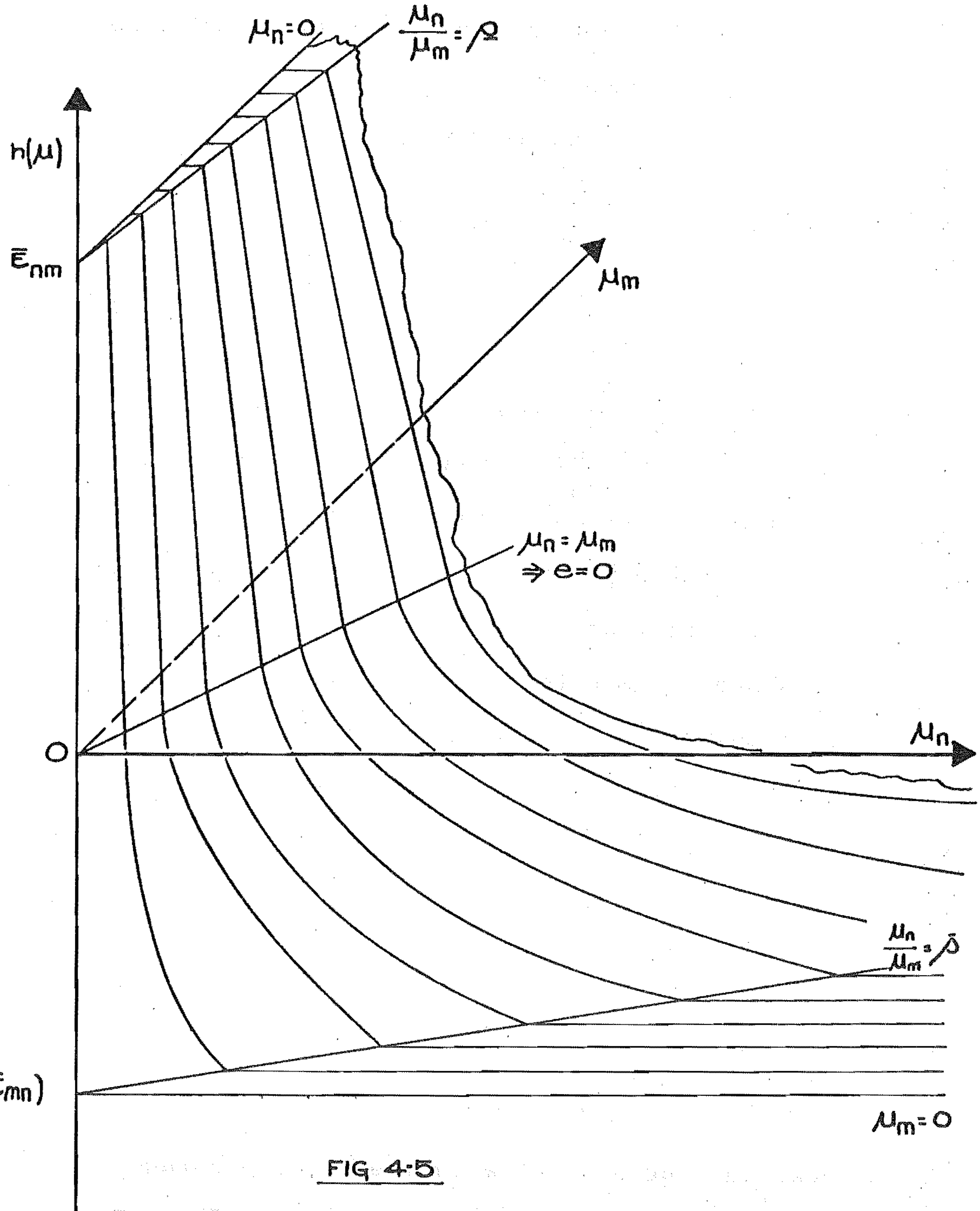


FIG 4·5

EXCHANGE RESPONSE SURFACE
FOR $L(e) = \alpha e^2$

In our consideration of the thermal problem we were interested in the behaviour of the "generation response function" ($g_i^{r*}(\mu_i)$). Here we will be interested in the behaviour of the "net transfer function" given by:

$$h(\mu) = e^*_{nm}(\mu_n,\mu_m) - f^*_{mn}(\mu_m,\mu_n) \qquad \text{(E-8)}$$

Figures (4-3) - (4-5) demonstrate this function for the quadratic case. Figure (4-3) shows $h(\mu)$ as a function of $\left(\frac{\mu_n}{\mu_m}\right)$. If $\mu_m = \mu_n$ then there is no transmission in either direction. As $\left(\frac{\mu_n}{\mu_m}\right)$ decreases from that point it becomes profitable to transmit more and more energy from n to m until, at a ratio $\left(\frac{\mu_n}{\mu_m}\right)=\underline{\rho}$, the exchange reaches its maximum in that direction. On the other hand, if $\left(\frac{\mu_n}{\mu_m}\right)$ rises, the transfer from n to m rises linearly, the energy received at m being reduced by the losses. Figure (4-4) summarises this behaviour in the different sectors of (μ_n,μ_m) space, while Figure (4-5) shows the whole response surface.

Some lines, particularly DC lines of the type installed in the NZED, do not have pure square law losses. We allow the more general loss formula:

$$L_{nm}(e_{nm}) = \alpha_{nm}e^2_{nm} + \beta_{nm}e_{nm} \qquad \text{(E-9)}$$

This is shown in Figure (4-6) with the corresponding "energy received" function, f(e), in Figure (4-7). For this type of line:

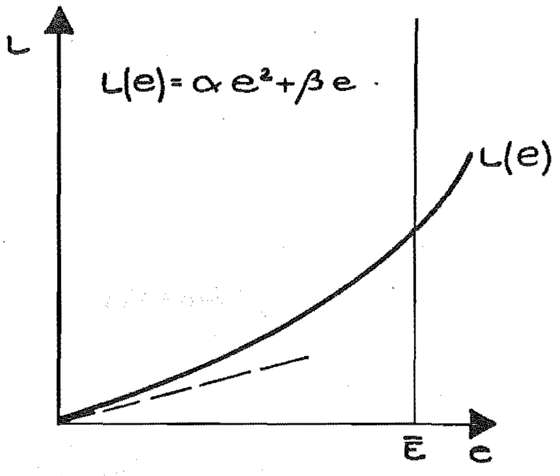


FIG (4-6):
LOSS FUNCTION.

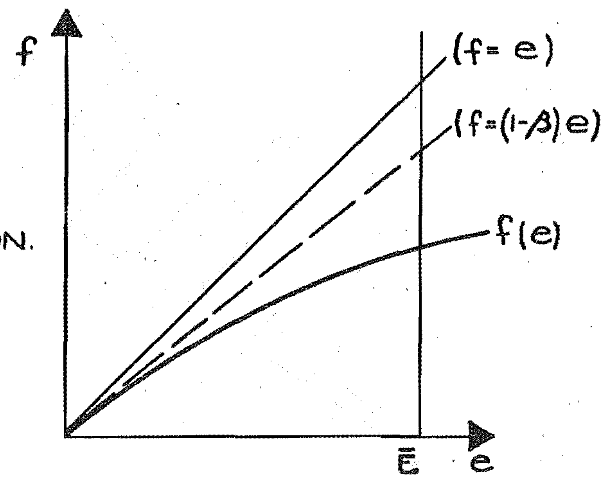


FIG (4-7):
ENERGY RECEIVED FUNCTION.

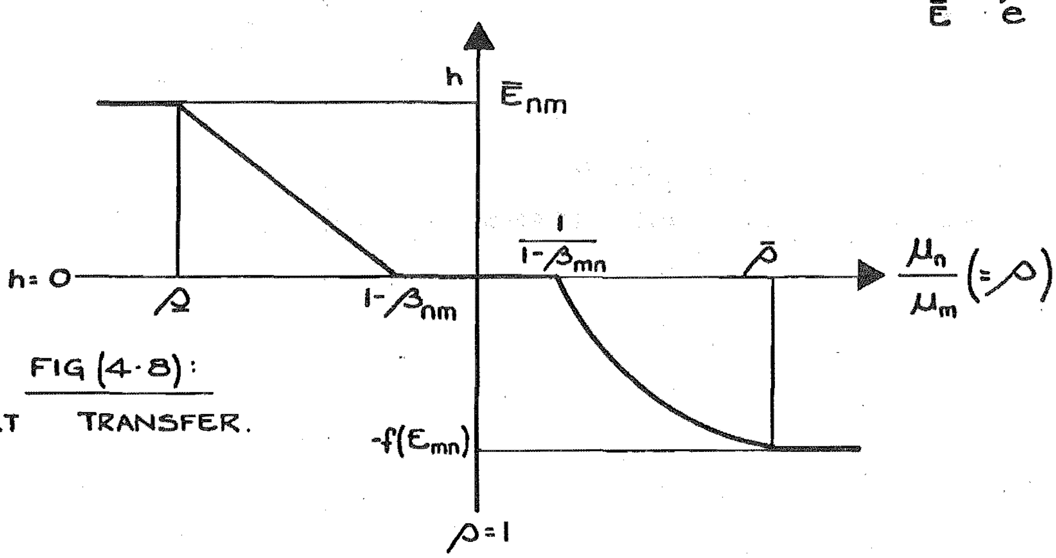


FIG (4-8):
NET TRANSFER.

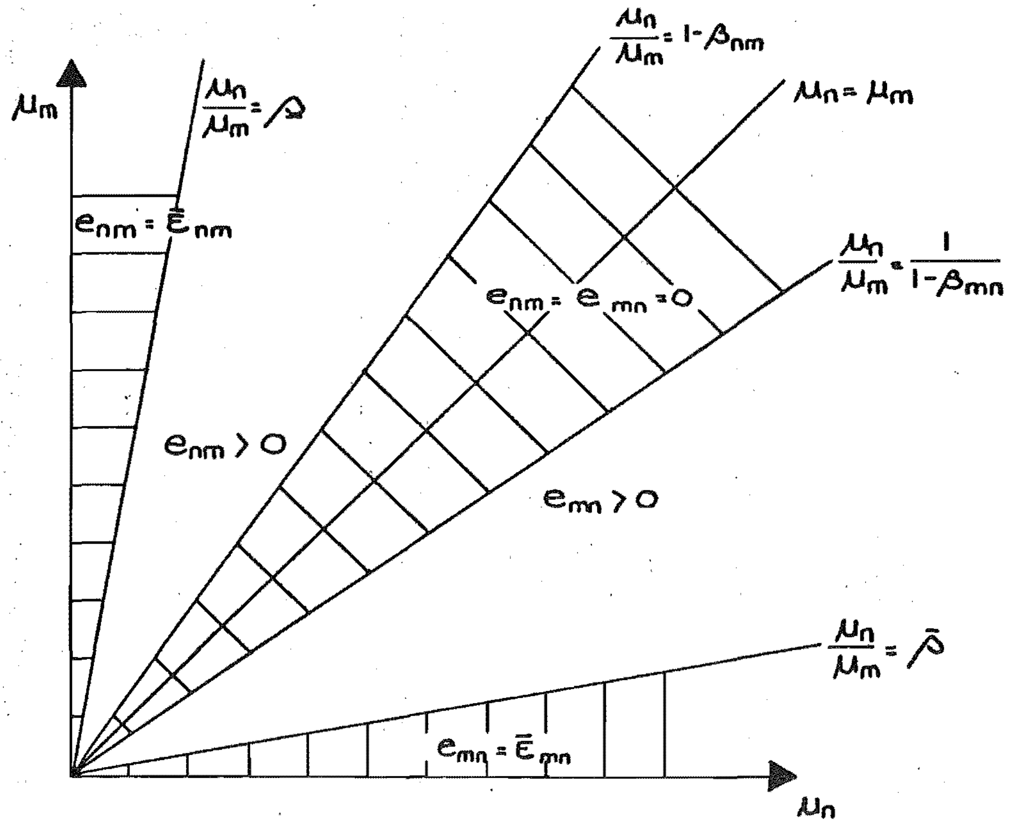


FIG (4-9):

PROJECTION OF NET TRANSFER FUNCTION.

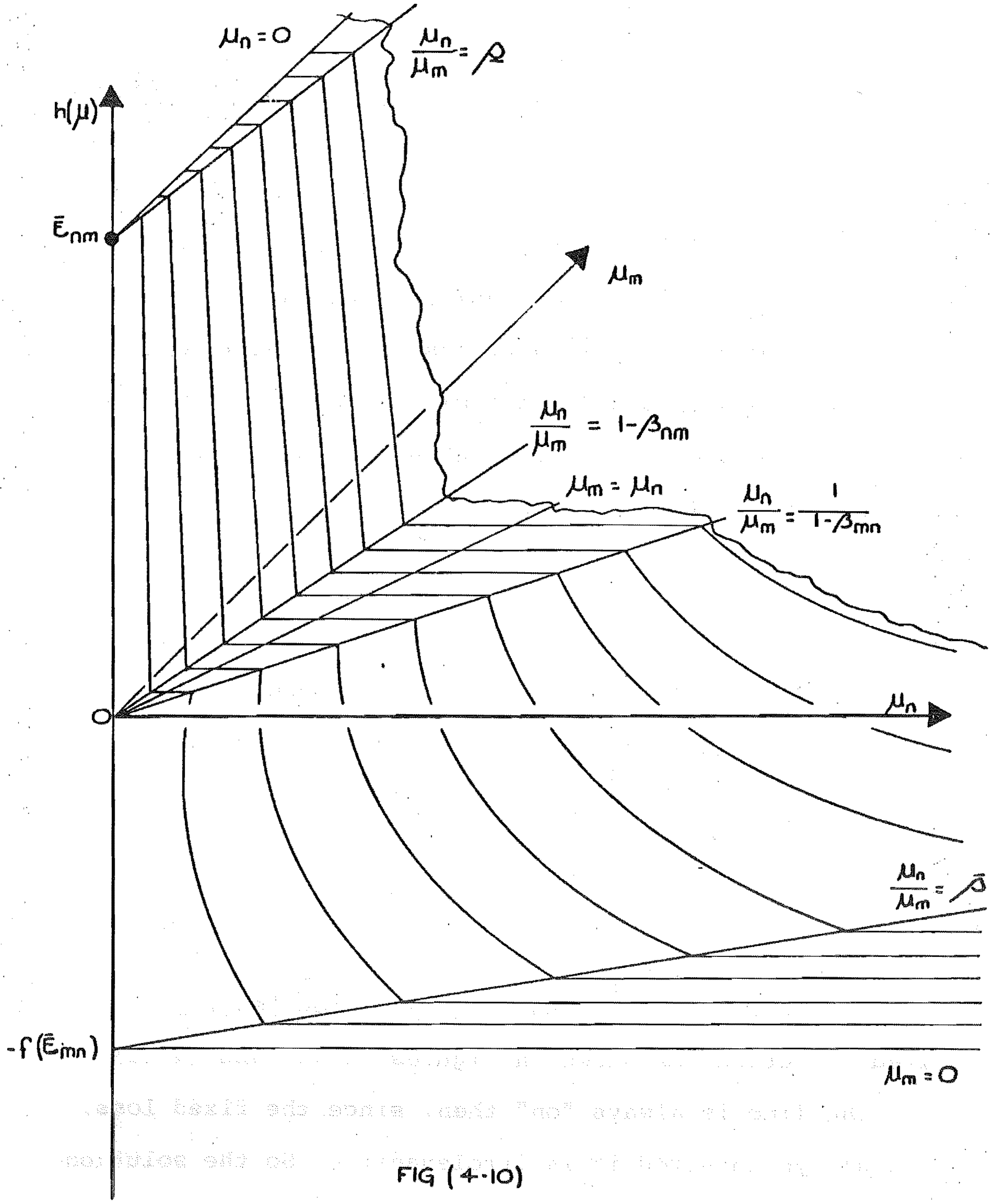


FIG (4-10)

EXCHANGE RESPONSE SURFACE
FOR $L(e) = \alpha e^2 + \beta e$

(E-3) ⇒

$$2\alpha_{nm}\tilde{e}_{nm} + \beta_{nm} = \frac{\mu_m-\mu_n}{\mu_n}$$

$$\Rightarrow \tilde{e}_{nm} = \frac{\mu_m(1-\beta_{nm})-\mu_n}{2\alpha_{nm}\mu_n} \qquad \text{(E-10)}$$

The response surface for such a line is summarised by Figures (4-8) - (4-10). Here, owing to the constant loss factor, β_{nm}, it is unprofitable to transmit in any direction if the difference in prices is too small to cover the minimum marginal loss in that direction, β_{nm}. Thus the response curve has a flat portion when the price ratio $\left(\frac{\mu_n}{\mu_m}\right)$ is in between $(1 - \beta_{nm})$ and $\frac{1}{1-\beta_{mn}}$.

4.3 NON-CONVEX LOSSES.

Lines such as the NZED DC link may consume a certain amount of energy when they are "on" even if no energy is being transmitted. We will deal with a loss function of the form:

$$L_{nm}(e_{nm}) = \begin{cases} \alpha_{nm}e_{nm}^2+\beta_{nm}e_{nm}+\gamma & \text{if } e_{nm} > 0 \\ 0 & \text{if } e_{nm} = 0 \end{cases} \qquad \text{(E-11)}$$

Such a loss function with its associated "energy received" function is shown in Figures (4-11) and (4-12). Now, if the line is always "on" then, since the fixed loss, γ_{nm}, is always incurred it is irrelevant. So the solution to the problem would be identical to that for (E-9).

However we have the option of switching the line "off", in which case we incur no losses. Just as in the thermal

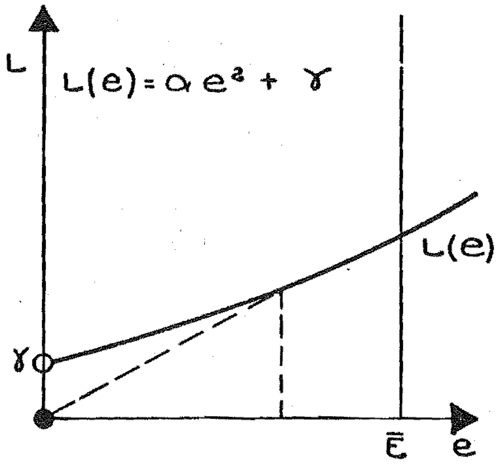


FIG (4·11):
LOSS FUNCTION

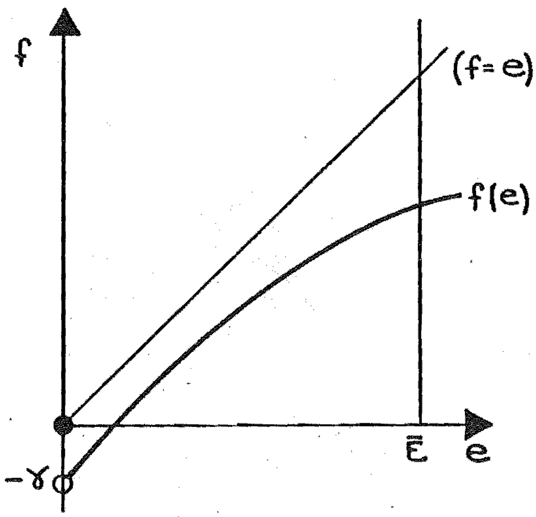


FIG (4·12):
ENERGY RECEIVED FUNCTION

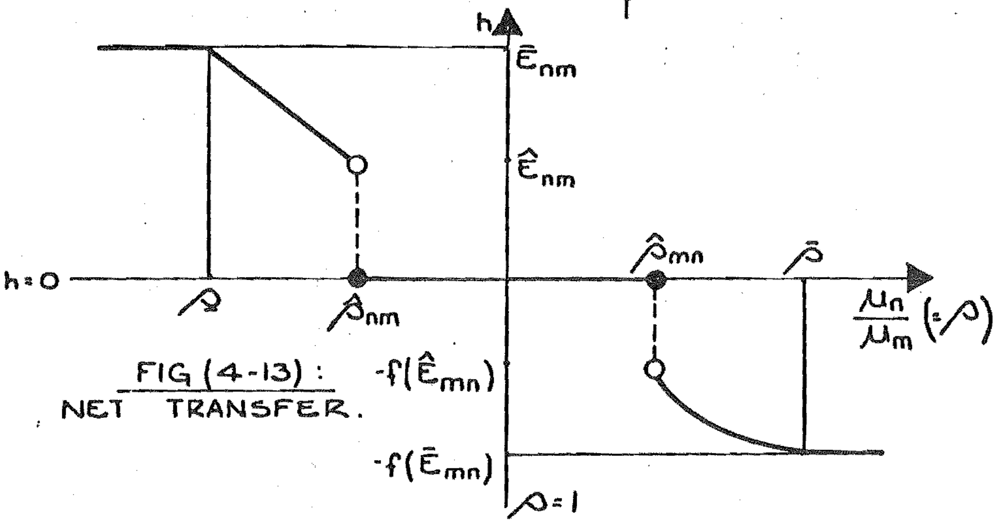


FIG (4-13):
NET TRANSFER.

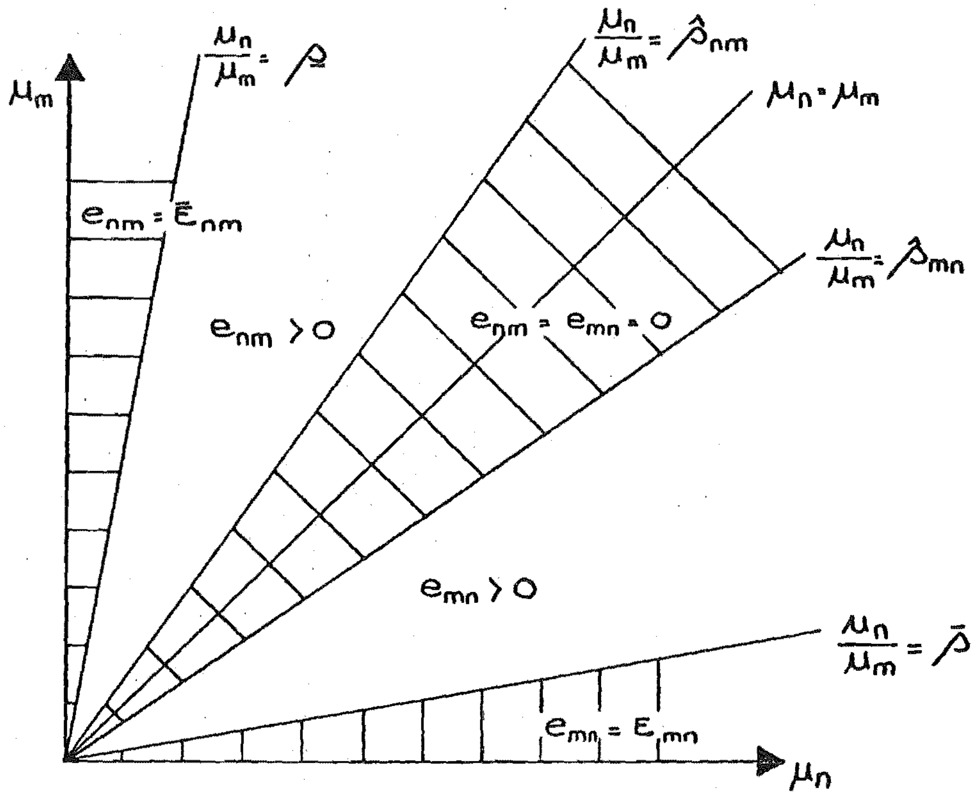


FIG (4.14):
PROJECTION OF NET TRANSFER FUNCTION.

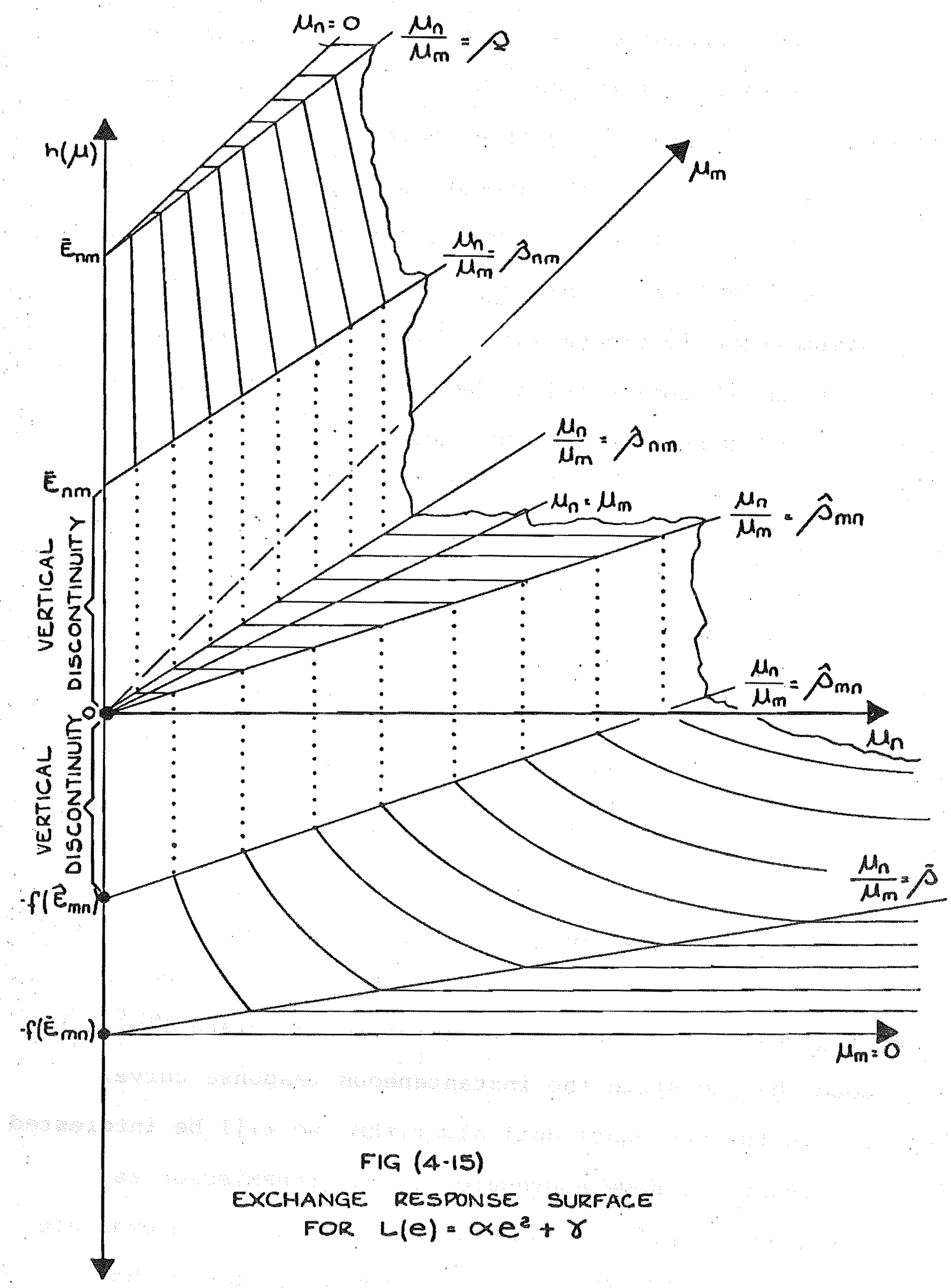


FIG (4-15)
EXCHANGE RESPONSE SURFACE
FOR $L(e) = \alpha e^2 + \gamma$

case, we would not choose to transmit in either direction if the efficiency of the line were less than its maximum in that direction. So we again find ourselves with a discontinuous response curve and a 'duality gap' for our global algorithm. As in that case, we can "patch" the response surface with a very steep portion.

Figures (4-13) - (4-15) summarise the resultant response surface. In drawing these figures we have assumed, for simplicity, that β_{nm} (the linear coefficient) = 0. The minimum economic transmission level, $\hat{E}_{nm}$, is found, just as in the thermal problem, by maximising the average benefit from transmission. This results in:

$$\hat{E}_{nm} = \sqrt{\frac{\gamma_{nm}}{\alpha_{nm}}} \tag{E-12}$$

$\hat{\rho}_{nm}$ is the corresponding price ratio.

4.4 CONCLUSIONS

Here we have shown that the solution of the instantaneous exchange sub-problem is not difficult. The result is a response surface giving the optimal instantaneous transmission between two regions as a function of the "prices", μ, pertaining to those regions. It is probably easier to calculate $e^*(\mu_i^r, \mu_j^r)$ as needed than to store the instantaneous response curve. However, in the aggregate dual algorithm, we will be interested in the response of some aggregate of the transmission as a function of the price parameters λ. So we should evaluate $e^*(\lambda_n, \lambda_m)$ over a grid of λ_n and λ_m. However, noting that

$e^*(\mu_n, \mu_m)$ can be expressed as a function of the ratio of the prices μ_n, μ_m ((E-8)), we should only need to evaluate e^* for a number of ratios of λ_n and λ_m. We expect that this response surface will become progressively smoother as the detail of the μ curve increases, just as did the thermal response curve.

If other costs or losses are incurred by the transmission process, due, for instance, to maintenance or start-up/shut-down procedures, then these can be incorporated in the same way as they were in the thermal model. A similar algorithm to that of [41] could be developed.

CHAPTER 5

THE LOCAL HYDRO PROBLEM

5.1 INTRODUCTION

We are concerned here with the problem of managing the release pattern for a given "valley" so as to maximise the "profit" from its hydro-electric generation (at the prices (μ) supplied by the dual algorithm). We assume here that the inflows F_h are known in advance - Chapter 8 deals with a more realistic stochastic model. This is the most complex of our sub-models, involving, as it does, the interaction between several nodes over the entire planning horizon.

Recall from Section 2.2.3 our problem, PAH_h, (or PH_h), may be stated as:

$$\text{Find } \underset{q_h}{\text{MAX}} \sum_{r=1}^{R} \mu_h^r g_{hH}^r(q_h) \qquad (C\text{-}22'')_h$$

$$\text{Such that:} \quad q_h \varepsilon \mathcal{Q}_h \qquad (C\text{-}15)_h$$

Where, in the aggregate model, the μ prices have been determined from the λ price parameters.

In Section 5.2 we detail the nature of both the objective $(C\text{-}22'')_h$ and the constraints $(C\text{-}15)_h$. Then, in Section 5.3, we aggregate the instants into periods as in the global problem. This results in a long-term sub-model and a set of short-term sub-models. The bulk of this chapter (Sections 5.4 and 5.5) deals with the solution of these problems.

5.2 COMPLETE HYDRO VALLEY MODEL

We deal first with a general representation for the physical setup of a valley.

We concern ourselves here solely with a specific valley h, and so drop this subscript, indexing the nodes within the valley by $j = 1,..,J$ (where index J represents the eventual sink for all flows). These nodes may involve any or all of hydro-electric generation plants, pumping stations or reservoirs of various types. Since there are currently no pumping stations in existence or planned for New Zealand we omit discussion of these here (although see [51])

Each reservoir (j) has, at the end of instant r, volume s_j^r in storage. There are physical limits on the minimum and maximum permissible storage levels and these may be further restricted, at various times of the year, for purposes of flood control or aesthetic appeal. We express these limits by:

$$\underline{S}_j^r \leqslant s_j^r \leqslant \bar{S}_j^r \qquad \text{for all } j = 1,\ldots J-1$$
$$r = 1,\ldots R \qquad \text{(H-1)}$$

(Note that we often loosely refer to the whole set of lower bounds $((\underline{S}_j^r)^{r=1,\ldots R})$ for a reservoir as "the lower constraint". Similarly we refer to "the upper constraint").

We let A_j be the set of all reservoirs immediately upstream from j in the valley. Thus releases from these reservoirs are discharged into reservoir j. We assume that there is only one reservoir immediately below j.

We let F_j^r be the uncontrollable inflow into reservoir j in instant r (assuming here that F_j^r is known in advance), and q_j^r be the controlled release from j in r. There are physical and operational limits on q_j^r expressed by:

$$\underline{Q}_j^r \leqslant q_j^r \leqslant \bar{Q}_j^r \qquad \text{for all } j = 1,\ldots J-1 \quad r = 1,\ldots R \qquad \text{(H-2)}$$

If $i \varepsilon A_j$ then the water released from i may take some time to flow into j. We denote this delay time by w_i (uniquely defined for all $i \varepsilon h$, since there is always only one reservoir immediately downstream from i).

Then we have a water balance equation linking the instants:

$$s_j^r = s_j^{r-1} + F_j^r + \sum_{i \varepsilon A_j} \left(q_i^{(r-w_i)} \right) - q_j^r \qquad \text{for all } j = 1,\ldots J-1 \quad r = 1,\ldots R \qquad \text{(H-3)}$$

Here we have assumed that the instants are sufficiently short so that w_i can be reasonably approximated by an integer. This is relaxed in Section 5.5.2.

We also require that:

$$s_j^o = S_j^o \qquad \text{(H-4)}$$

$$s_j^R = S_j^R \qquad \text{for all } j = 1,\ldots J-1 \qquad \text{(H-5)}$$

This completes our description of the hydraulic interconnections between the reservoirs and we turn our attention to the behaviour of the generating plant.

The output of j at instant r is given by a function, g_j^r, of the release, q_j^r, and (possibly) the volume of water in storage, s_j^r, (in as much as it affects the 'head' at the station).

Thus we have:

$$g_j^r = g_j^r(q_j^r, s_j^r) \qquad \text{for all } j = 1,\dots J-1$$
$$r = 1,\dots R \qquad \text{(H-6)}$$

As discussed in Section 2.1 we will require the concavity of g^r as a function of the independent variables (q_j^p, for all $p=1,\dots,r$; since q_j^p affects s_j^r).

Firstly, generators are typically characterised by concave efficiency curves so that, eventually, as more and more water is released the productivity of release drops off. Although this may not hold for low machine loadings (and hence for various corresponding station output levels) we can overcome this difficulty, just as in the thermal case, by patching the generation curve with straight line segments as in Figure (5-1). Generation in this region is supposed to be achieved by generating at one end point for part of the time and at the other for the remainder. Alternatively, we can fit an approximate generation curve (say a quadratic as in Figure (5-2)). The EDF takes the former approach, the NZED, in their current short-term scheduling model, the latter. Both result in a suitable concave generation function for g_j^r as a function of the release in period r, q_j^r. However g_j^r is not generally a concave function of q_j^p, $p < r$. If we let the station head be given by the function, $H_j^r(s_j^r)$ then it is clear that:

$$\frac{\partial H_j^r}{\partial s_j^r} > 0 \qquad \text{(H-7)}$$

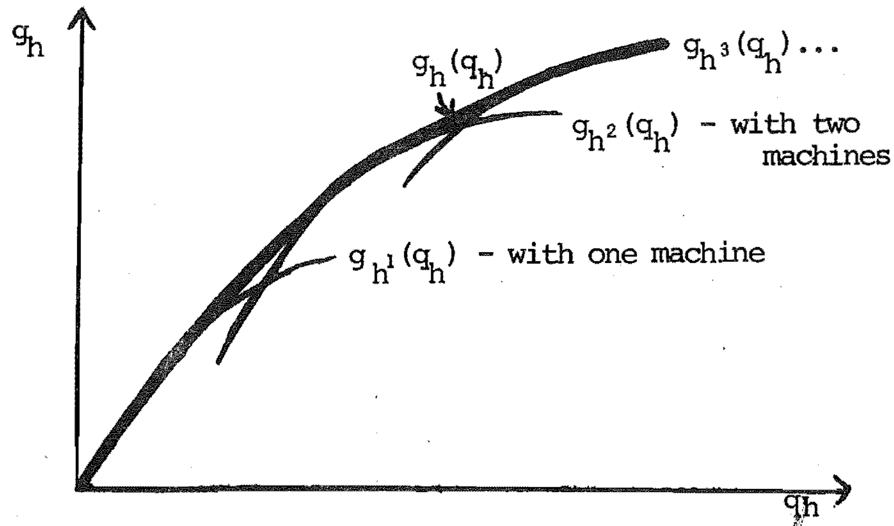


FIGURE (5-1): Concave approximation for g_h (cf. EDF)

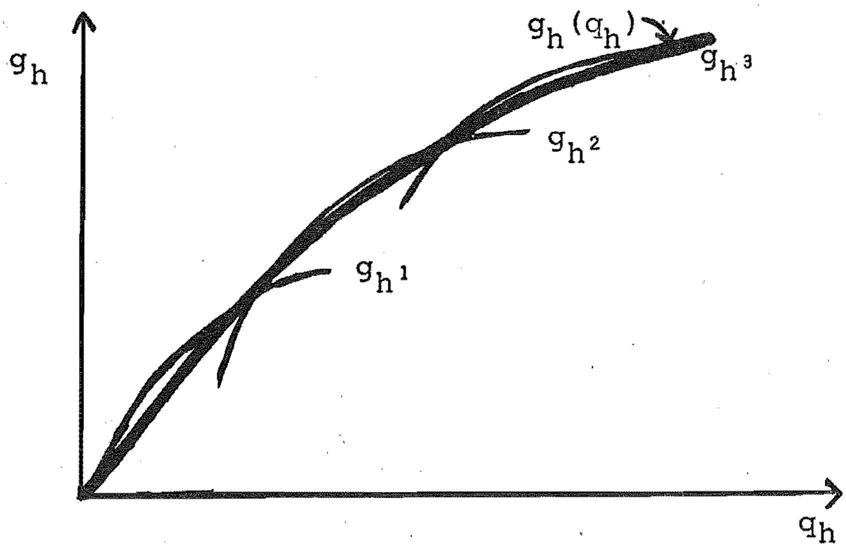


FIGURE (5-2): Concave approximation for g_h (cf. NZED)

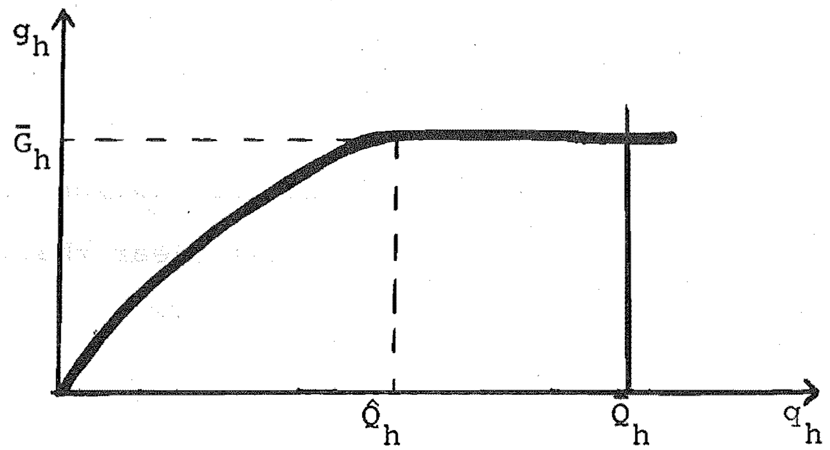


FIGURE (5-3): Interpretation of release limits and:

$$\frac{\partial^2 H_j^r}{\partial (s_j^r)^2} \leqslant 0. \tag{H-8}$$

(With: $\dfrac{\partial^2 H_j^r}{\partial (s_j^r)^2} = 0$ corresponding to reservoir j having vertical sides.) So the head is a concave function of the storage volume. Further, generation is an approximately linear function of head ([13]) and therefore concave in storage volume. Now, from (H-3) and (H-4) we have that:

$$s_j^r = s_j^o + \sum_{p=1}^{r} \left[F_j^p + \sum_{k \varepsilon A_j} \left(q_k^{(p-w_k)} \right) - q_j^p \right] \tag{H-9}$$

So the generation in instant r, g_j^r, is a convex function in the earlier releases, q_j^p, $p < r$. However, this problem is not serious because:

(a) the convexity is very minor by comparison with the concavity of g_j^r as a function of q_j^r. (cf. [25] ,p23, and [54], Section 8, p.14).

(b) As is pointed out in Section 2.2 the head effect itself is not present in any significant New Zealand long-term reservoir. However we will develop an algorithm for short-term reservoir management which allows for variable head.

(c) As in the thermal and exchange sub-models, the global optimisation can proceed provided the "dual objective function, $P(\lambda)$, is convex. The contribution of the hydro sub-model to this function is the (negative of the) "profit" function given by $(C-22)'_h$, so that provided this is concave our global algorithm will work.

We can allow for spill in a number of ways. We have opted here (see Figure (5-3)) for allowing $\bar{Q}_j^r$ to represent the maximum allowable total release from the station.

If $\hat{Q}_j^r(s_j^r)$ is the maximum utilisable release, at storage level s_j^r, then:

$$\frac{\partial g_j^r}{\partial q_j^r} = 0 \quad \text{for all} \quad q_j^r \geqslant \hat{Q}_j^r(s_j^r). \tag{H-10}$$

(Here $\hat{Q}_j^r$ may depend on s_j^r if the station throughput is limited by its generators and a head effect is present.)

Now we can state our problem as:

$$\text{(PAH) Find} \quad \underset{q}{\text{MAX}} \sum_{r=1}^{R} \sum_{j=1}^{J-1} \mu_j^r(\lambda)\left[g_j^r(q_j^r, s_j^r)\right] \tag{H-0}$$

Such that:

$$\underline{S}_j^r \leqslant s_j^r \leqslant \bar{S}_j^r \tag{H-1}$$

$$\underline{Q}_j^r \leqslant q_j^r \leqslant \bar{Q}_j^r \tag{H-2}$$

$$s_j^r = s_j^{r-1} + F_j^r + \sum_{i\varepsilon A_j} \left(q_j^{(r-w_i)}\right) - q_j^r \tag{H-3}$$

$$\text{for all } j = 1, \ldots J-1$$
$$r = 1, \ldots R$$

And:

$$s_j^o = S_j^o \qquad \text{for all } j = 1, \ldots J-1 \tag{H-4}$$

$$s_j^R = S_j^R \qquad \text{for all } j = 1, \ldots J-1 \tag{H-5}$$

This problem is similar to that faced by the EDF in their P2 program (see Appendix F2). However the use of "instants" rather than "periods" complicates any attempt to decompose this problem across time by assigning multipliers to the constraints (H-1) or (H-3). This is so because temporal separability is destroyed by the fact that water released by one station in one instant may result in generation at stations downstream for several instants thereafter. Rather than detailing such a decomposition we proceed, in Section 5.3, to aggregate this model in the same way in which we aggregated the complete model.

5.3 APPROXIMATE SOLUTIONS VIA AGGREGATION

Just as in the complete model we will divide our model into two: an aggregate "long-term" model, and a "short-term" sub-model. To do this we aggregate the instants into periods compatible with those of the aggregated complete model. Thus we will be able to deal in our long-term model with the representative prices, λ, utilising the detailed μ prices only in the short-term sub-models. In using these approximate μ prices derived from λ we are again assuming that the solutions so produced will be satisfactory in the national problem (i.e., Assumption AA).

We will also require that, in the long-term model, only certain approximations to the constraints (H-1) and (H-2) be satisfied, specifically that the storage constraints (H-1) be met at the end of each period. This requirement involves the following assumption:

(HAI) That if, in the solution of the long-term model, the constraints (H-1) are met at the beginning and end of a period then the short-term model will be able to manage the release pattern so as to satisfy those constraints during the period.

We now make a distinction between two types of reservoir: long-term controllable reservoirs, which can have significant variations from week to week in absolute storage volume, and short-term controllable reservoirs which cannot. We let K be the set of all long-term controllable reservoirs and J be the set of all short-term controllable reservoirs, indexing these sets by k and j respectively. The significant feature of a short-term controllable reservoir is that we can expect that all inflows received during any period are released during that period. So we make the following assumption:

(HAII) For all $j \varepsilon J$, the following approximation is sufficiently accurate:

$$\sum_{r \varepsilon t}\left(F_j^r + \sum_{i \varepsilon A_j} q_i^{r-w_i} - q_j^r\right) \approx 0 \quad \text{for all } t = 1,\ldots T \tag{H-11}$$

Hence we have:

$$s_j^t \approx s_j^{t-1} \qquad \text{for all } t = 1,\ldots T \tag{H-12}$$

(since s_j^{t-1} is the initial level of reservoir j).

By choosing our periods appropriately we can improve our model with respect to its conformity to this assumption.

In particular, if we end each period on Sunday night then the volume of water from the previous period's discretionary release, $(q_h^{t-1} - \underline{Q}_h^{t-1})$, remaining in the system will be minimised.

We wish to be able to deal, in the long-term problem, with a network consisting only of long-term controllable reservoirs. For this network the set $\hat{AK}_j$ plays the role of the set A_j in the original model. Similarly, for the short-term model, we wish to deal with a network consisting only of short-term controllable reservoirs, defining $\hat{AJ}_j$ to play the role of A_j. More formally, let:

$$AJ_j^o = \{j\} \qquad \text{for all } j\varepsilon h. \tag{H-13}$$

Recursively define:

$$AJ_j^i = \bigcup_{k\varepsilon AJ_j^{i-1}} (A_k \backslash K) \tag{H-14}$$

$$AK_j^i = \bigcup_{k\varepsilon AJ_j^{i-1}} (A_k \cap K) \tag{H-15}$$

Then: $$\hat{AK}_j = \bigcup_{i>0} AK_j^i \tag{H-16}$$

is the set of all long-term controllable reservoirs, upstream from reservoir j, whose releases in any period will, after possibly passing through several short-term reservoirs, flow into reservoir j in the same period (since there is no other long term controllable reservoir between them and reservoir j on the chain).

Also:

$$\hat{AJ}_j = \bigcup_{i>0} AJ_j^i \tag{H-17}$$

is the set of all non long-term controllable reservoirs upstream from j, the inflows into which, in any period, must flow down into reservoir j during that period since there is no long-term controllable reservoir between them and j on the chain.

The practical application of these definitions is quite trivial as is demonstrated by the example in Figure (5-4).

Let us characterise period t by:

$$t = (\underline{r}^t, \ldots \bar{r}^t) \tag{H-18}$$

Then, for each $k \varepsilon K \setminus \{J\}$, we define:

$$q_k^t = \sum_{r \varepsilon t} q_k^r \qquad \text{for all } t=1,\ldots T \tag{H-19}$$

$$s_k^t = s_k^{\bar{r}^t} \qquad \text{for all } t=1,\ldots T \tag{H-20}$$

$$F_k^t = \sum_{r \varepsilon t} \left[F_k^r + \sum_{j \varepsilon \hat{AJ}_k} F_j^r \right] \qquad \text{for all } t=1,\ldots T \tag{H-21}$$

So we have that:

$$s_k^t = s_k^{t-1} + F_k^t - q_k^t + \sum_{\ell \varepsilon \hat{AK}_k} q_\ell^t \qquad \text{for all } t = 1,\ldots T \tag{H-22}$$

And we require that:

$$\underline{s}_k^t \leqslant s_k^t \leqslant \bar{s}_k^t \qquad \text{for all } t = 1,\ldots T \tag{H-23}$$

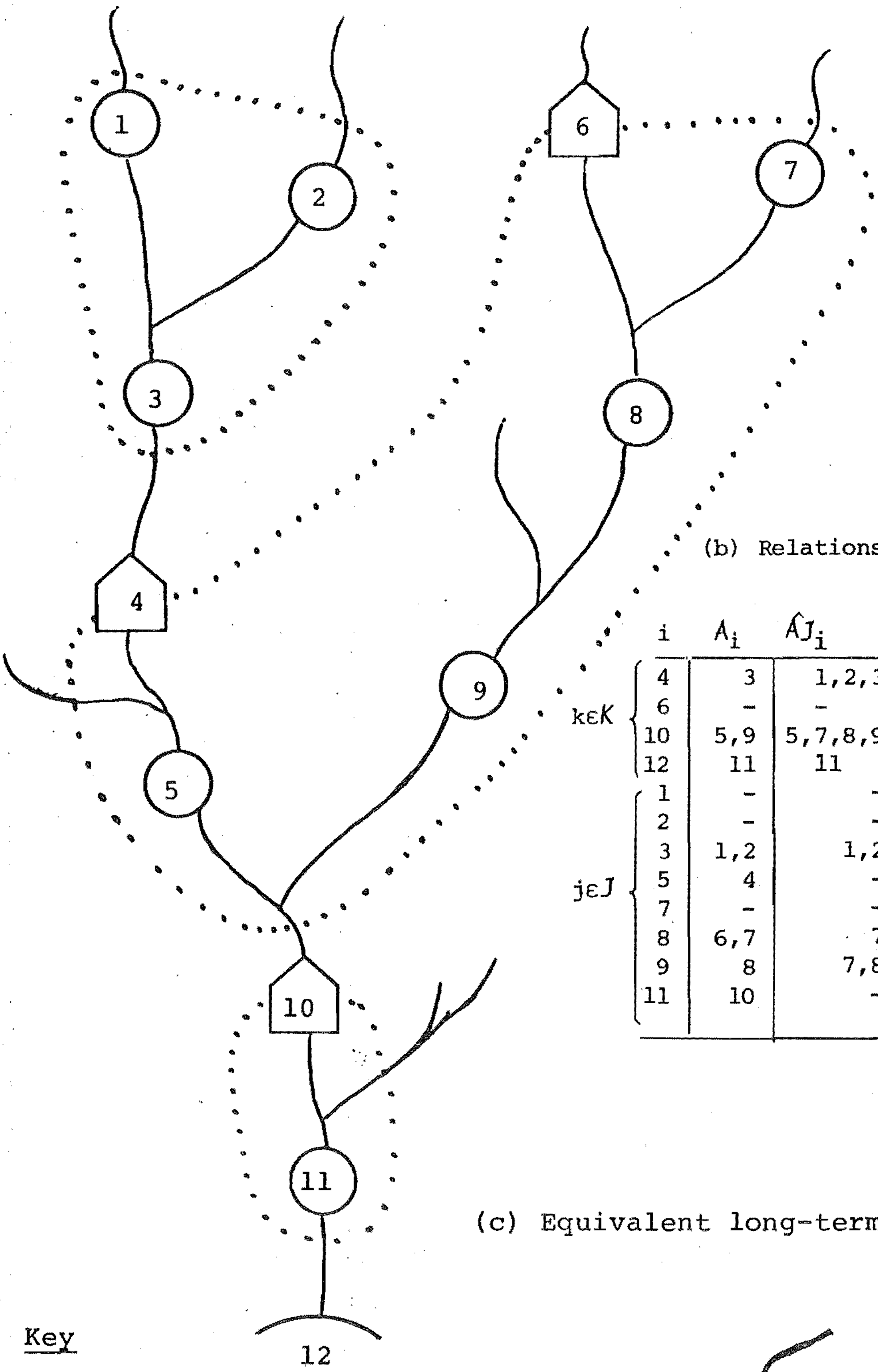
where:

$$\underline{s}_k^t = \underline{s}_k^{\bar{r}^t} \tag{H-24}$$

$$\bar{s}_k^t = \bar{s}_k^{\bar{r}^t} \tag{H-25}$$

We also require that:

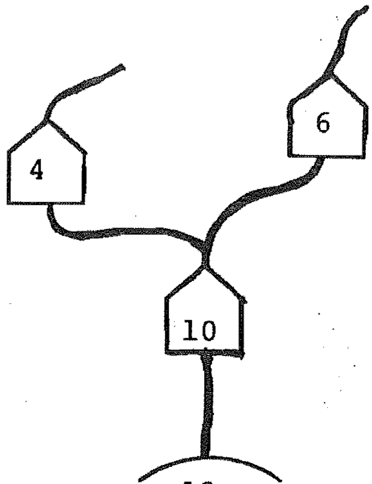
(a) Complete valley system



(b) Relationship table

	i	A_i	$\hat{J}_i$	$\hat{K}_i$
$k \epsilon K$	4	3	1,2,3	-
	6	-	-	-
	10	5,9	5,7,8,9	4,6
	12	11	11	10
$j \epsilon J$	1	-	-	-
	2	-	-	-
	3	1,2	1,2	-
	5	4	-	4
	7	-	-	-
	8	6,7	7	6
	9	8	7,8	6
	11	10	-	10

(c) Equivalent long-term system



Key

: rivers

: Short-term sub-systems

: Long-term reservoir

: Short-term reservoir

FIGURE (5-4): Hydro valley

$$\underline{Q}_k^t \leqslant q_k^t \leqslant \bar{Q}_k^t \qquad \text{for all } t = 1,\ldots T \qquad \text{(H-26)}$$

where:

$$\underline{Q}_k^t = \sum_{r\varepsilon t} \underline{Q}_k^r \qquad \text{(H-27)}$$

$$\bar{Q}_k^t = \sum_{r\varepsilon t} \bar{Q}_k^r \qquad \text{(H-28)}$$

We can now restate our problem (PAH) as:

$$\text{Find } \underset{q}{\text{MAX}} \sum_{t=1}^{T} \sum_{k\varepsilon K} \left\{ \sum_{r\varepsilon t} \mu^r \left[\sum_{j\varepsilon \hat{AJ}_k \cup \hat{AK}_k} \left(g_j^r(q_j^r, s_j^r) \right) \right] \right\} \qquad \text{(H-29)} \quad \text{(cf. (H-0))}$$

Such that, for each $k\varepsilon K$, we have as initial and final conditions

$$s_k^o = S_k^o \qquad \text{(H-30)}$$

$$s_k^T = S_k^T \qquad \text{(H-31)}$$

And, for each $k\varepsilon K$, $t=1,\ldots T$, the following "long-term constraints" hold:

$$\underline{S}_k^t \leqslant s_k^t \leqslant \bar{S}_k^t \qquad \text{(H-23)}$$

$$\underline{Q}_k^t \leqslant q_k^t \leqslant \bar{Q}_k^t \qquad \text{(H-26)}$$

$$s_k^t = s_k^{t-1} + F_k^t - q_k^t + \sum_{\ell\varepsilon \hat{AK}_k} (q_\ell^t) \qquad \text{(H-22)}$$

$$s_j^t = S_j^t \text{ (fixed)} \qquad \text{For each } j\varepsilon\ \hat{AJ}_k \qquad \text{(H-32)}$$

And also, for all $j\varepsilon AJ_k$, the following "short-term constraints":

$$s_j^r = s_j^{r-1} + F_j^r - q_j^r + \sum_{i\varepsilon A_j} q_i^{r-w_i} \qquad \text{(H-33)}$$

$$\underline{Q}_j^r \leqslant q_j^r \leqslant \bar{Q}_j^r \qquad \text{(H-34)}$$

$$\underline{S}_j^r \leqslant s_j^r \leqslant \bar{S}_j^r \qquad \text{(H-35)}$$

If we define our short-term hydro scheduling problem, $PASH_k$, as:

$$\text{Find} \quad \underset{q}{MAX} \quad \sum_{r\varepsilon t} \mu^r \left[\sum_{j\varepsilon(A\hat{J}_k \cup A\hat{K}_k)} g_j^r(q_j^r, s_j^r) \right] \qquad \text{(H-36)} \quad \text{(cf.(H-29))}$$

Such that, for each $j\varepsilon A\hat{J}_k$ (i.e., for each short-term reservoir in the sub-system):

$$s_j^{\bar{r}^t} = s_j^t = S_j^t \quad \text{(fixed)} \qquad \text{(H-37)}$$

and:

$$s_j^{(\underline{r}^t - 1)} = s_j^{\bar{r}^{(t-1)}} = s_j^{t-1} = S_j^{t-1} \quad \text{(fixed)} \qquad \text{(H-38)}$$

And, for all $r\varepsilon t$:

$$s_j^r = s_j^{r-1} + F_j^r - q_j^r + \sum_{i\varepsilon A_j} q_i^{r-w_i} \qquad \text{((H-33)=)} \quad \text{(H-39)}$$

$$\underline{S}_j^r \leqslant s_j^r \leqslant \bar{S}_j^r \qquad \text{((H-35)=)} \quad \text{(H-40)}$$

$$\underline{Q}_j^r \leqslant q_j^r \leqslant \bar{Q}_j^r \qquad \text{((H-34)=)} \quad \text{(H-41)}$$

Also, for each $\ell\varepsilon A\hat{K}_k$ (i.e., for each long-term reservoir in the sub-system).

$$\sum_{r\varepsilon t} q_\ell^r = Q_\ell^t \quad \text{(given)} \qquad \text{(H-42)}$$

$$s_\ell^{\underline{r}^t} = S_\ell^{t-1} \quad \text{(given)} \left(\Rightarrow s_\ell^{\bar{r}^t} = S_\ell^t \right) \qquad \text{(H-43)}$$

Note that, for $j\varepsilon J$, S_j^t must be set at a level which will allow reasonable operation in the next period. We let $(\underset{\sim}{q}, \underset{\sim}{s})_k = (q_\ell, s_\ell)_{\ell\varepsilon A\hat{K}_k}$, i.e. the vector of releases from the long-term reservoirs involved in problem $PASH_k$. Then, if we can solve $PASH_k$, we can find the optimal generation:

$$g_j^{r*}(\underset{\sim}{q}, \underset{\sim}{s})_k \quad \text{for all } r\varepsilon t, j\varepsilon A\hat{J}_k \cup A\hat{K}_k \,. \qquad \text{(H-44)}$$

The profit from g_j^{r*} will be: $\mu^r g_j^{r*}$.

So we can define the total generation from the release (and storage) pattern $(\underset{\sim}{q},\underset{\sim}{s})_k$ to be:

$$g_k^{r*}(\underset{\sim}{q},\underset{\sim}{s})_k = \sum_{j\varepsilon \hat{A}J_k \cup \hat{A}K_k} g_j^{r*}(\underset{\sim}{q},\underset{\sim}{s})_k \quad \text{for all } r\varepsilon t. \tag{H-45}$$

The profit from this total generation is:

$$\pi_k^{t*}(\underset{\sim}{q},\underset{\sim}{s})_k = \sum_{r\varepsilon t} \mu^r g_k^{r*}(\underset{\sim}{q},\underset{\sim}{s})_k \tag{H-46}$$

Now we can define our long-term hydro scheduling problem, PALH, as:

$$\text{Find } \underset{q}{\text{MAX}} \sum_{t=1}^{T} \sum_{k\varepsilon K} \pi_k^{t*}(\underset{\sim}{q},\underset{\sim}{s})_k \tag{H-47}$$

(cf. (H-29))

Such that, for each $k\varepsilon K\backslash\{J\}$:

$$s_k^o = S_k^o \qquad ((\text{H-30})=) \tag{H-48}$$

$$s_k^T = S_k^T \qquad ((\text{H-31})=) \tag{H-49}$$

For all $t = 1,\ldots T$:

$$s_k^t = s_k^{t-1} + F_k^t + \sum_{\ell\varepsilon \hat{A}K_k} (q_\ell^t) - q_k^t \qquad ((\text{H-22})=) \tag{H-50}$$

$$\underline{S}_k^t \leqslant s_k^t \leqslant \bar{S}_k^t \qquad ((\text{H-23})=) \tag{H-51}$$

$$\underline{Q}_k^t \leqslant q_k^t \leqslant \bar{Q}_k^t \qquad ((\text{H-26})=) \tag{H-52}$$

Now, apart from the explicit form of the profit expression, (H-46), in the objective, this problem is in a form identical to that studied by the EDF in their GRAF model ([1]) or to a deterministic version of that solved by P2 in SGEP.

(See Appendix A). We turn our attention in the next section to its solution. In the following section we deal with the solution of the local short-term scheduling problem, PASH.

5.4 THE LONG-TERM HYDRO PROBLEM.

5.4.1 Introduction

We consider here the solution of problem PALH - the determination of a sequence of releases for each long-term controllable reservoir in a river system so as to maximise the benefit from hydro generation. This model assumes a given set of inflows and (λ) prices, and also that the releases will be utilised as specified by the short-term scheduling model. We will first look at suitable algorithms for river systems involving only one long-term controllable reservoir then consider appropriate generalisations for multiple reservoir systems.

If we have only one long-term controllable reservoir, then we can use some form of dynamic programming, or some form of non-linear programming, or a specialised method of some kind. A deterministic dynamic programming formulation was put forward by Little ([32]) and various studies since have made use of this method. A (stochastic) dynamic programming model of the NZED system is described in [16]. Rosentahl has proposed a non-linear network flow formulation for application to the multi-reservoir TVA system ([54]), and in the next section we propose a rather different network flow formulation for application to the multi-reservoir short-term scheduling problem. Either of these could be adapted for this long-term problem.

The EDF have applied non-linear programming/optimal control techniques of various kinds. Their original model used a specialised trajectory method or "shooting method" described below. A later model used an optimal control approach. We have adopted the former method.

We first outline a simplified version of the EDF optimal control model which is more fully described in Appendix A. We use this model to derive the properties of the optimal trajectory required by the shooting method which is then described.

5.4.2 A Lagrangian Approach

First, let us restate our problem, PALH, for the single reservoir case with no head effect in the top reservoir:

$$\text{Find} \quad \underset{q}{\text{MAX}} \sum_{t=1}^{T} \pi^{t*}(q^t) \qquad \text{("profits")} \qquad (\text{H-}47')$$

Such that:

$$s^o = S^o \qquad \text{(initial storage fixed)} \qquad (\text{H-}48')$$

$$s^T = S^T \qquad \text{(final storage fixed)} \qquad (\text{H-}49')$$

For all $t = 1,\dots T$

$$s^t = s^{t-1} + F^t - q^t \qquad \text{(flows balance)} \qquad (\text{H-}50')$$

$$\underline{S}^t \leqslant s^t \leqslant \bar{S}^t \qquad \text{(storage is feasible)} \qquad (\text{H-}51')$$

$$\underline{Q}^t \leqslant q^t \leqslant \bar{Q}^t \qquad \text{(releases feasible)} \qquad (\text{H-}52')$$

Firstly we substitute (H-50′) and (H-48′) into (H-51′) and (H-49′) and so eliminate the storage (s) as a variable, getting:

$$\underline{s}^t \leqslant s^o + \sum_{r=1}^{t} (F^r - q^r) \leqslant \bar{s}^t \qquad \text{(H-51'')}$$

$$\Rightarrow s^o + \sum_{r=1}^{t} F^r - \bar{s}^t \leqslant \sum_{r=1}^{t} q^r \leqslant s^o + \sum_{r=1}^{t} F^r - \underline{s}^t \qquad \text{(H-53)}$$

and:

$$\sum_{r=1}^{T} q^r = s^o + \sum_{r=1}^{T} F^r - s^T \qquad \text{(H-54)}$$

Now we can form a Lagrangian:

$$\begin{aligned} \mathcal{L}_H(q,\delta,\gamma,\sigma) = \sum_{t=1}^{T} \Big[& \pi^{t*}(q^t) \\ & - \gamma^t(\underline{s}^t - \sum_{r=1}^{t} F^r - s^o + \sum_{r=1}^{t} q^r) \\ & + \delta^t(\bar{s}^t - \sum_{r=1} F^r - s^o + \sum_{r=1} q^r) \Big] \\ & - \sigma^T(s^T - \sum_{t=1}^{T} F^t - s^o + \sum_{t=1}^{T} q^t) \end{aligned} \qquad \text{(H-55)}$$

Here we can think of σ^T as the marginal value of water in storage at time T while the multipliers γ and δ penalise violations of the bounds.

This problem is very similar in form to the original problem, PC. The feasible region is clearly convex, as are the constraints, (H-52′), so that, provided the profit function π supplied by PASH is concave, we can again apply Karlin's Theorem. Thus we could again apply an iterative scheme whereby a dual problem adjusts the multipliers (γ, δ, σ) and a Lagrangian problem determines the optimal system response.

The Lagrangian problem, PALH′, could be stated as:

$$\text{Find } \underset{q}{\text{MAX}} \sum_{t=1}^{T} \left[\pi^{t*}(q^t) - \left(\sum_{r=1}^{t} q^r \right) (\gamma^t - \delta^t) \right] - \sigma^T \sum_{t=1}^{T} q^t \qquad \text{(H-55′)}$$

Such that (H-52′) holds.

Here, since for any iteration of the dual problem the multipliers (γ, δ, σ) are fixed, we can ignore the remainder of $\mathcal{L}_H$.

Now, rearranging (H-55′), we get:

$$\text{(H-55′)} = \sum_{t=1}^{T} \left[\pi^{t*}(q^t) - \psi^t q^t \right] \qquad \text{(H-56)}$$

Where: $$\psi^t = \sigma^T + \sum_{r=t}^{T} (\gamma^r - \delta^r) \qquad \text{(H-57)}$$

Here ψ^t can be thought of as the marginal value of water in storage at the end of period t.

Now (H-56) is temporally separable and so is the constraint set (H-52′). So we can break PALH′ into T one-period sub-problems PALH′^{t}:

$$\text{Find } \underset{q^t}{\text{MAX}} \; \pi^{t*}(q^t) - \psi^t q^t \qquad \text{(H-56′)}^t$$

$$\text{Such that: } \underline{Q}^t \leqslant q^t \leqslant \bar{Q}^t \qquad \text{(H-52′)}^t$$

Provided that the function π^{t^*} as determined by PASH is concave (and differentiable), this problem $(PALH')^t$ can easily be solved by finding $\tilde{q}$ such that:

$$\left.\frac{\partial \pi^{t^*}(q^t)}{\partial q^t}\right|_{\tilde{q}} = \psi^t \tag{H-58}$$

Then setting:

$$q^{t^*}(\psi^t) = \text{MAX}\{\text{MIN}\{\tilde{q},\bar{Q}^t\},\underline{Q}^t\} \tag{H-59}$$

In economic terms we are equating the marginal value of water released in period t with the marginal value of water stored at the end of period t. Equivalently, we may think of a "manager" buying water for current use from the stock in storage. Then (H-58) expresses the standard economic formula for maximum profit production - equating marginal revenue with marginal cost (ψ).

A model of this type has, in fact, been implemented by the EDF. We have, however, implemented their earlier 'trajectory method' which seems quite appropriate for our problem.

5.4.3 The Trajectory Method

This method utilises several known mathematical properties of the optimal trajectory to search for this optimum. The method is presented in [1] without reference to the Lagrangian approach. We will derive the required properties from the solution to the Lagrangian problem just discussed. These properties guarantee the optimality of our trajectory method under rather more general conditions than those of [1].

Let us suppose that we have found the optimal multipliers ($\tilde{\delta},\tilde{\gamma},\tilde{\sigma}$ and hence $\tilde{\psi}$) and the corresponding trajectory $s^*(\tilde{\psi})$. Now we will have (by the saddle point orthogonality condition (e.g.[30], Theorem 8.1)):

$$\underline{s}^t < s^{t*}(\psi) < \bar{s}^t \Rightarrow \tilde{\delta}^t = \tilde{\gamma}^t = 0 \qquad \text{(H-60)}$$

Hence, if the optimal trajectory is not constrained between period r and period t, then, from (H-57), their water values are equal, i.e.:

$$\underline{s}^p < s^{p*}(\tilde{\psi}) < \bar{s}^p \quad \text{for all } p \varepsilon (r,t-1)$$
$$\Rightarrow \tilde{\psi}^r = \tilde{\psi}^t \qquad \text{(H-61)}$$

$$\left[\Rightarrow \frac{\partial \pi^{r*}(q^r)}{\partial q^r} \Bigg|_{\tilde{q}(\tilde{\psi}^t)} = \frac{\partial \pi^{t*}(q^t)}{\partial q^t} \Bigg|_{\tilde{q}(\tilde{\psi}^t)} \qquad \text{(H-62)} \right]$$

So that, if it were not for the constraints, the optimum would involve equating the marginal value of production in all periods.

More generally, we can utilise (H-58)(with (H-48$'$)) to determine, for any water value $\hat{\psi}$, a trial trajectory $s(\hat{\psi})$. Where:

$$s^{t*}(\hat{\psi}) = s^{t-1*}(\hat{\psi}) + F^t - q^{t*}(\hat{\psi}) \qquad \text{(H-63)}$$

This trajectory would be the "optimal" trajectory for water value $\hat{\psi}$ if there were no constraints, (H-51).

Now, if the optimal trajectory corresponding to $(\tilde{\gamma},\tilde{\delta},\tilde{\sigma})$ is unconstrained, then we will have:

$$\tilde{\gamma}^t = \tilde{\delta}^t = 0 \qquad \text{for all } t = 1,\ldots T \qquad \text{(H-64)}$$

so that:

$$\tilde{\psi}^t = \tilde{\sigma}^T \qquad \text{for all } t = 1,\ldots T \qquad \text{(H-65)}$$

On the other hand, if, for instance, the upper storage constraint is active in period t. Then:

$$\tilde{\delta}^t > 0 \qquad \text{(H-66)}$$

So: $$\tilde{\psi}^r > \tilde{\psi}^{t-} \qquad \text{for all } r \geqslant t,\ r < \bar{t} \qquad \text{(H-67)}$$

Where $\tilde{\psi}^{t-}$ is the water value for the trajectory arc preceding t and $\bar{t}$ is the first period, after t, in which the optimal trajectory is constrained below.

$$\bar{t} = \mathrm{MIN}\{T, r \,\big|\, r > t \text{ and } s^{r*}(\tilde{\psi}) = \underline{s}^r\} \qquad \text{(H-68)}$$

Now, for any $r \varepsilon [t,\bar{t})$:

$$\bar{s}^r \geqslant s^{r*}(\tilde{\psi}) = s^{t*}(\tilde{\psi}) + \sum_{p=t}^{r} \left[F^p - q^{p*}(\tilde{\psi}^p)\right]$$

$$\geqslant s^{t*}(\tilde{\psi}) + \sum_{p=t}^{r} \left[F^p - q^{p*}(\tilde{\psi}^{t-})\right] \qquad \text{(H-69)}$$

i.e. $$s^{t*}(\tilde{\psi}) + \sum_{p=t}^{r} \left[F^p - q^{p*}(\tilde{\psi}^{t-})\right] \leqslant \bar{s}^r \qquad \text{(H-69')}$$

for all $r \varepsilon [t,\bar{t})$

(since (H-67) $\Rightarrow q^{p*}(\tilde{\psi}^{t-1}) > q^{p*}(\tilde{\psi}^r)$ for all $r \varepsilon [t,\bar{t})$ by the concavity of π, via (H-58)).

This condition is equivalent, in our discrete time framework, to the "tangency" condition of [1].

This may be seen as follows: suppose that we were to form a trial trajectory with water value $\tilde{\psi}^{t-}$, as a continuation of the optimal trajectory arc ending at t. Then it would just graze the "upper storage constraint" in period t, never rising above it subsequently (unless a lower constraint is reached first). Heuristically, if a reservoir manager expects that the next constraint he encounters will be an upper constraint, he sets a water value low enough to ensure that, when his valley is managed for constant marginal profit (equal to that value), the storage trajectory will just touch the constraint. After this period he will raise his water value, keeping the reservoir level up so as to avoid running out of water.

A similar 'tangency' condition applies to the lower constraint. Also we have the following 'principle of optimality'.

"If $s^*(\tilde{\psi})$ is the optimal trajectory then $(s^{r*}(\tilde{\psi}))_{r=t}^{T}$ is the optimal trajectory from $s^{t*}(\tilde{\psi})$), at t, to S^T at T."

For convenience we will suppose that the profit functions, π, are strictly concave, so that the optimal release q^* determined by (H-58) and (H-59) is unique. Now we may use the known properties of the, unique, optimal trajectory to construct it. Such an algorithm may be simply stated:

(1) Set: $\underline{t} = 0$

(2) Choose $\psi^{\underline{t}} > 0$

(3) Form a trial trajectory $s^{t*}(\psi^{\underline{t}})$ using (H-63).

(4) (a) IF the optimal trajectory, from $\underline{t}$ on, is not clearly constrained then GO TO (5)

ELSE DO (b) and (c).

(b) Adjust $\psi^{\underline{t}}$ until the trajectory $s^{*}(\psi^{\underline{t}})$ is tangential to the first constraint it meets (in period $\bar{t}$).

The arc from $\underline{t}$ to $\bar{t}$ is now optimal so...

(c) Let $\underline{t} = \bar{t}$, GO TO (2)

(5) IF $s^{T*}(\psi^{\underline{t}}) = S^{T*}$ THEN STOP,

ELSE adjust $\psi^{\underline{t}}$ so as to bring $s^{T*}(\psi^{\underline{t}})$ closer to S^{T}.
GO TO (3).

Here, in Step (4a), the optimal trajectory is clearly constrained if, for some $r > \underline{t}$:

$$\begin{aligned}
&(((s^{r*}(\psi^{\underline{t}}) > \bar{S}^{r}) \\
\text{AND } &((s^{T*}(\psi^{\underline{t}}) \leqslant S^{T}) \text{ OR } (s^{p*}(\psi^{\underline{t}}) \leqslant \underline{S}^{p} \text{ for some } p > r))) \\
\text{OR } &((s^{r*}(\psi^{\underline{t}}) < \underline{S}^{r}) \\
\text{AND } &((s^{T*}(\psi^{\underline{t}}) \geqslant S^{T}) \text{ OR } (s^{p*}(\psi^{\underline{t}}) \geqslant \bar{S}^{p} \text{ for some } p > r))))
\end{aligned} \quad \text{(H-70)}$$

Also Step 4(b), for the case of an upper constraint being active in t, could be expanded to:

(i) Find p such that: $s^{p*}(\psi^{\underline{t}}) - \bar{S}^{p} = \underset{r > \underline{t}}{\text{MAX}} \{s^{r*}(\psi^{\underline{t}}) - \bar{S}^{r}\}$

(ii) IF $s^{p*}(\psi^{\underline{t}}) = \bar{S}^{p}$ THEN GO TO (v),

ELSE adjust $\psi^{\underline{t}}$ so as to reduce $(s^{p*}(\psi^{\underline{t}}) - \bar{S}^{p})$

(lower $\psi^{\underline{t}}$ if $s^{p}(\psi^{\underline{t}}) > \bar{S}^{p}$ otherwise raise it).

(iii) Form the trial trajectory $s^{*}(\psi^{\underline{t}})$

(iv) GO TO (i)

(v) Set $\bar{t} = p$, CONTINUE.

During this process we must continually check that the optimal trajectory does not violate any lower constraint previous to $\bar{t}$. We can treat lower constraints similarly.

This then gives us a complete algorithm for finding the optimal trajectory for a single long-term controllable storage reservoir. It can easily be generalised, as in Appendix A, to cope with reservoirs with significant head effects. A flow chart for this process is shown in Figure (5-5), while a typical set of trial trajectories is summarised in Figure (5-6).

5.4.4 Conclusions

The trajectory method outlined above has been found to perform very well in optimising the long-term release pattern of a single long-term controllable reservoir with a valley of stations downstream from it. It requires as input only the inflow values, the aggregate constraints and the profit functions determined by the short-term problems (PASH). Details of our implementation can be found in Section 7.4.

The multi-reservoir problem is much more complex. In dynamic programming the dimensionality of the state space must increase by one for each extra reservoir included. This is likely to impose a prohibitive computational burden for even a two reservoir problem which is to be solved several times as a sub-problem of the whole optimisation.

With the trajectory method we must use successive approximations. We would, for each reservoir, for a given set of water values in the reservoir immediately downstream and a given set of releases from the reservoirs immediately upstream, determine an optimal trajectory. This would provide a release pattern to be assumed in the optimisation

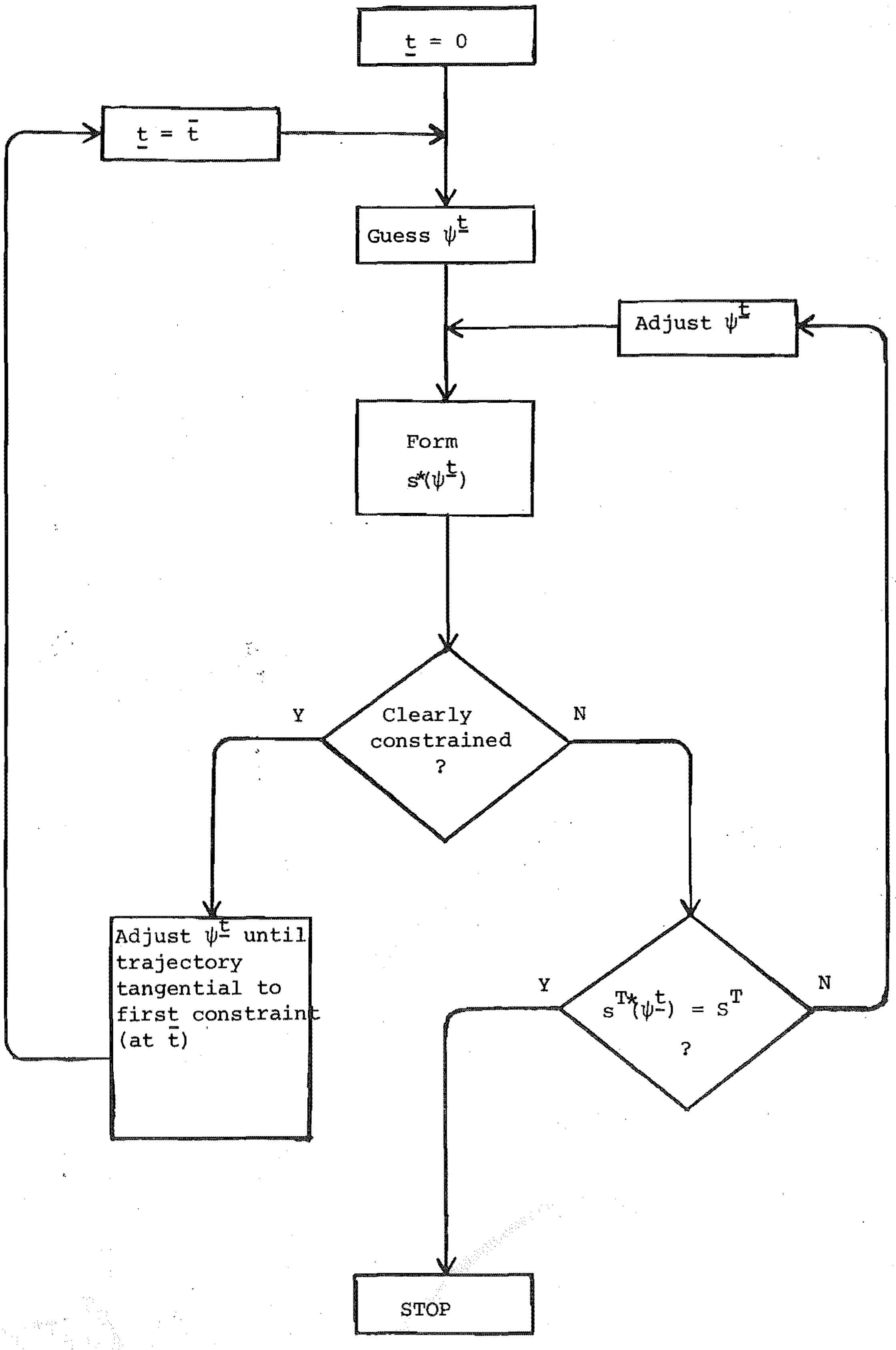


FIGURE (5-5): Flow chart for long-term hydro problem (PALH)

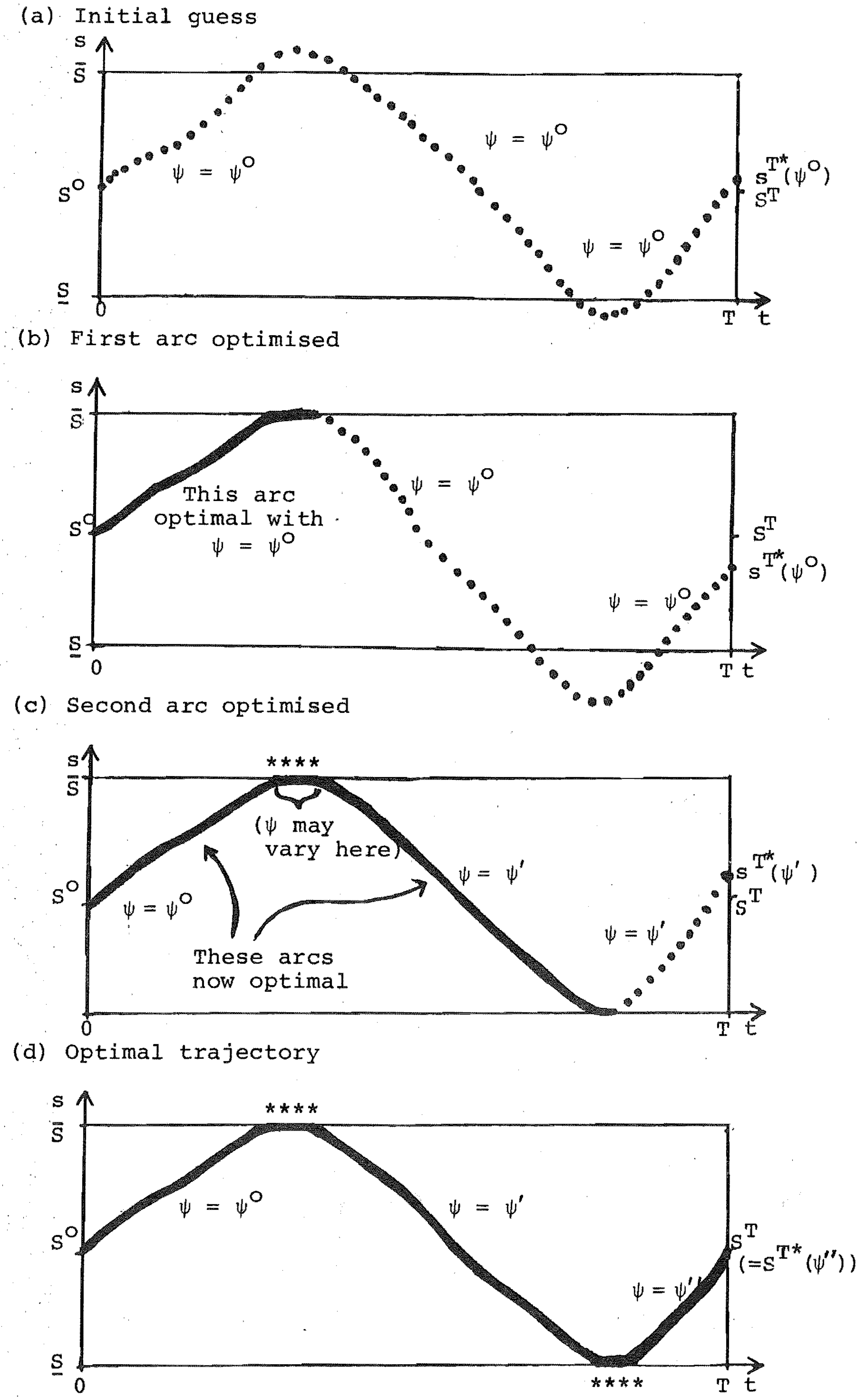


FIGURE (5-6): Example trial trajectories in PALH.

of the downstream reservoir (if any), and water values to be assumed in the optimisation of the upstream reservoirs (if any). We would optimise each of the reservoirs in turn until convergence. This procedure is not likely to be much more attractive computationally than multi-dimensional dynamic programming.

The optimal control approach was introduced by the EDF to overcome this limitation in the trajectory method. This method can, at least in theory, adjust the dual variables for all the long term controllable reservoirs in a valley simultaneously. However, according to [56], it had not at that time (1973) been applied in practice to valleys involving more than two such reservoirs. Hydro-Quebec ([25]) have aggregated the reservoirs in each valley using common-sense rules (resulting in no more than two aggregate reservoirs per valley) then applied a reduced gradient method. The Pacific North West Model ([22]) uses conjugate gradients on a rather more aggregated model without decomposition. Finally the Tenessee Valley Authority's latest model ([54]) uses a non-linear network flow formulation and applies a reduced gradient method to a six reservoir system. This last model is in fact essentially equivalent to the network flow formulation we have proposed for the short term model. It is clear that the dynamic programming solution developed there could also be generalised to handle the long-term problem.

Thus a variety of approaches to this rather difficult problem have been tried with varying degrees of success.

It would appear likely that the most suitable choice of model in a particular instance is highly dependent on such factors as the topology of the river system and the degree of correlation between its inflows. It may well be best to have different models for different river systems within the overall system. Thus we could use simple heuristic rules on some river systems so as to reduce them to single reservoir systems, while possibly optimising others by more sophisticated methods. Fortunately New Zealand has only one system which contains more than one long-term controllable reservoir. The Waitaki valley includes lakes Tekapo and Pukaki, but most (90%) of its generating capacity is downstream of both. We intend to reduce this, in our model, to a single reservoir system and apply the trajectory method just outlined.

5.5 SHORT-TERM HYDRO SCHEDULING

5.5.1 Introduction

We consider here the solution of the short-term hydro scheduling problem (PASH) for some particular period and valley. Thus we are given the tributary inflows (F), prices (μ) and a specified total release (q_{ℓ}^{t}) from each of the long-term controllable reservoirs involved. Our problem is to determine a schedule for the entire valley so as to maximise the gain from utilising the specified releases (at the prices μ).

As we have previously noted, this problem can be broken into a number of smaller problems ($PASH_k$, $k \varepsilon K$), each involving determination of a schedule for the reservoirs in $\hat{A}J_k$ given the releases from $\hat{A}K_k$. In other words, we treat reservoir k (where k = J corresponds to the ultimate destination of the flows) as a sink. We also consider all those long-term reservoirs whose releases flow into reservoirs whose releases flow into reservoir k (without passing through any intervening long-term controllable reservoir) as sources, with fixed total release for the period. We then try to optimise the utilisation of these releases as they pass through the intervening short-term controllable reservoirs. Since we will only need to consider such individual sub-systems we will simplify our notation by dropping the subscript k and letting: L represent $\hat{A}K_k$, N represent $\hat{A}J_k$. We also index the instants in t by r = 1,...R. So our problem is:

$$\text{Find } \underset{q}{\text{MAX}} \sum_{r \varepsilon t} \mu^r \left(\sum_{n \varepsilon N \cup L} g_n^r(q_n^r, s_n^r) \right) \qquad \text{("Profits")} \qquad \text{(H-36')}$$

Such that, for each $n \varepsilon N$:

$$s_n^o = S_n^o \qquad \text{(Fixed initial level)} \qquad \text{(H-37')}$$

$$s_n^R = S_n^R \qquad \text{(Fixed final level)} \qquad \text{(H-38')}$$

For all $r \varepsilon t$:

$$s_n^r = s_n^{r-1} + F_n^r + \sum_{i \varepsilon A_n} q_i^{(r-w_i)} - q_n^r \qquad \text{(H-39')}$$

(Flows balance)

$$\underline{s}_n^r \leqslant s_n^r \leqslant \bar{s}_n^r \qquad \text{(Storage feasible)} \qquad \text{(H-40')}$$

$$\underline{Q}_n^r \leqslant q_n^r \leqslant \bar{Q}_n^r \qquad \text{(Releases feasible)} \qquad \text{(H-41}'\text{)}$$

Also, for all $\ell \varepsilon L$:

$$\sum_{r \varepsilon t} q_\ell^r = Q_\ell^t \quad \text{(fixed)} \qquad \text{(H-42}'\text{)}$$

$$s_\ell^o = S_\ell^o \quad \text{(fixed)} \qquad \text{(H-43}'\text{)}$$

This problem can be considered as a scaled down version of the long-term scheduling problem. Its solution is complicated, however, by the greater degree of interdependence between instants (as indicated by constraints (H-39$'$)) than between the periods of the long-term problem. Also there are generally many more reservoirs involved.

Thus the dimensionality of this problem is so great as to be likely to render both dynamic programming and non-linear programming computationally infeasible. In fact, a dynamic programming model developed for NZED was abandoned for this reason ([23]). Faced with this difficulty, both the NZED and the EDF have produced heuristic short-term scheduling programs. The EDF use their P1 program for the purpose of building up curves to summarise the response of each river system to different releases from a single top reservoir. This is done for various given tributary inflow sequences and price vectors (see Appendix A). The NZED have a short-term scheduling program ([42]) which, for a given release pattern from the long-term reservoirs, attempts to schedule production from downstream stations so as to produce as much energy as possible in peak periods. This latter method is inappropriate for our purposes because its approach is incompatible with ours. The basic philosophy of our model is to achieve a feasible solution by adjusting prices until the optimal system response to those prices meets the demand constraints. The NZED, on the other hand, being primarily concerned with feasibility in the short-term, co-ordinates the output of the valleys via quantities. Each successive valley is scheduled so as to attempt to satisfy the demand (especially peak) remaining unsatisfied by the schedules determined for the valleys already scheduled. Thus a feasible solution is generally obtained. However the schedules derived for a particular valley are highly dependent on those for all other valleys. This is quite incompatible with our decomposition framework.

We propose here a network flow formulation which uses dynamic programming to find the optimal schedule without the disadvantage of high dimensionality. The disadvantage of the method is that it requires that we schedule each successive increment of the release individually. This is, however, entirely appropriate to the purpose of deriving curves representing the output and profit as functions of the total release. We restrict our attention here (cf.,[49]) to a system containing only one reservoir in L and a single unbranched chain of stations below it. Initially, we will also ignore the possibility that head variations significantly affect the output.

We first describe the method (Section 5.5.2) then the way in which it should be utilised by the long-term scheduling program (Section 5.5.3).

5.5.2 Solution Algorithm

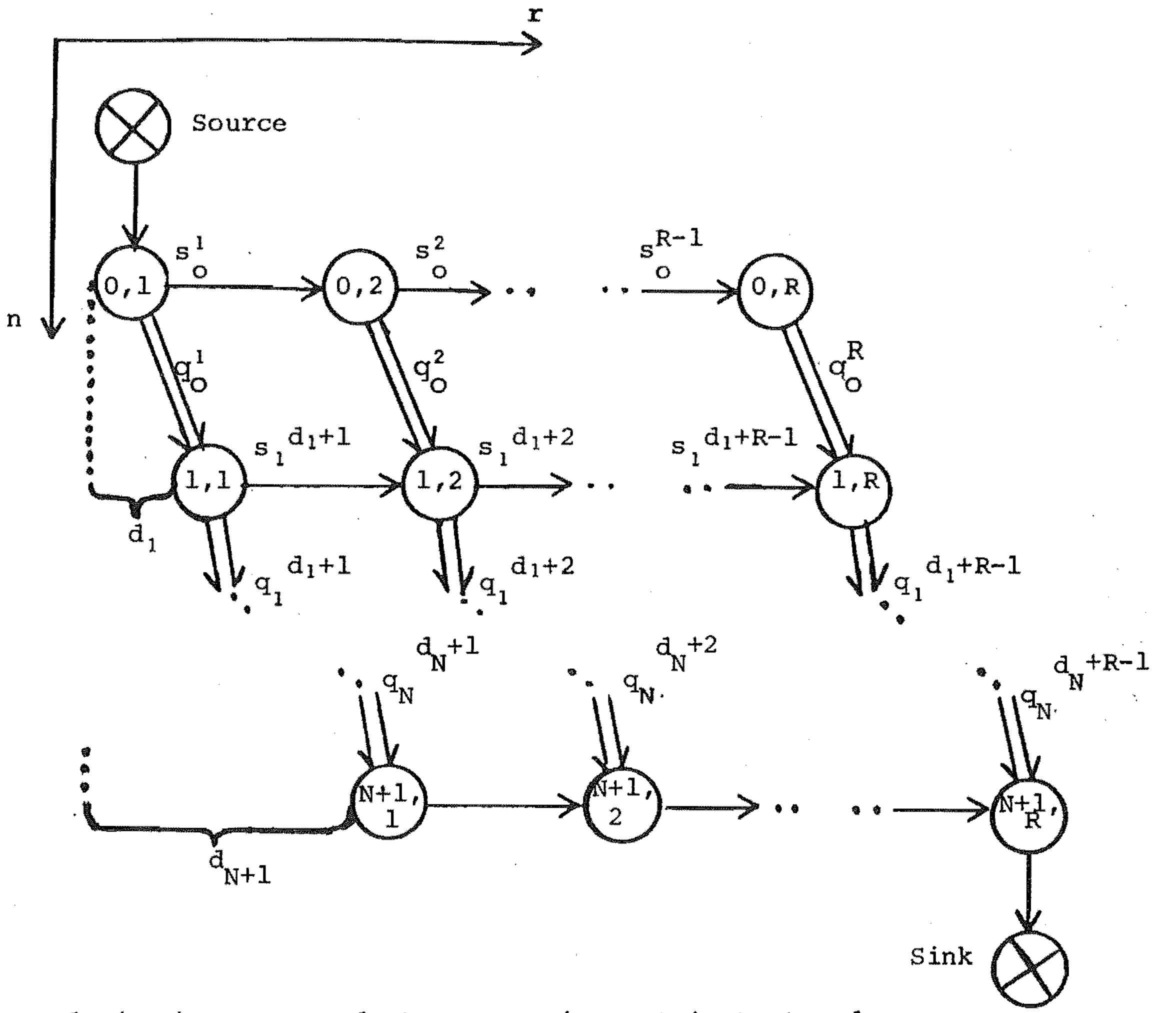
We could formulate our short-term scheduling problem, PASH, as a non-linear network flow problem similar to that of [54]. However we utilise a piece-wise linear approximation to the non-linear generation functions, g(q). Thus we may introduce the linear network shown in Figure (5-7). Here we have divided the total release, q_o, into many "increments", Δ. These increments are chosen to be small enough so that all the relevant quantities, $q, \underline{Q}, \bar{Q}, s, \underline{S}, \bar{S}$ may be reasonably expressed in terms of integral numbers of increments. Given concave generation functions each successive such increment must be released, at each station, with lower productivity than its predecessor. Hence the multiple arcs shown in Figure (5-7)(b), each with capacity Δ and its own "marginal profit":

$$\mu^r \frac{\partial g_n^r}{\partial q_n^r}\bigg|_{q_n^r = i\Delta} \qquad \text{for arc } i \qquad \text{(H-71)}$$

Now, if a total release of Q_o from the top reservoir is specified by a long-term program, we can determine its optimal downstream utilisation by finding the optimal flow through this network with the capacity of the initial arc restricted to be Q_o. In fact the following simple algorithm gives us, not only this optimal utilisation, but also that for any smaller flow ([27], p169). Thus it is ideal for building up the kind of "profit curves" required by our long-term algorithm.

(1) Let $q_n^r = 0$ for all n,r. Let $s_n^r = S_n^o$ (fixed) for all r.

(a) THE NETWORK



Here: node (n,r) corresponds to reservoir n at instant $r+d_n$

$$\text{where:} \quad d_n = \sum_{k=0}^{n-1} w_k$$

Also: we assume that S_n^o is injected at node $(n,1)$ and S_n^R extracted at node (n,R)

Storage arcs have limits $\underline{S}$ and $\bar{S}$.

(b) DETAIL OF MULTIPLE PELEASE ARCS

Here arc i has:

Capacity Δ

Profit $\Delta\mu^r \left.\dfrac{\partial g_n^r}{\partial q_n^r}\right|_{i\Delta}$

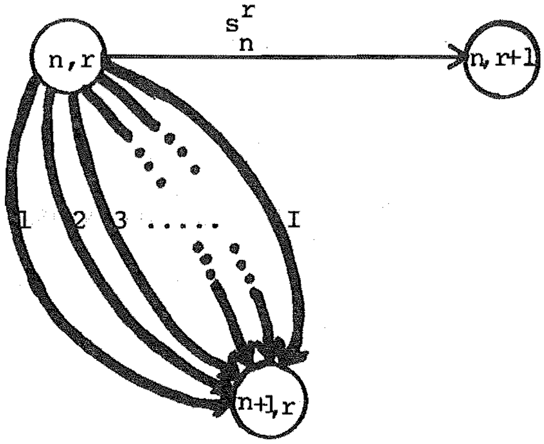


FIGURE (5-7): Short-term network

(2) Find a maximum profit, flow-augmenting path and increase flow along it to its capacity (i.e. Δ).

(3) If $q_o < Q_o$ THEN GO TO (2)
ELSE STOP

Here, Step (2), the determination of a maximum profit flow-augmenting path, can be achieved by means of the following dynamic programming algorithm.

Let us suppose that we have already derived an optimal schedule for the utilisation of a total release $\tilde{q}$ (given the prices μ and the inflows F). We now wish to schedule a further increment of release Δ, assuming that the schedule for the remainder of the release $(\tilde{q})$ remains as it is. Then, since the increment Δ is assumed to pass through the whole chain during this period ((H-10)), we need only decide the instant in which it is to be released from each station in the chain. This problem can be formulated as a dynamic program with N stages and a one dimensional state space with R states in each stage. We first detail a simple algorithm for this problem, then consider some variations on this scheme designed to better model reality.

Before formally stating our algorithm, consider the problem of determining the release instant of an increment of water arriving at reservoir n in instant r. We suppose that we know, for each possible release instant, the total benefits which would be derived, at this and all downstream stations, from a release in this instant. If this is summarised by the function $V_n(\tilde{q},r)$ (where $\tilde{q}$ is the already scheduled release), and the storage time graph for the reservoir is as in Figure (5-8), then this increment of water can be released in any instant p such that:

(i) $\tilde{q}_n^p \leqslant \bar{Q}_n^p - \Delta$ (H-72)

(ii) $\underline{p}_n^r < p \leq \bar{p}_n^r$ (H-73)

This second condition results from the following:

It is clear that the increment may be stored from instant r to be released in any future instant, p, provided the upper storage constraint is not violated (dashed trajectory segment $(r, \bar{p}_n^r)$). It is also true that the increment may be effectively "released" in any instant, previous to instant r, such that water is being held in storage (in the current solution for $\tilde{q}$) from that instant until r or after. In reality an amount of water equal to Δ is released in instant p, resulting in the decreased storage shown by the dotted trajectory segment $(\underline{p}_n^r, r)$. This decreased storage persists until instant r, when the arrival of Δ restores the storage trajectory to its original level. The increment Δ is then eventually released in place of the increment which is now released in instant p. (This arrangement corresponds, in the network formulation, to a flow augmenting path involving a reduced flow on some arcs - specifically those corresponding to the transfer of water in storage between each of the instants between p and r). So we can define:

$$\bar{p}_n^r = \underset{p>r}{\mathrm{MIN}} \{p \mid \tilde{s}_n^p > \bar{s}_n^p - \Delta\} \qquad \text{(H-74)}$$

$$\underline{p}_n^r = \underset{p<r}{\mathrm{MAX}} \{p \mid \tilde{s}_n^p < \underline{s}_n^p + \Delta\} + 1 \qquad \text{(H-75)}$$

(since we are concerned only with storage at the end of each instant).

Then we must obviously "release" Δ in the instant $p_n^r \varepsilon (\underline{p}_n^r, \bar{p}_n^r)$, with $\tilde{q}_n^{p_n^r} \leq \bar{Q}^{p_n^r} - \Delta$, which has maximum $V_n(\tilde{q}, p_n^r)$. The algorithm for scheduling Δ down the entire chain becomes:

1. Set n = N.

   Compute, for all $r\varepsilon t$:

$$V_N^r = \Delta\mu^r\left(\frac{\partial g_N^r}{\partial q_N^r}\bigg|_{\tilde{q}_N^r}\right) \tag{H-76}$$

2. Set n = n - 1.

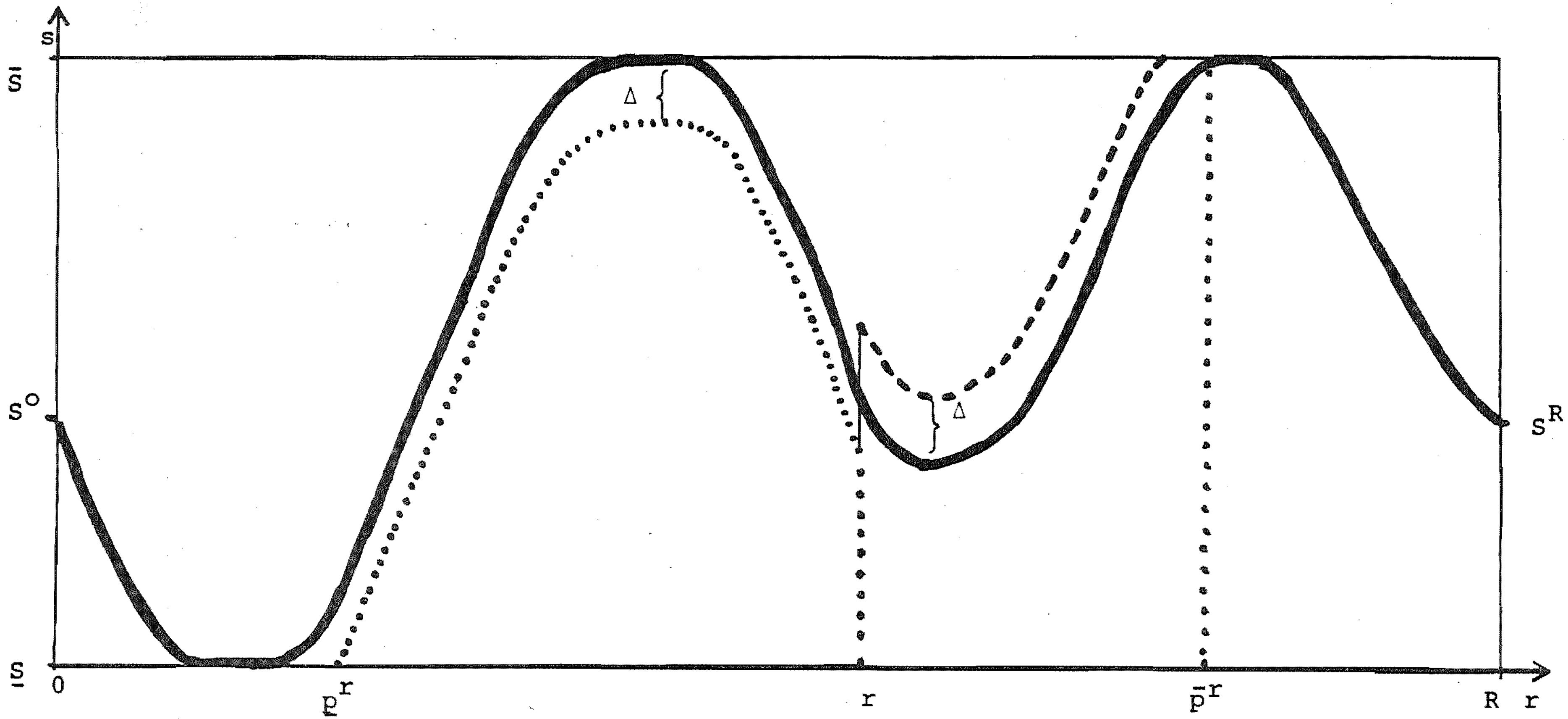
   Compute, for all $r\varepsilon t$:

$$V_n^r = \Delta\mu^r\left[\frac{\partial g_n^r}{\partial g_n^r}\bigg|_{(\tilde{q}_n^r,\tilde{s}_n^r)}\right] + \underset{\left\{p\varepsilon\left(\underline{p}_{n+1}^{r+w_n},\ \bar{p}_{n+1}^{r+w_n}\right)\ \middle|\ \tilde{q}_{n+1}^p \leqslant \tilde{Q}_{n+1}^p - \Delta\right\}}{\mathrm{MAX}} \left[V_{n+1}^{r+w_n}\right] \tag{H-77}$$

3. IF n > 0 THEN GO TO 2, otherwise the optimal schedule of Δ corresponds to the maximum V_0^0.

Our procedure for scheduling the entire release q_o is then to divide it into increments (Δ) and apply this method to each successive Δ assuming the schedule just determined for all preceding increments. We are assuming here (HAII) that each reservoir ($n\varepsilon N$) has a fixed initial and final level for the period. A reasonable level can be determined by experience.

So, for the initial solution with $\tilde{q} = 0$, we assume that this level is maintained throughout the period. We then schedule the uncontrollable inflows, starting from the "bottom" station and working upstream. At each stage of this process we schedule the release pattern (through all downstream stations) of the inflows (increment by increment) into the station in question, just as if they were planned releases from an upstream station. When all



optimal storage trajectory for $\tilde{q}$

•••• reduced storage if Δ released before r

- - - increased storage if Δ released after r

(Δ may be released in any instant between $\underline{p}^r$ and $\bar{p}^r$)

FIGURE (5-8): Feasible release instants for an increment Δ arriving in instant r.

such inflows have been scheduled we proceed to schedule the planned release from the top reservoir, increment by increment. This process is summarised by the flow chart in Figure (5-9).

We have, so far, ignored several real life complications. We consider two of these here. Some others are discussed in [49].

(a) The head of each station may affect the energy output from that station. We can then schedule each successive increment Δ using a dynamic programming recursion similar to (H-77). We define:

$$V_n^r = \Delta\mu^r \left(\frac{\partial g_n^r}{\partial q_n^r} \bigg|_{(\hat{q}_n^r, \tilde{s}_n^r)} \right)$$

$$+ \underset{\left\{ p \in \left(\underline{p}_{n+1}^{r+w_n},\ \bar{p}_{n+1}^{r+w_n} \right) \,\middle|\, \tilde{q}_{n+1}^p \leqslant \bar{Q}_{n+1}^p - \Delta \right\}}{\mathrm{MAX}} \Bigg[V_{n+1}^p \tag{H-77'}$$

$$+ \sum_{t=r+w_n}^{p-1} \Delta\mu^t \left(\frac{\partial g_{n+1}}{\partial s_{n+1}} \bigg|_{(\tilde{q}_{n+1}^t, \tilde{s}_{n+1}^t)} \right)$$

$$- \sum_{t=p}^{r+w_n-1} \Delta\mu^t \left(\frac{\partial g_{n+1}}{\partial s_{n+1}} \bigg|_{(\tilde{q}_{n+1}^t, \tilde{s}_{n+1}^t)} \right) \Bigg] \tag{H-78}$$

Here the term (H-78) summarises the head effect.

((†) here f_n^r is the total tributary inflow to n in r which has already been scheduled)

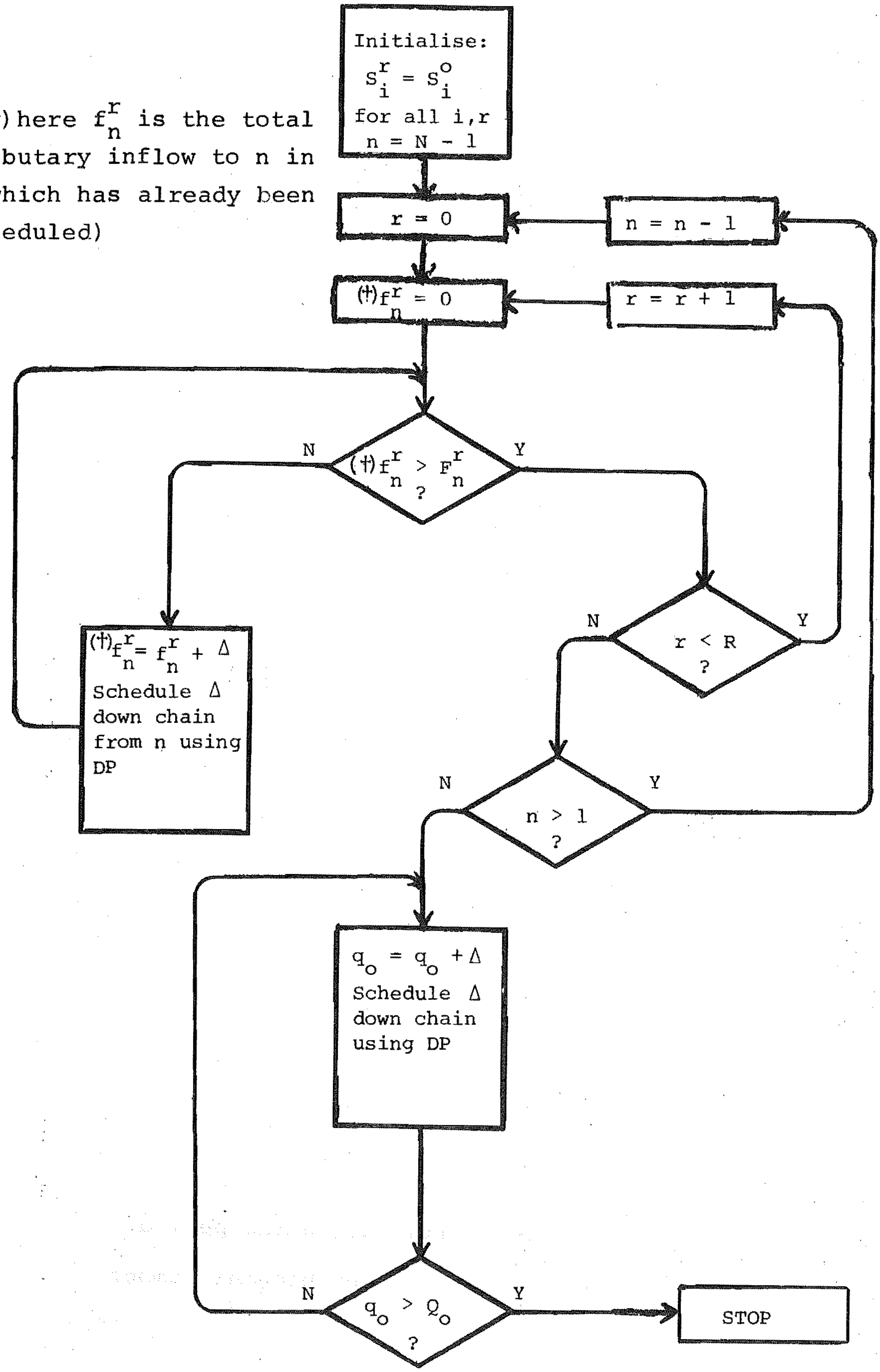


FIGURE (5-9): Flow chart for short-term hydro problem.

If an extra increment of water is stored from instant $r + w_n$ until instant (p-1) then it will increase the head in all intervening periods, so increasing output. Similarly, if an increment less is stored between p and $(r + w_n - 1)$ then output will be decreased for those instants.

As we have already noted the inclusion of this head effect introduces the possibility of non-convexity into the problem. Thus we cannot guarantee that scheduling successive release increments will produce an optimal overall solution, or that the resultant return function will be concave. However, since the head effect is minor, we do not expect this to cause any major difficulty. Preliminary experience (see Section 7.6) with the model has confirmed this expectation.

At some stations there may also be a "tail-water elevation" effect. Here the storage level at one reservoir affects the "effective head" at the previous station. We can account for this by the inclusion of a term analogous to (H-78) in our dynamic programming recursion.

(b) As we have formulated the problem releases from the top reservoir in late instants of the period may not arrive in downstream reservoirs during the period. Also, delay times, w_n, may not be an integral number of "instants". We avoid these problems by eliminating the delay times from our consideration. We first define:

$$d_n = \sum_{i=0}^{n-1} w_i \quad \text{for all } n = 1,\ldots N \tag{H-79}$$

Then transform the price curve, letting:

$$\mu_n^r = \mu_n^{\left((r+d_n)_{\text{MOD } R}\right)} \tag{H-80}$$

Here we interpolate as necessary. We have assumed that the prices in the early instants of the next period will be much the same as those in the corresponding portion of the current period (hence the "MOD R"). Now we can optimise as before, except that we have no delays and must price the output from station n in instant r at the price μ_n^r.

Later, after having performed the entire optimisation, we transform the generation curves back so that:

$$g^r = \sum_{n=1}^{N} g_n^{\left((r-d_n)_{\text{MOD } R}\right)} \tag{H-81}$$

(interpolating as necessary).

(c) Further to the above we may have minimum flow requirements, restrictions on flow changes, differing initial and final storage levels, side chains and pumped storage plants. Each of these can be accommodated in a straightforward manner as detailed in [49]. There we also consider various devices which may be employed to reduce the computation involved.

We conclude this section with a simple example. We deal with a river chain consisting of two stations in series. We simplify the arithmetic by assuming that, not only is there no head effect, but also that the generators in both stations run at the same constant efficiency. We allow 8 "instants" in the period with a delay of two instants between the two stations. So as to simplify the visual presentation we extend the price curve (Figure (5-10)(a)) rather than deal with different prices for each station. The relevant data for the two stations is given by the following:

$$g_i(q_i) = q_i \quad \text{for } i=0,1 \tag{H-82}$$

$$\hat{Q}_0 = \bar{Q}_0 = \bar{Q}_1 = 3 \tag{H-83}$$

$$\hat{Q}_1 = 2 \tag{H-84}$$

$$\bar{S}_1 = 3 \tag{H-85}$$

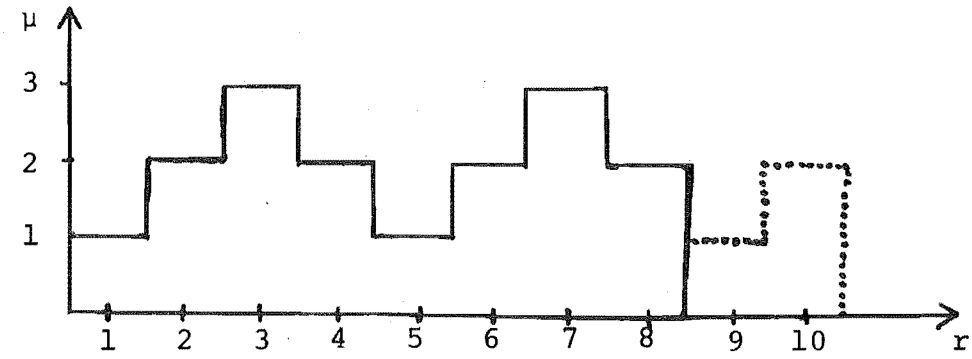
$$w_0 = 2 \tag{H-86}$$

$$\Delta = 1 \tag{H-87}$$

We assume that the initial storage level in reservoir 1 is zero and that that level must be restored after the final instant. (Also we have no tributary flows.) We only show some typical iterations. Each square in the Figures (5-10)(c) and (5-11)(c) corresponds to one increment of release, flow or storage for one instant. The release increments will be numbered according to the order in which they were assigned. Flow and storage are merely indicated by an X.

Firstly, consideration of the prices given by Figure (5-10)(a) allows us to deduce the (marginal) value functions shown in Figure (5-10)(b). The optimal release pattern

(a) Price curve



(b) Value functions

r	1	2	3	4	5	6	7	8	9	10
V_o^r	1	2	3	2	1	2	3	2	-	-
$\hat{V}_1^r$	-	-	3	3	3	2	2	2	2	2
V_1^r	-	-	3	2	1	2	3	2	1	2

($\hat{V}^r$ is the maximum (feasible) value from utilization of Δ arriving in r)

(c) Release-storage pattern

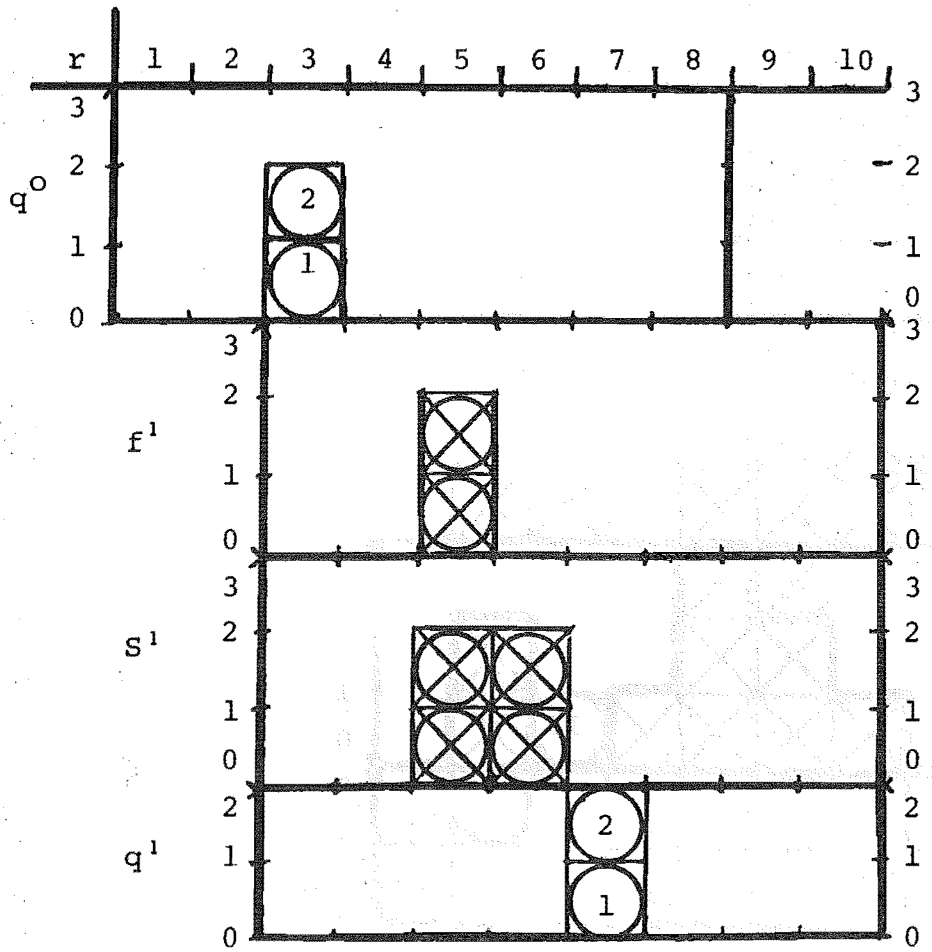


FIGURE (5-10): Initial iterations (see text).

(a) Feasible release instants

r	1	2	3	4	5	6	7	8	9	10
$\underline{p}_1^r$	-	-	3	4	4	4	4	4	9	9
$\bar{p}_1^r$	-	-	5	5	5	6	10	10	10	10

(b) Value functions

r	1	2	3	4	5	6	7	8	9	10
v_o^r	1	0	0	2	1	2	0	2	-	-
$\hat{v}_1^r$	-	-	1	1	1	1	1	1	1	0
v_1^r	-	-	0	0	1	0	0	0	1	0
μ^r	1	2	3	2	1	2	3	2	1	2

(c) Release-storage pattern

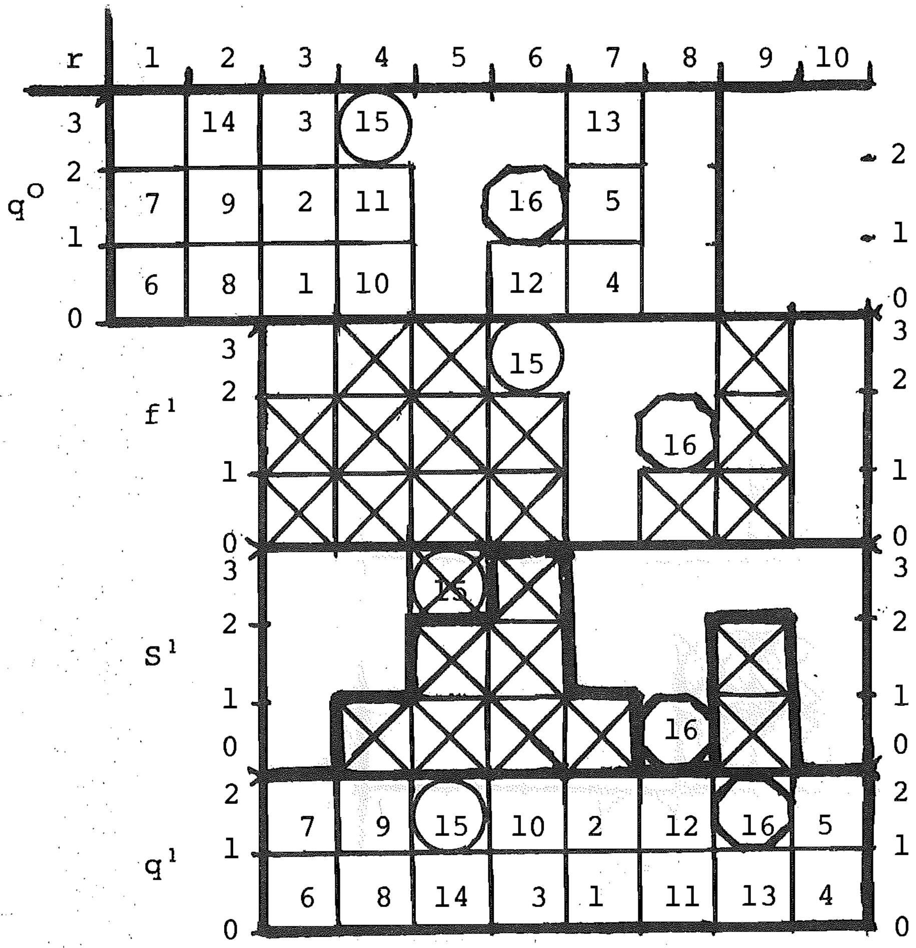


FIGURE (5-11): Further iterations (see text).

for the first increment then corresponds to the maximum (marginal) value of release, 6 , and involves release from the top reservoir in instant 3 ($\mu^3 = 3$). This is shown by the circled "path" in Figure (5-10)(b). This involves arrival at the downstream reservoir in instant 5 and release from that reservoir in instant 7 ($\mu^7 = 3$). The second increment should clearly be given the same schedule, resulting in the pattern of releases and storage shown in Figure (5-10)(c).

Continuing in this fashion, the optimal schedules for the first fourteen increments lead to the release/storage pattern of Figure (5-11)(c). Consideration of the storage pattern shown there gives the range of feasible release instants, for each instant in which an increment might arrive at the lower reservoir, shown in Figure (5-11)(a). Then we can derive the (marginal) value functions of Figure 5-11(b). Now the circled instants in that figure give an optimal schedule for the release of increment 15. This involves an initial release in instant 4 ($\mu^4 = 2$) resulting in arrival at the lower reservoir in instant 6. This arriving increment can then be best utilised by "releasing" it in instant 5 ($\mu^5 = 1$) This reduces the storage in (i.e. at the end of) that instant, resulting in the storage trajectory shown by the heavy outline. A further increment, number 16, can be released at the same marginal value, as is shown by the octagons in Figure (5-11). Any further releases must be spilled by the downstream station so that the problem becomes trivial. The marginal value of release curve for this example is shown in Figure (5-12) and the corresponding total value function in Figure (5-13).

5.5.3 Utilisation of the Algorithm

This algorithm can be used to build up curves, such as those in Figures (5-12) and (5-13), describing the output of the river system (or aggregates thereof) and the "profit" therefrom as a function of total release(s) from the long-term controllable reservoir(s). Such curves should be derived for each of several different aggregate price vectors, λ_n (implying μ_n), and inflow vectors, F_h. These curves, once prepared, can then be used in the long-term algorithm to summarise the behaviour of the river system in the short term. We note that, if there is no head effect, then each increment scheduled produces no higher profit than the previous one, so that our profit function will in fact be concave (and this must be true for any optimal short-term scheduling program). If we do have a head effect there is a theoretical possibility that one increment may have a slightly greater productivity than its predecessor. However both theory and experience suggest that this effect is very minor. It would seem appropriate, for the purposes of long-term planning, to modify the curves where necessary so that they are concave.

The task of computing and storing these output and

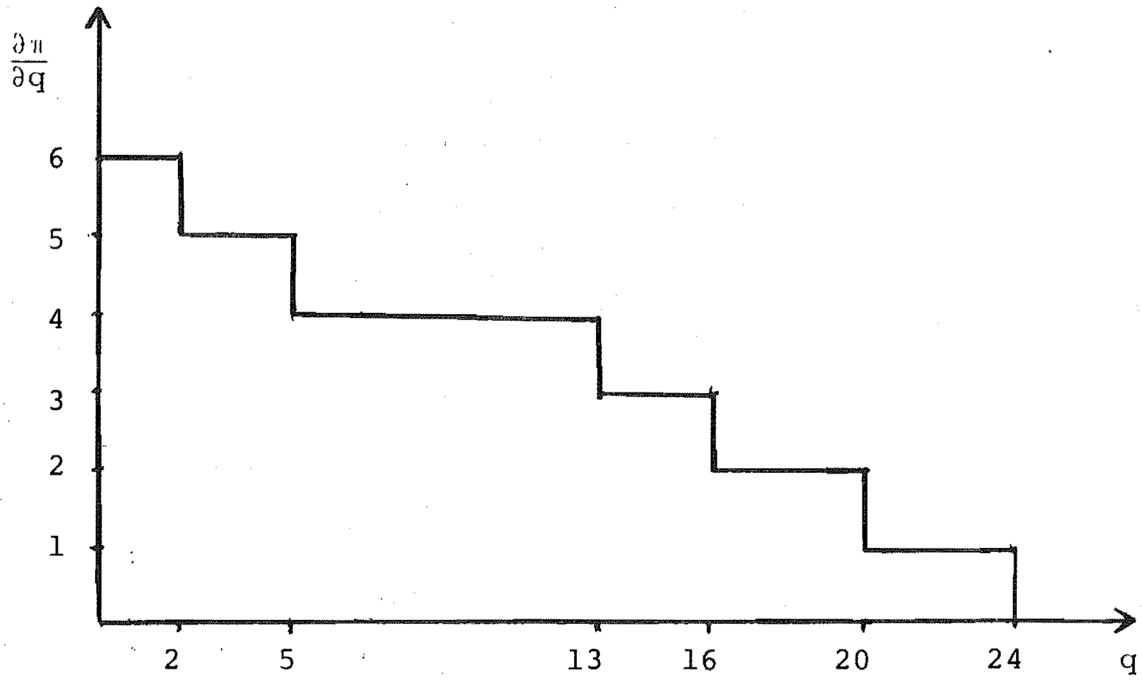


FIGURE (5-12): "Marginal value of release" curve

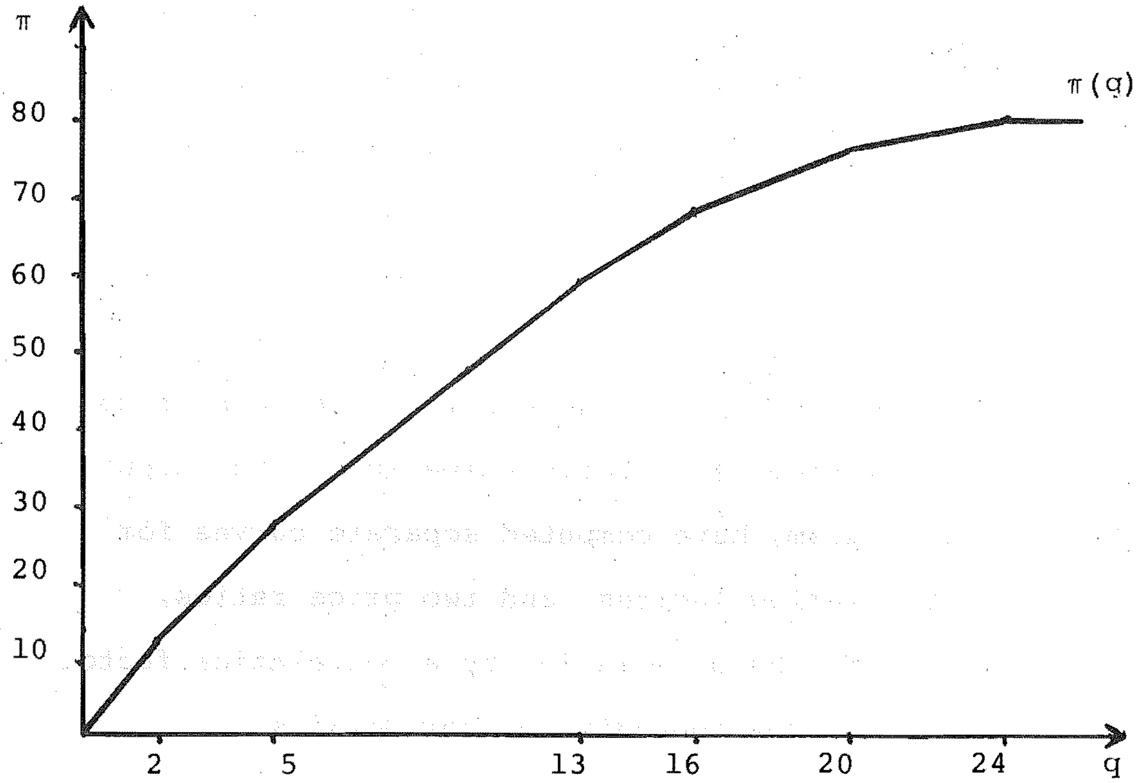


FIGURE (5-13): "Value of release" curve

profit curves is, depending on the degree of precision required, a potentially massive one. If the head in each of the L long-term controllable reservoirs has an effect and we were to allow the inflows to each of the N reservoirs to vary independently and had $\lambda = (\lambda^1, \ldots\ldots \lambda^k)$, then we would need to evaluate the output curve (or surface) over an N+K+2L dimensional grid. It is clear, however, that it is only the relative magnitudes of the prices which affect the solution, so that we can reduce the dimensionality of the problem by one. For New Zealand rivers it is a reasonable approximation to assume that the inflows at different reservoirs are sufficiently highly correlated to be represented by one inflow index. Also, the head of none of our long-term controllable reservoirs has a significant effect on valley output. Thus our grid is reduced to K+L dimensions. All of the sub-systems to be scheduled by this algorithm have L=1, so, allowing K=2, we require a 3 dimensional grid.

Furthermore, for the purposes of long-term scheduling, we will be content with a fairly coarse discretisation, relying on interpolation where necessary. The EDF, faced with a similar problem in suitably summarising the output from their P1 program, have computed separate curves for many (about 100) inflow indices and two price ratios, while accounting for head variation by a correlation factor where necessary. (See Appendix A for details).

In the remainder of this section we consider the effect of outages. In general the unavailability of equipment at some plants will have no effect when release is so low that the optimal schedule does not require those machines. But it will have an increasing effect as releases increase past that point to their maximum level. We could roughly account for this by assuming the same release schedule as for the totally available system, limiting the contribution of the particular stations as in Figure (5-14)(a),(b). This gives an overall system profit curve π' as in Figure (5-14)(c). However we should properly re-optimise the system with these restrictions. The resultant profit curve, π'', will lie between π and π' as the schedule is re-arranged to extract maximum profit from the restricted system. If we know that these units will be unavailable in a particular period, owing to planned maintenance or current break-down, then we should use the curve π'' (and its corresponding output curves) to represent the system in that period (or π' if π'' is unavailable). However, for an instant in which no outages are definitely known, we must consider only the probability of an unforseen breakdown. Thus, if we derive the relevant performance curves for each pattern of plant availability and determine the probability of each such pattern, we should combine these curves to produce an expected performance curve and use this to summarise the behaviour of the system in our long-term scheduling program.

5.5.4 Conclusions

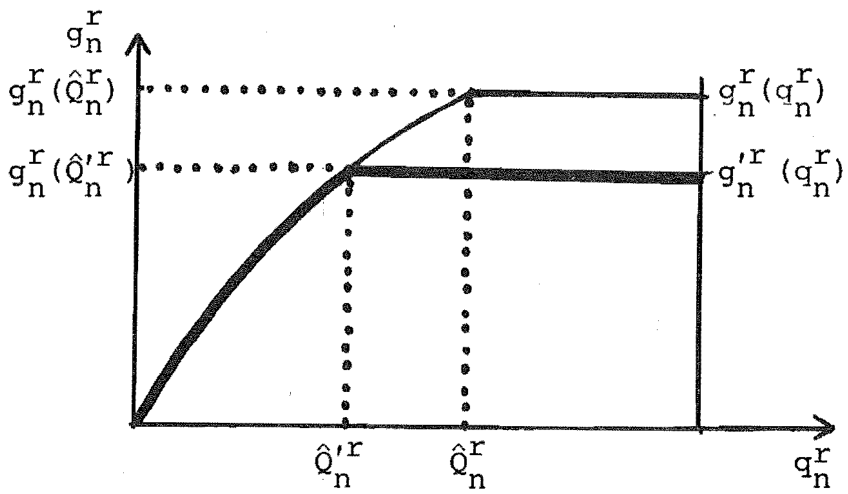
We have developed a new procedure for the solution of the short-term hydro scheduling sub-problem. The method is capable of producing the system response curves required by the long-term hydro scheduling sub-problem. This model involves the optimisation of a fairly realistic model of each river valley. So we expect the accuracy of this method to be greater than that of the heuristics currently in use. This model has, in fact, been partially implemented and computation times have been quite reasonable for all river systems involved. This is discussed further in Section 7.6.

We have developed this model in order to provide input for long-term optimisation. However it could also be used to improve short-term scheduling. In this context it would be important to be able to derive the optimal solution for one set of prices from that for another without re-starting the whole process. This whole subject is discussed in [49].

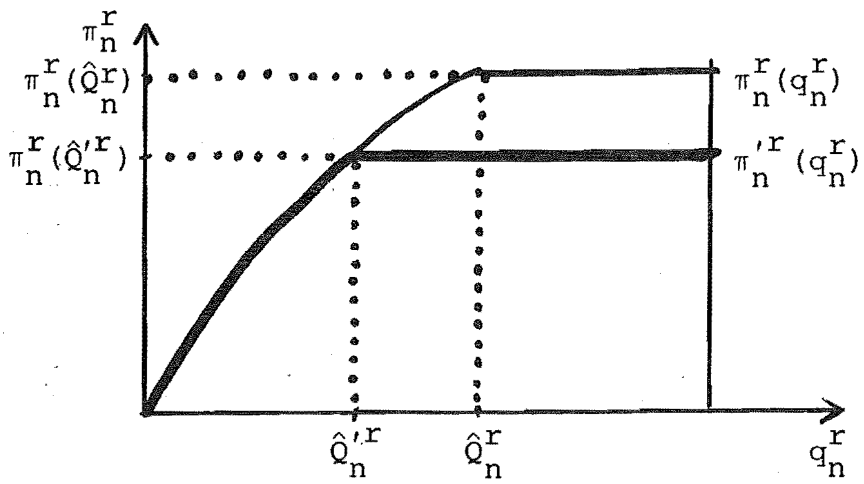
5.6 CONCLUSIONS

We have described a fairly general optimisation model of a local hydro "valley". The direct optimisation of this model is not attempted. From this model we have developed a long-term and a short-term model. The long-term model can be solved by established techniques developed by the EDF. For the solution of the short-term problem we have developed a new network formulation which can be used to develop response curves which can be stored in tabular form. These curves are to be used

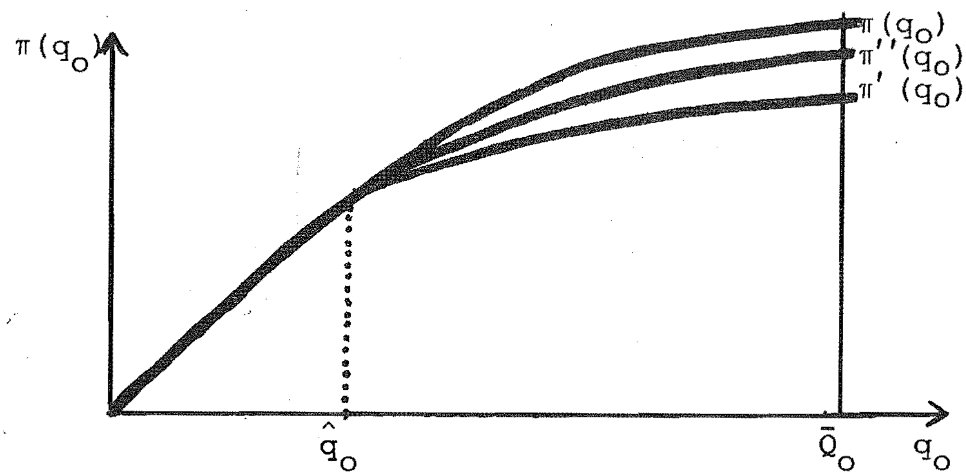
(a) Restricted generation curve for a station



(b) Corresponding profit curve for the station.



(c) Corresponding profit curve for the chain.



<u>FIGURE (5-14)</u>: Modified profit curves.

in the long-term model to determine both the levels and value of generation which would result from a particular total release for a period, if it were used optimally during the period.

This overall approach appears to produce (approximate) optimal solutions to the local hydro problem in a reasonable time. While the short-term problems need only be solved once, the long-term problem must be solved at each iteration of the dual problem (PAD), so as to give the optimal response of each hydro valley to the λ prices.

The derivation of detailed (μ) prices from the aggregate (λ) prices, discussed in Section 6.2, is here incorporated into the short-term models.

CHAPTER 6

THE DUAL PROBLEM

6.1 INTRODUCTION

The complete dual problem (DC) is:

$$\text{Find} \quad \underset{\mu}{\text{MAX}} \quad \mathcal{P}_c(\mu) \qquad \text{(C-17}'\text{)}$$

$$\text{Such that:} \quad \mu_i^r \geqslant 0 \qquad \text{for all } i \varepsilon \mathcal{I} \text{ (nodes)}, \quad r=1,\ldots R \text{ (instants)} \qquad \text{(C-18)}$$

Here $\mathcal{P}_c(\mu)$ would be determined by solving the (complete) Lagrangian problem, PC$'$. $\mathcal{P}_c(\mu)$ is the net cost of optimal system operation if energy surpluses and deficits are valued at the μ prices. Then the dual problem (DC) is assigned the task of adjusting the μ prices so as to ensure that all demands are met.

Also, in Section 2.4, we have developed an aggregated model for practical application. The dual problem (DA$''$) for this model can be stated:

$$\text{Find} \quad \underset{\lambda}{\text{MAX}} \quad \mathcal{P}_A(\lambda) \qquad \text{(A-15)}$$

$$\text{Such that:} \quad \lambda_n^{tk} \geqslant 0 \quad \text{for all } n = 1,\ldots N \text{ (regions)}, \quad t = 1,\ldots T \text{ (periods)}, \quad k = 1,\ldots K. \text{ (load segments)} \qquad \text{(A-16)}$$

From Section 2.3.1 we have the following general algorithm for this problem:

(1) Initialise λ.

(2) :

2.1 Derive $\mu(\lambda)$.

2.2 Solve the modified aggregate Lagrangian problem, PA″, to determine $z^*(\mu(\lambda))$.

2.3 Aggregate the responses to get:

$g_{nT}^{tk*}(\lambda)$, $g_{nH}^{tk*}(\lambda)$, $e_{nm}^{tk*}(\lambda)$, $L_{nn}^{tk*}(\lambda)$

for $n,m = 1,\ldots N$, $t = 1,\ldots T$, $k = 1,\ldots K$.

(3)

$$\text{If: } \left\{ \left| g_{nT}^{tk*}(\lambda)+g_{nH}^{tk*}(\lambda)-L_{nn}^{tk*}(\lambda)+\sum_{m=1}^{N}\left(f_{mn}^{tk*}(\lambda)-e_{nm}^{tk*}(\lambda)\right)-D_{n}^{tk}\right| < \varepsilon \right.$$

$$\left. \text{for all } n = 1,\ldots N,\ t = 1,\ldots T,\ k = 1,\ldots K \right\} \tag{A-7$'$}$$

THEN STOP

ELSE GO TO (4)

(4)

Adjust λ

GO TO (2)

In this chapter we deal first with Step 2.1, the derivation of the detailed prices from the aggregate prices, and some related issues.

Then our major concern will be with Step 4, the adjustment of the λ prices.

6.2 DERIVING DETAILED PRICES FROM AGGREGATE PRICES

6.2.1 Introduction

Our whole approach to obtaining approximate solutions to our problem has been based upon the assumption (AA):

> That, for each node $i \varepsilon n$, for any period t, the optimal vector of multipliers, $\tilde{\mu}_i^t$, can be constructed with adequate accuracy from the optimal vector of aggregate multipliers, $\tilde{\lambda}_n^t$, in the sense that optimal generation and transmission schedules based on the approximate price vector, $\mu(\tilde{\lambda})$, so constructed come acceptably close to satisfying conditions (C-12) (i.e., to meeting detailed load patterns).

We turn our attention here to the general approach by which we intend to develop a suitable mechanism for this derivation. Firstly we consider the problem of deriving, for a particular node, an appropriate set of detailed prices, $(\mu^r)^{r \varepsilon t}$, for the instants in period t. For this problem we propose to adopt a feedback mechanism which continually improves the accuracy of the detailed prices. Secondly we consider the problem of determining, for each instant r, the best vector of detailed prices $(\mu_i^r)_{i \varepsilon n}$, for the nodes in a region n. We avoid this problem by assuming "traditional" transmission patterns.

6.2.2 Temporal Price Distribution

Here we ignore the question of the spatial price distribution and, assuming that we are dealing with one node (index n), consider the problem of deriving the price vector, $(\mu_n^r)^{r \varepsilon t}$. We obviously wish to form $\mu_n(\lambda_n^t)$

in such a way as to force the energy supply to match the energy demand as nearly as possible in each instant (at least at the optimum $\tilde{\lambda}_n^t$). We expect that the price curve will more or less follow the load curve. However it would seem that the only reasonable way to determine the exact price vector is by experience - we propose the following general approach:

(1) Assume some general form for $\mu(\lambda)$.

(e.g., if λ_n^{kt} is "price" for energy in segment k, at week t, at node n, let $\mu_n^r(\lambda_n^t) = \lambda_n^{kt}$ for all $r \varepsilon$ segment k, week t).

(2) Perform entire optimisation under this assumption getting $\tilde{\lambda}, \tilde{\mu}(\tilde{\lambda}), \tilde{g}, \tilde{e}$ etc.

(3) Compare supply curves derived from this optimisation with load curves.

(4) If the correspondence is "satisfactory" then stop. Otherwise adjust the form of $\mu(\lambda)$ so as to improve the correspondence (on average) and go to (2)

In order to achieve a really accurate form for the price curve it would be necessary to apply this method for a number of different inflow sequences, load curves etc., - in fact we should use the stochastic model of Chapter 9. Then Step (3), effectively involving the simulation of short-term management for the whole of each inflow sequence (say one year), would involve a considerable computational burden. It is not envisaged that this task be undertaken at any one time but rather that, as the model is implemented, such a feedback mechanism be employed to continually increase its accuracy.

The accuracy of the representation arrived at will depend on the flexibility allowed by the particular aggregation chosen. Flexibility can be increased, either by allowing more aggregate "price parameters" or by allowing a greater variety of curves for different regions and seasons. In the long run the latter scheme would seem to have great computational advantages, since it not only requires the adjustment of fewer dual variables, but also reduces the dimensionality of the 'response surfaces' to be evaluated and stored. For example, if we have four load segments in the period with "prices" $(\lambda_n^1, \lambda_n^2, \lambda_n^3, \lambda_n^4)$, then, if we allow all these to vary independently, we must evaluate and store the regional response surfaces over a four-dimensional grid. Suppose, however, that we can divide the year into four seasons and ascertain that, for a particular region and season, each period's aggregate price vector displays (approximately) the same fixed relationship between $\lambda_n^1, \lambda_n^2, \lambda_n^3$ and λ_n^4. Then we could represent λ_n by just one parameter, λ_n' $(\lambda_n = \lambda_n(\lambda_n'))$, and evaluate and store a response surface for each region and season over a grid in only one dimension (λ'). Thus, with four such one-dimensional surfaces, we could summarize all that was in the original four-dimensional surface. Also we would only have to adjust the λ' parameters in the dual optimisation.

While this seems desirable in the long term, in order to be able to determine what relationships do in fact hold within the price vectors, we will need to gain considerable experience from using the, inherently more flexible, scheme of allowing a number of "price parameters" to vary independently.

Thus, to re-iterate, we have an interlocking hierarchy of models. Our long-term optimisation works out optimal prices (λ) and water values (ψ) so as to satisfy certain aggregate constraints on the generation. It does this using as input the optimal short-term management (as summarised by the relevant curves) of the short-term hydro, thermal and exchange sub-models. These sub-models, in turn, derive from λ the required detailed price vectors, $\mu(\lambda)$, and optimise their output to these. We may then compare the output from the sub-models with the actual load curve which they were supposed to meet and make appropriate adjustments to the price derivation scheme. Note that if these sub-models have also been incorporated into a short-term scheduling model of the type discussed in [49] then, in that model, the μ prices would be manipulated directly so as to ensure that all demands are met exactly in each instant. The optimum detailed price vector(μ^*) derived from the short-term model would then be the true detailed price vector which should have been derived from $\tilde{\lambda}$. Hence, if $\mu^* \neq \mu(\tilde{\lambda})$, we should modify our derivation of μ from λ, so as to ensure that (on average):

$$\mu(\tilde{\lambda}) \approx \mu^* \qquad \text{(D-1)}$$

This more accurate representation of the price curve (however it is derived) in turn increases the accuracy of the short-term sub-models and hence of the long-term optimisation.

6.2.3 Spatial Price Distribution, Intra-Regional Exchange and Regional Aggregation

In accordance with our general approach we should, properly, develop from the aggregate price vector, λ_n^t, for region n, period t, a spatial price distribution, $(\mu_i^r)_{i\varepsilon n}$, for each instant $r\varepsilon t$. Then we should optimise the output schedule at each node i to the prices $(\mu_i^r)^{r\varepsilon t}$ and the transfer schedule between each pair of nodes i and $j\varepsilon n$ to the prices $(\mu_i^r,\mu_j^r)^{r\varepsilon t}$. The derivation of this spatial price distribution from the aggregate price vector should be done, as for the temporal price distribution, on the basis of the experience gained from application of the model. However, particularly in a sparse system such as that of the NZED (see Figure (1-2)), local transmission patterns are very largely determined by the topology of the network and the relative magnitudes of outputs, demands, and line capacities. Thus, rather than arriving at a transmission pattern from a spatial price distribution, it would seem more appropriate to derive a price distribution from the transmission pattern.

For instance, suppose that a hydro plant (h) is connected to just one node (i) by a transmission line. Then, if h is to generate $\tilde{g}_h^r$ in instant r, all of $\tilde{g}_h^r$ must be transmitted to i. If L^r_{hi} is the loss function for this line, then, since $\tilde{e}^r_{hi} = \tilde{g}^r_h$, we have that:

$$\left.\frac{\partial L^r_{hi}}{\partial e^r_{hi}}\right|_{\tilde{g}^r_h} = \frac{\tilde{\mu}^r_i - \tilde{\mu}^r_h}{\tilde{\mu}^r_i} \qquad \text{(D-2)}$$

So that:

$$\tilde{\mu}^r_h = \tilde{\mu}^r_i \left(1 - \left.\frac{\partial L_{hi}}{\partial e^r_{hi}}\right|_{\tilde{g}^r_h}\right) \qquad \text{(D-3)}$$

So, if the price vector at node i is μ_i, we should optimise the production at node h to the vector, $\mu_h(\mu_i)$, given by equation (D-3).

We might just as well, however, 'collapse' the line from h to i, treating h as if it was at location i and subtracting the losses incurred between h and i. Thus we can define a new generation function, g'_h, by:

$$g'^r_h = g^r_h - L^r_{hi}(g^r_h) \qquad \text{(D-4)}$$

and henceforth deal with this generation function.

We can similarly collapse lines leading to load centres by adding the losses to the loads and arrive at a reduced network. The largest regions with which we shall ever be concerned are the North and South Islands of New Zealand. Figure (1-2) shows the major components of these regional systems. Collapsing lines we get rather simple networks. Moreover, for any particular demand pattern, we can easily collapse these lines further to a single node (assuming that local loads are met first). (e.g., See Figure (7-1)).

After aggregation the region may be represented by a sub-network as in Figure (6-1). The response surfaces for the various stations in the region can be aggregated. Figure (6-2) shows a typical (instantaneous) regional thermal response curve. Apart from variations in equipment and demand patterns, this curve remains constant for all periods. A corresponding hydro response curve may also be formed (on the basis of water values) from the previous iteration. However, since this obviously changes from period to period and iteration to iteration (as the water values change), it must be recomputed for each period as it is required. Also this curve must be modified considerably if any reservoir is in storage constraint (see Section 6.3.7). Note that this aggregation has eliminated the L_{nn}^{tk*} term in (A-7).

Provided that demand patterns do not vary too widely (apart from the variation reflected in λ), we can store a regional aggregate thermal response curve, such as that of Figure (6-2), which is valid for all (or many) periods. However, since we can never do this for the hydro response curve, the advantages may not be great. Again the most sensible procedure would seem to be to experiment with a model with a larger number of regions (which really can be treated just like individual nodes) to gain experience as to the best aggregation procedure.

These aggregate response curves must be modified slightly so as to ensure unique solutions to our problem. Firstly, some sections of the response curve may be flat. We introduce a slight (rising) slope to these so as to ensure that solutions are unique and that prices are always adjusted in the right direction. Secondly, we add a fictitious "shortage extension" corresponding to the shortage nodes of the complete problem. This ensures that feasible solutions always exist.

6.2.4 Conclusion

It is clear that we cannot ensure that detailed short-term requirements are met merely by adjusting aggregate prices. This problem could be overcome, as in [6], by using a national short-term sub-model which directly attempts to ensure that they are met. This however does not allow us to decompose the problem. We have chosen to adopt a scheme in which a vector of detailed prices is derived from the aggregate prices so as to ensure that generation levels are appropriate to each time of day. We cannot prove the existence of an optimal detailed price derivation mechanism or provide any explicit formulae. However we have outlined a feedback mechanism by which this derivation may constantly be improved to the extent to which a particular aggregation scheme allows this.

Our approach is based on that of the EDF. They, however, do not derive detailed price vectors but rather "price duration curves" (see Section A.3.4 of Appendix A). Our approach is intended to extend theirs in two respects. Firstly it is more general in that we have not assumed any fixed relationship between the prices. However,

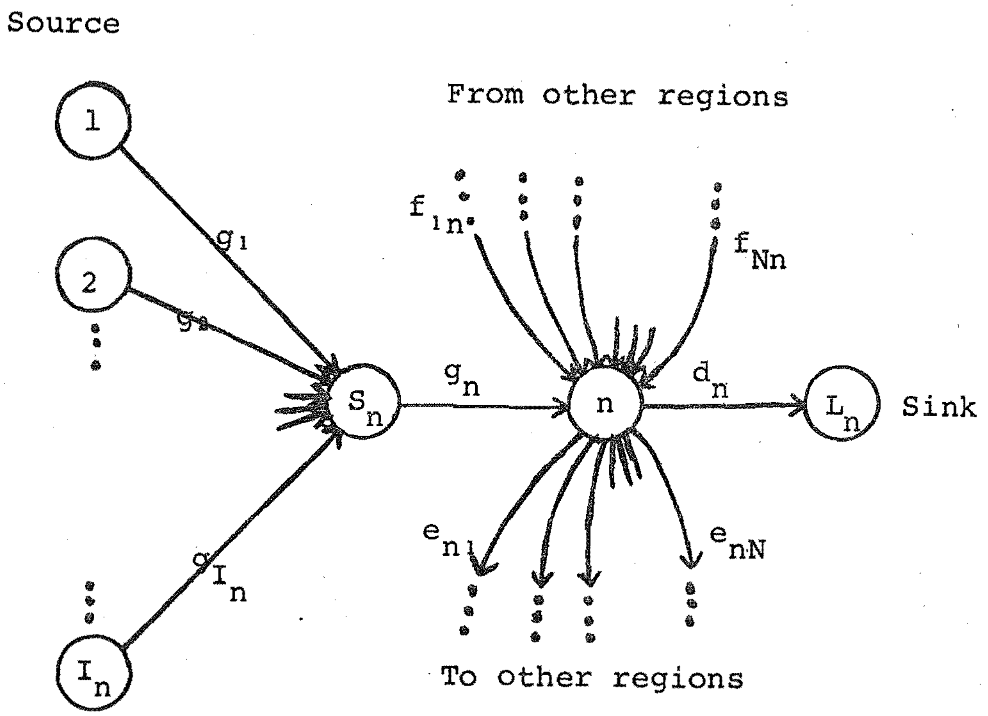


FIGURE (6-1): Regional sub-system

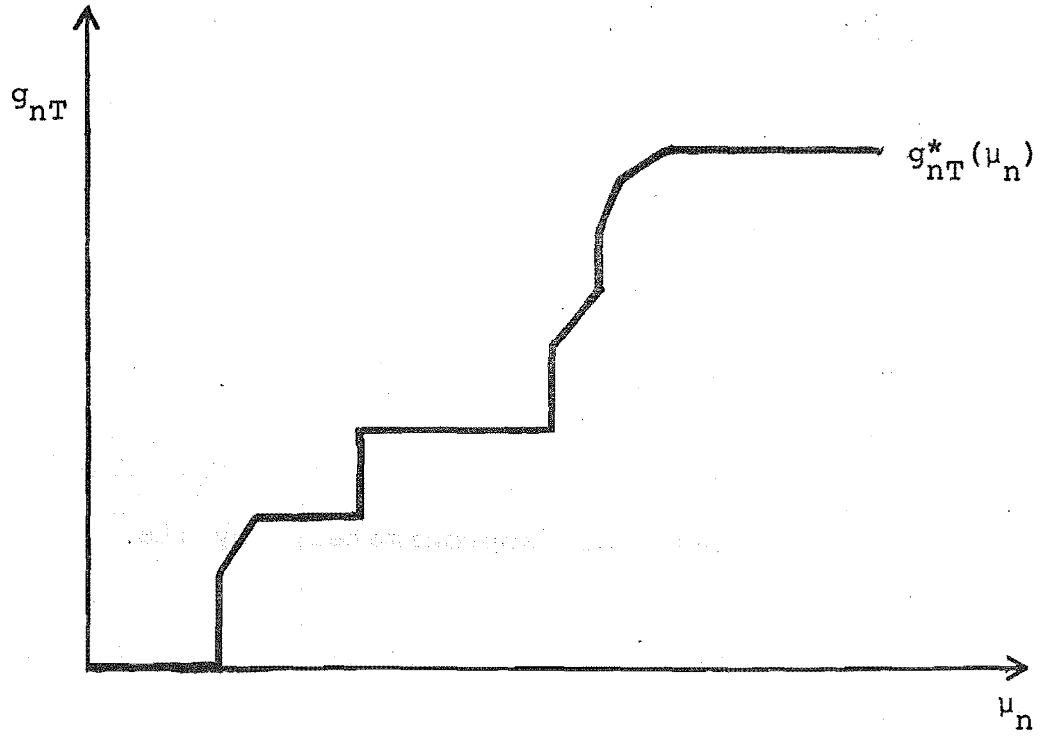


FIGURE (6-2): Thermal response curve

when we have gained sufficient experience it will probably be desirable to adopt specific simple formulae for price derivation, perhaps similar to those of the EDF. Secondly our approach allows more detailed optimisation of the short-term operation of the sub-systems. In particular we can explicitly model dynamic phenomena such as delay times and start-up costs. (See, e.g., Section 5.5). Our scheme does not, however, involve any more computation time in the main optimisation - the extra detail being confined to the sub-models used to prepare input tables.

Rather than extend our scheme to produce a spatial price distribution we intend to assume that the traditional transmission patterns will be adopted within each region. In this case each region may reasonably be represented as a node having an aggregate generation response function.

6.3 PRICE ADJUSTMENT

6.3.1 Introduction

We deal here with the adjustment of the λ prices in the solution of the aggregate dual problem (PA) with objective $P_A(\lambda)$.

$P_A(\lambda)$ can be determined by solving the aggregate Lagrangian problem (PA$'$) or (approximately) by its modification (PA''). This latter problem is identical in form to PC', evaluating the optimal response of the system to the detailed prices, μ, derived from the aggregate prices, λ. $P_A(\lambda)$ is then determined by aggregation from this detailed response. So we expect that the aggregate dual function will exhibit very similar behaviour to the complete dual function. In this section we will deal with the aggregate dual problem as if it were the complete dual problem. The properties we shall derive would hold exactly if we were to determine P_A via the solution of PA' but only approximately if we use PA''. In this latter case discrepancies could arise if μ is not a linear function of λ. However, given assumption (AA), any such discrepancies will not be major. We refer here to $P(\lambda)$, dropping the subscript A since our comments apply at least equally to $P_C(\mu)$. Also, for simplicity, we restrict our attention to a single - segment model.

At each iteration we must find a new set of λ prices which results in a system response which more nearly satisfies the aggregate constraints (A-8) (or equivalently raises $P(\lambda)$). In view of the effort involved in evaluating $P(\lambda)$ we wish to minimise the number of iterations of the algorithm by making a good choice of λ at each iteration. However we need to beware that the computational burden involved in adjusting λ does not outweigh the advantages of an accurate adjustment process. In choosing an appropriate method the behaviour of $P(\lambda)$

is of paramount importance. In the next subsection we outline some properties of $P(\lambda)$. Then we develop a solution procedure utilising these properties. Next we describe a slightly different dual problem and outline a method, based on this, which has proved to converge very quickly. Finally we outline a scheme to improve convergence in periods when the storage trajectory for some reservoir is in constraint.

6.3.2 Properties of $P(\lambda)$

We have already shown (Section 2.3.2) that $P(\lambda)$ is concave. Moreover, by Corollaries 1 and 2 to Theorem 8.5.5 of [30] we have that, provided:

(i) Z is closed and bounded.

(ii) C_n^t is continuous and strictly concave on Z, for each n = 1,...N, t = 1,...T.

Then:

(a) $P(\lambda)$ is differentiable, for all $\lambda \geqslant 0$

(b) $$\left.\frac{\partial P}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} = D_n^t - \left[g_{nT}^{t\,*}(\tilde{\lambda}) + \sum_{h \in n} g_h^{t\,*}(\tilde{\lambda}) + \sum_{m=1}^{N} (f_{mn}^{t\,*}(\tilde{\lambda}) - e_{nm}^t(\tilde{\lambda})) \right] \quad \text{(D-5)}$$

Thus, since our problem may be modified so as to satisfy (i) and (ii), we may apply, to the dual problem, any procedure capable of maximising a concave differentiable objective.

We further note that, given the modifications outlined in Chapters 3 to 5, the "response functions" appearing in (D-5) are themselves differentiable and so we can form second partial derivatives.

From (D-5) and (C-29)$'$ - (C-31)$'$ we have that:

$$\left.\frac{\partial P(\lambda)}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} = D_n^t - \left[g_{nT}^{t*}(\tilde{\lambda}_n^t) + \sum_{h \varepsilon n} g_h^{t*}(\tilde{\lambda}_n) + \sum_{m=1}^{N} \left(f_{mn}^{t*}(\tilde{\lambda}_m^t, \tilde{\lambda}_n^t) - e_{nm}^{t*}(\tilde{\lambda}_n^t, \tilde{\lambda}_m^t) \right) \right] \tag{D-6}$$

Hence:

$$\frac{\partial^2 P(\lambda)}{\partial \lambda_n^t \partial \lambda_m^r} = \begin{cases} -\left[\left.\dfrac{\partial g_{nT}^{t*}}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} + \displaystyle\sum_{h \varepsilon n} \left.\dfrac{g_h^{t*}}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} + \displaystyle\sum_{m=1}^{N} \left(\left.\dfrac{\partial f_{mn}^{t*}}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} - \left.\dfrac{\partial e_{nm}^{t*}}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} \right) \right] & \text{if } m=n, r=t \quad \text{(D-7)} \\[2ex] -\displaystyle\sum_{h \varepsilon n} \left.\dfrac{g_h^{t*}}{\partial \lambda_n^t}\right|_{\tilde{\lambda}} & \text{if } m=n, r \neq t \quad \text{(D-8)} \\[2ex] \left.\dfrac{\partial e_{nm}^{t*}}{\partial \lambda_m^t}\right|_{\tilde{\lambda}} - \left.\dfrac{\partial f_{mn}^{t*}}{\partial \lambda_m^t}\right|_{\tilde{\lambda}} & \text{if } m \neq n, r=t \quad \text{(D-9)} \\[2ex] 0 & \text{if } m \neq n, r \neq t \quad \text{(D-10)} \end{cases}$$

Thus $H(\lambda)$, the Hessian matrix of second order partial derivatives of $P(\lambda)$, has the structure shown in Figure (6-3). All of the entries in $H(\lambda)$ can easily be determined from the sub-models.

6.3.3 A Solution Strategy

Firstly, since $P(\lambda)$ is differentiable, we could apply a gradient method (or steepest ascent method) to our problem.

This would involve substituting the following for Step (4).

(4')

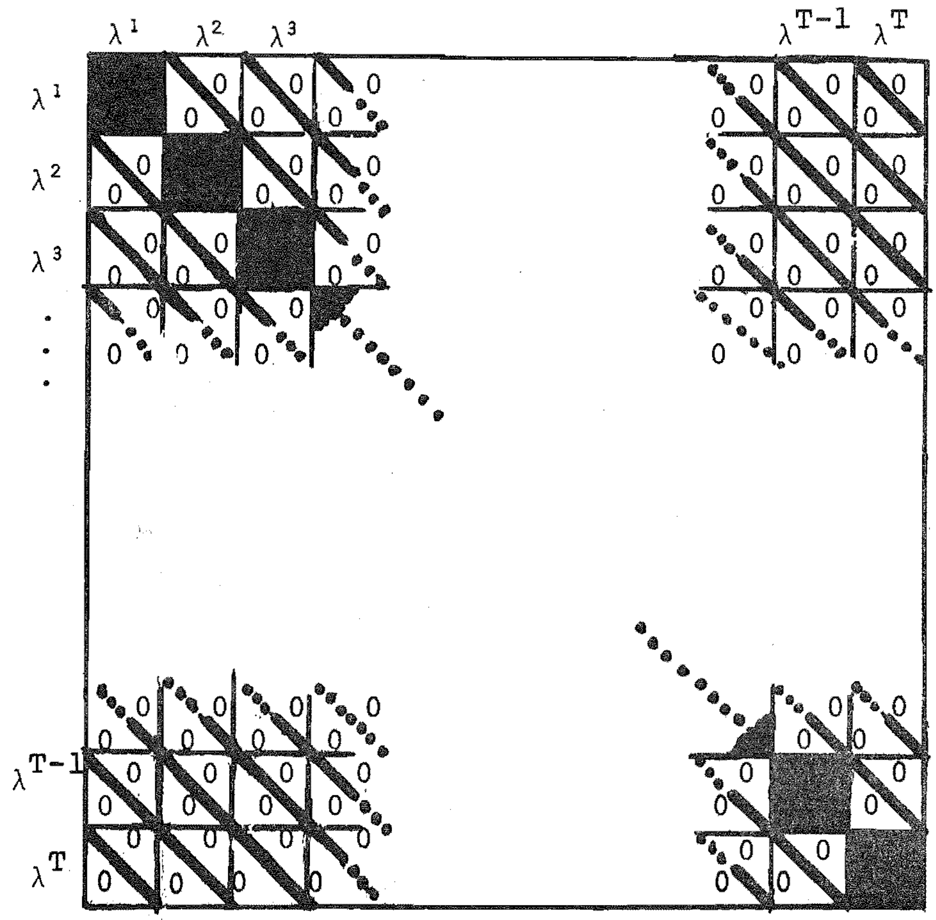
$$\text{Let: } \lambda_n^t = \lambda_n^t - \theta\left(g_{nT}^{t*}(\tilde{\lambda})+g_{nH}^{t*}(\tilde{\lambda}) + \sum_{n=1}^{N}(f_{mn}^{t*}(\tilde{\lambda})-e_{mn}^{t*}(\tilde{\lambda}))-D_n^t\right) \tag{D-11}$$

Where $\tilde{\lambda}$ is the current solution and θ, the "step size", is to be determined so as to increase $P(\lambda)$.

Here we adjust the price, λ_n^t, up (or down) in proportion to the excess demand (or supply) indicated in that region, for that period, at the current prices $\tilde{\lambda}$. This straightforward scheme has been suggested by the EDF. It has, however, some disadvantages:

(a) In our multi-regional model the excess demand in any region is dependent not only on its own price but also on those of its neighbours. So even the general direction of price movement indicated by (D-11) may be incorrect.

(b) The determination of a suitable θ is made rather difficult by the erratic nature of the response surfaces. Ignoring the transmission system, consider the portion of a regional generation response curve shown in Figure (6-2). We could use a single fixed θ for all iterations as in Uzawa's algorithm ([64]). Then, provided θ is small enough, the algorithm will converge. However, in order to ensure convergence in regions, and periods, where the optimum involves generation in a very steep portion of the response curve, we will require a very small θ. Such a small θ will cause very slow convergence in regions, and



 NxN block given by: $\mathrm{diag}\left(\frac{-\partial g_n^{r*}}{\partial \lambda_n^t}\right)_{n=1}^{N}$ ((D-8))

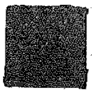
 NxN block, the entries for which are given by the equations (D-7) and (D-9).

FIGURE (6-3): Hessian matrix $H(\lambda)$.

periods, where the optimum lies in (nearly) flat portions of the response curve. Further, this problem will occur (for some region and/or period) at all stages of the algorithm, so that there is little to be gained from allowing θ to vary from iteration to iteration.

The former problem is peculiar to our multi-load model while the latter is probably not important in a large system with many stations (resulting in a much smoother generation response curve). As we develop a more detailed representation of the short-term price structure the response surfaces for our own system will become much smoother. However, at least at present, this type of algorithm is not suitable for our model.

Instead of the gradient method just discussed we use an adaptation of Newton's method. The gradient method can be classified as a "first order method". In it we move λ towards where the optimal λ would be if $P(\lambda)$ was linear (and hence $z^*(\lambda)$ constant). Newton's method is a "second order method". In it we try to move λ towards where the optimal λ would be if $P(\lambda)$ were quadratic (and hence $z^*(\lambda)$ linear). Second order methods generally exhibit much better convergence properties, especially near the optimum, than do first order methods. Both $e^*(\lambda)$ and $g_T^*(\lambda)$ are, in fact, both (locally) linear and $g_H(\lambda)$ is nearly so. So, if we can perform the calculations required by Newton's method, we will expect rapid convergence.

Using Newton's method we replace Step (4) by:

(4′)

$$\lambda_n^t = \lambda_n^t - \theta\left[H(\tilde{\lambda})\right]^{-1} \nabla P(\tilde{\lambda}) \qquad \text{(D-12)}$$

Where:

$\tilde{\lambda}$ is the current λ and the "step size", θ, is to be determined. $\nabla P(\tilde{\lambda})$ is given by (D-6) and $H(\tilde{\lambda})$ by (D-7) - (D-10).

This method avoids both of the difficulties ((a) and (b)) in the gradient method. The inter-regional interaction is accounted for by the off-diagonal terms, (D-9), in the diagonal blocks of the Hessian. Also the varying slope of the generation response curve is accounted for by the diagonal terms (D-8). For the thermal system this accounting is (locally) exact, for the hydro system not quite so.

However this method has two major problems - the practical evaluation of the search direction, $\delta(\tilde{\lambda})$, and the determination of a suitable step size, θ. We deal with these two problems in the next two sections.

6.3.4 The Search Direction

In Newton's method the search direction, $\delta(\tilde{\lambda})$, is given by:

$$\delta(\tilde{\lambda}) = - [H(\tilde{\lambda})]^{-1} \nabla P(\tilde{\lambda}) \qquad \text{(D-12')}$$

This involves the evaluation and inversion of the Hessian matrix, which may be quite large, say 100 x 100, even for a two-region single-segment model. Moreover, as we discuss in the next section, the search direction is only valid locally - the Hessian changing significantly each time a "corner" of the generation response curve (or "edge" of the transmission response curve) is reached.

Many schemes have been devised, in a general context, to avoid explicitly evaluating and inverting such Hessian matrices. However such schemes usually rely on building up an approximation to the inverse on the basis of "past experience". Owing to the erratic nature of the response curve such methods are not at all appropriate. Fortunately the evaluation of the Hessian is not difficult, all the entries being easily obtainable from the solutions to the sub-problems. We will utilise the structure of the matrix to obtain an approximate inverse with little effort.

Firstly, if we were to ignore the terms introduced by the hydro sector, the matrix would have a simple block-diagonal structure. In this case we need only invert T small (NxN) matrices - an easy task.

However the inter-period interaction in the hydro sector introduces widely dispersed off-diagonal terms. These terms, while individually small, do have a significant effect. From Chapter 5 it may be seen that this interaction can be entirely summarised in terms of the water value ψ.

In relation to this, two extreme cases are relevant. Firstly, suppose that the optimal overall levels of hydro and thermal generation have been determined for a particular region, containing valley h, for all periods in a free arc of the storage trajectory for h. Then we expect the water value for that arc, ψ_h, to remain constant and there will be no inter-period interaction. This is because the minor adjustments in λ, necessary to balance hydro generation at h optimally between periods in that arc, cancel out. So we could summarise the hydro response, for any period t in that arc, by letting:

$$\frac{\partial g_h^{t*}}{\partial \lambda_n^r} = \begin{cases} \dfrac{\partial g_h^{t*}}{\partial \lambda_n^t} & \text{if } r=t \\ 0 & \text{if } r\neq t \end{cases} \tag{D-13}$$

On the other hand, if the optimal balance of hydro generation at $\hat{h}$ between periods in the arc has been found, then the changes in λ necessary to achieve an optimal overall balance between hydro and thermal production will result in changes in ψ_h, but leave the generation pattern within the arc unchanged. Thus we can ignore the hydro response, letting:

$$\frac{\partial g_h^{t*}(\lambda)}{\partial \lambda_n^r} = 0 \qquad \text{for all } r,t \text{ in that arc} \tag{D-14}$$

In either case we may break the whole Hessian into T, NxN, blocks (H^t, t=1,...T), and so replace Step (4) by:

$$(4''') \quad \text{Let: } \lambda^t = \tilde{\lambda}^t - \theta^t\Big([H^t(\tilde{\lambda})]^{-1} \nabla P^t(\tilde{\lambda})\Big) \tag{D-12''}$$

$$\text{for all } t = 1,\ldots T.$$

Where θ^t is to be determined.

In other words, we can replace our large price adjustment problem with a collection of rather small problems - one for each period. For the time being we assume (as in [47]) that this approach can be applied to the general case. In fact this would result in slow convergence (using (D-13)) or instability (using (D-14)). However these problems can be overcome by the method of Section 6.3.6, utilising a rather different Hessian. We turn our attention in the next section to the determination of θ^t - the step size.

6.3.5 The Step Size

We assume here that it is safe to ignore the inter-period interaction introduced by the hydro system. Accordingly we deal with a single-period price adjustment problem, dropping the superscript t. We shall also deal entirely with instantaneous response curves. Aggregated response curves will be smoother, with less well defined corners, but the same principles will apply.

Suppose that we have found a search direction, $\delta(\tilde{\lambda})$, which is locally optimal in the sense that $(\tilde{\lambda}+\delta(\tilde{\lambda}))$ would be the optimal solution to the local quadratic approximation to $P(\lambda)$. We wish to know how far to adjust λ in the direction $\delta(\tilde{\lambda})$. One simple answer to this problem would be to adjust λ until $P(\lambda)$ changes significantly. As we have seen $P(\lambda)$ will behave in a predictable manner until we reach a "corner" of a generation response curve (see Figure (6-2)) or an "edge" of an exchange response surface (see Figure (4-15)). At that point some of the entries in $H(\lambda)$ may change dramatically and so we should, properly, re-evaluate $H(\lambda)$ and compute a "new" locally optimal $\delta(\lambda)$ and resume the search. Of course, if the optimum for the local quadratic approximation (corresponding to $\theta=1$) is reached before any edge or corner, the process terminates. Then this λ is the one required and we return to the Lagrangian problem.

In order to apply this approach we must be able to recognise when a corner or edge is "reached". First we define:

$$\tilde{\lambda}^*(\theta) = \tilde{\lambda} + \theta\delta(\tilde{\lambda}) \tag{D-15}$$

Also, let λ_{nG} be the price corresponding to the next corner of the generation response curve for region n (in the direction of price movement indicated by $\delta(\tilde{\lambda})$). Then we can define:

$$\theta^*_{nG} = \theta \Big| \lambda_n(\theta) = \lambda_{nG} \tag{D-16}$$

Further, we can define:

$$e^*_{nm}(\tilde{\lambda},\theta) = e^*_{nm}(\tilde{\lambda} + \theta\delta(\tilde{\lambda})) \tag{D-17}$$

Then:
$$\dot{e}_{nm} = \frac{\partial e^*_{nm}}{\partial \theta} = \frac{\partial e^*_{nm}}{\partial \lambda_n}\frac{\partial \lambda_n}{\partial \theta} + \frac{\partial e^*_{nm}}{\partial \lambda_m}\frac{\partial \lambda_m}{\partial \theta}$$
$$= \delta_n(\tilde{\lambda})\frac{\partial e^*_{nm}}{\partial \lambda_n} + \delta_m(\tilde{\lambda})\frac{\partial e^*_{nm}}{\partial \lambda_m} \tag{D-18}$$

We can easily determine this direction of movement from the solution to the sub-models. Further, in the quadratic case, substituting (E-6) into (D-18), we get that e^*_{nm} rises if:

$$\tilde{\lambda}_m\delta_n(\tilde{\lambda}) > \tilde{\lambda}_n\delta_m(\tilde{\lambda}) \tag{D-19}$$

and falls otherwise. Thus we can determine the edge which will be encountered next in the direction given by $\delta(\tilde{\lambda})$. Let E be the corresponding transfer. Then we can easily solve for θ^*_{nmE}. In the pure quadratic case we have:

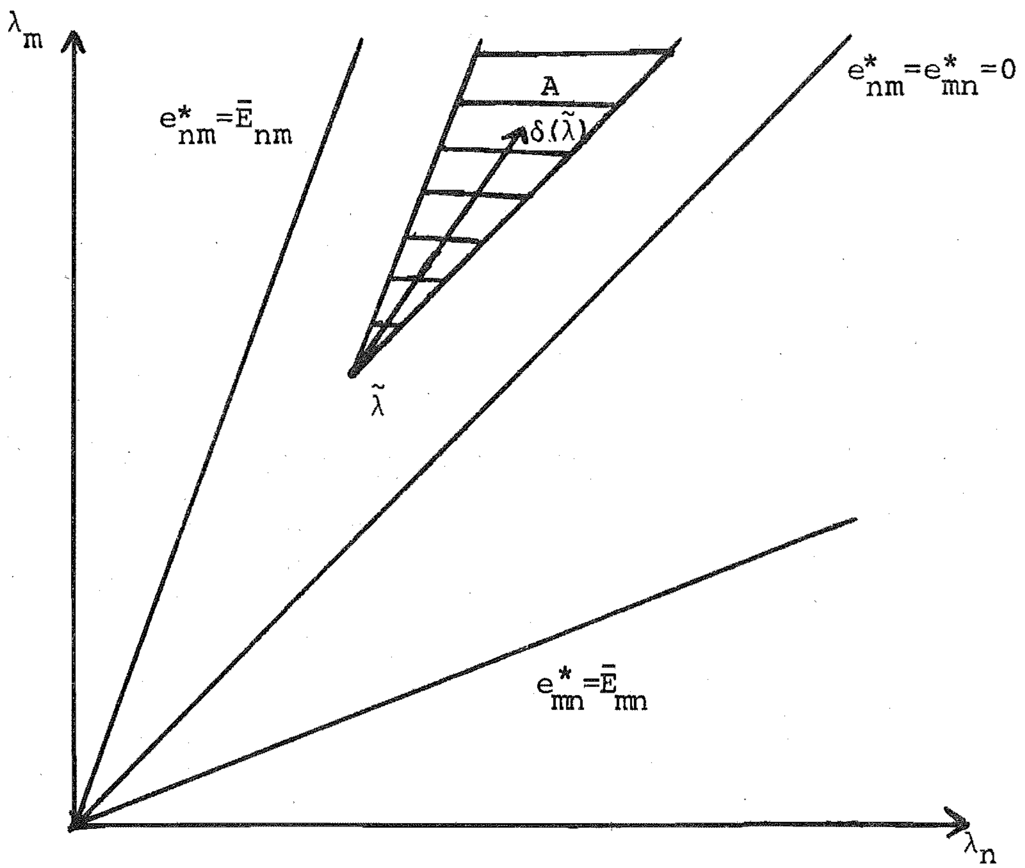


FIGURE (6-4): Edges of the exchange response surface.

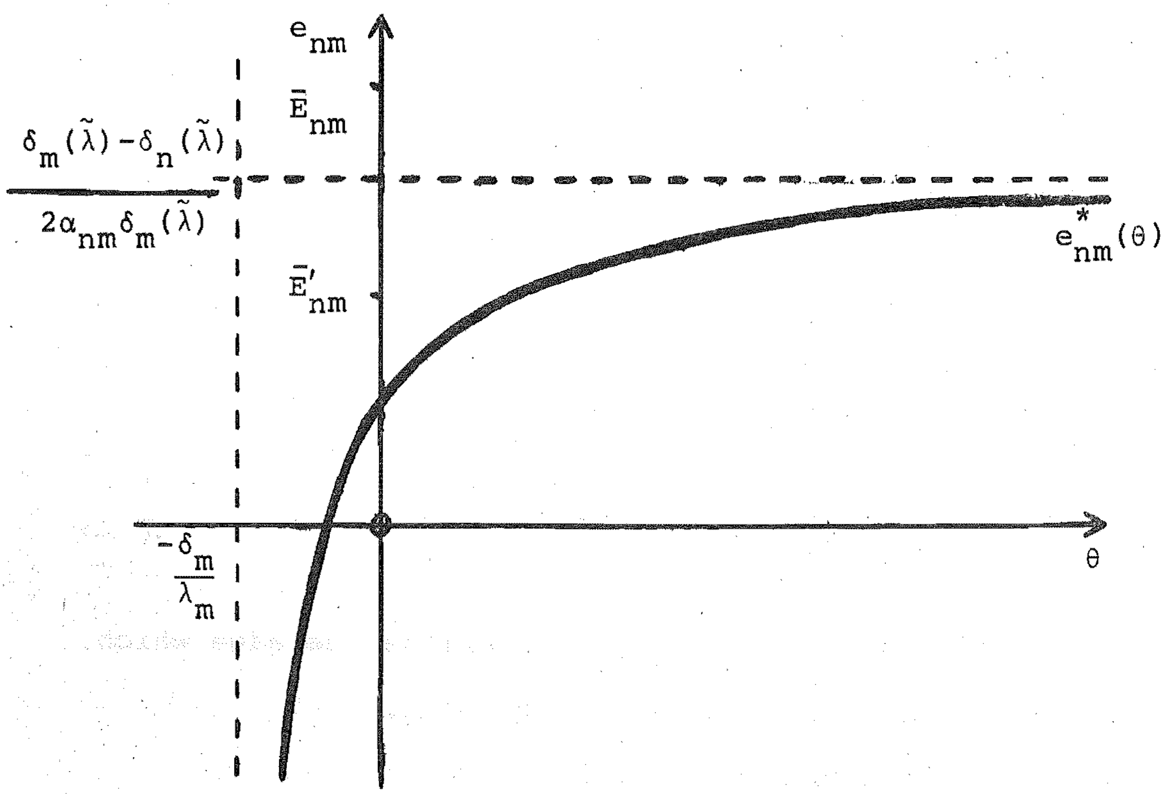


FIGURE (6-5): Variation of optimal exchange level with price adjustment.

$$\theta^*_{nmE} = \frac{\tilde{\lambda}_n - (1-2\alpha_{nm}E)\tilde{\lambda}_m}{(1-2\alpha_{nm}E)\delta_m(\tilde{\lambda}) - \delta_n(\tilde{\lambda})} \tag{D-20}$$

If this expression exists, and is positive, then it is the required θ. However θ^*_{nmE} may be undefined (if $\delta_m(\tilde{\lambda})(1-2\alpha_{nm}E) = \delta_n(\tilde{\lambda})$) or it may be negative. In order to understand these properties we must consider the nature of the function $e^*_{nm}(\tilde{\lambda},\theta)$.

In the pure quadratic case we have:

$$e^*_{nm}(\tilde{\lambda},\theta) = e^*_{nm}(\tilde{\lambda}+\theta\delta(\tilde{\lambda}))$$

$$= \frac{\tilde{\lambda}_m - \tilde{\lambda}_n + \theta(\delta_m(\tilde{\lambda}) - \delta_n(\tilde{\lambda}))}{2\alpha_{nm}(\tilde{\lambda}_m + \theta\delta_m(\tilde{\lambda}))} \tag{D-21}$$

This function is an hyperbola (See, e.g. Figure (6-5)) with vertical asymptote at:

$$\theta = -\frac{\tilde{\lambda}_m}{\delta_m(\tilde{\lambda})} \tag{D-22}$$

and horizontal asymptote at:

$$e_{nm} = \frac{\delta_m(\tilde{\lambda}) - \delta_n(\tilde{\lambda})}{2\alpha_{nm}\delta_m(\tilde{\lambda})} \tag{D-23}$$

The vertical asymptote results from the fact that the optimal transmission from m to n (cf., e_{nm}) will become idenfinitely large as the price in region m nears zero. The horizontal asymptote results from the fact that, as θ increases, the optimal transmission will approach a limit determined entirely by the search direction and independent of the starting value.

The vertical asymptote will never create any problems because, long before it is reached, the transmission level, $e^*_{nm}(\tilde{\lambda},\theta)$, must reach its limit, E.

Consider the situation in Figure (6-4) (cf. Figures (4-3) - (4-5)). Here we have drawn, in (λ_n, λ_m) space, the position of the edges of the response surface of a pure quadratic line, $\tilde{\lambda}$ being the current price vector. Suppose that the price λ_m is rising, so that the vertical asymptote of $e^*_{nm}(\tilde{\lambda},\rho)$ is to the left of the vertical axis (cf. (D-22)). Then we could draw the function $e^*_{nm}(\theta)$ as in Figure (6-5). Now it can be seen that if the adjustment vector lies within the cone (A), then the horizontal asymptote defined by (D-23) lies between 0 and $\bar{E}_{nm}$ (cf. $\bar{E}'_{nm}$). Clearly, if we adjust prices in such a direction, we will never meet an edge but the exchange level will tend asymptotically towards that given by (D-23). The negative solutions to (D-20) correspond to such directions of adjustment and so should be ignored.

So, since we can recognise the position of corners and edges, we can apply the approach outlined above. Then Step (4) for any particular period, would become:

(4''):

(4.1) Let $k=0$, $\lambda^k = \tilde{\lambda}$.

(4.2) $\delta^k(\lambda) = -[H(\lambda^k)]^{-1}\nabla P(\lambda^k)$ (D-12''')

(4.3) Let $\theta^k = \underset{n,m=1,\dots N}{\text{MIN}} \left\{1, \theta^*_{nG}(\lambda^k_n, \delta^k_n(\lambda)), \theta^*_{nmE}(\lambda^k, \delta^k(\lambda))\right\}$ (D-24)

(4.4) $\lambda^{k+1} = \lambda^k + \theta^k \delta^k(\lambda)$ (D-25)

(4.5) If $\theta^k = 1$ THEN STOP

ELSE $k = k + 1$, GO TO (4.2).

Thus we may have to invert our small Hessian block, H^t, several times in each iteration. It would be virtually impossible to apply this approach to the exact Hessian. Not only is the inversion of such a large matrix computationally burdensome, but we should have to invert the whole matrix (or update the inverse) every time we reached an edge or corner in any period. Thus we would face the same number of matrix inversions as in the approximate method - but each time we would have to deal with an NTxNT matrix instead of an NxN matrix. However the method as described, while computationally tractable, does not account properly for the inter-period interactions. This causes poor convergence but can be overcome by the modification described in the next section.

6.3.6 Accounting for Inter-Period Interaction

The scheme outlined in the previous sections rests on the assumption that we can ignore the inter-period interaction when adjusting prices. We have seen that this is true in two special cases but it is not true in general. In practice it has been found that if we assume that the price changes in various periods will balance out leaving the water value unchanged (i.e. use (D-13)), then, since the water values do change, convergence is very slow. On the other hand, if we assume that only the water value will change leaving the generation pattern unchanged (i.e. use (D-14)) then, since the generation does change very substantially, the algorithm becomes unstable.

Here we outline a scheme which has, in fact, proved to converge to the optimum very quickly. It involves two phases. In the first we use a "super-aggregated" model to determine the approximate average prices and water values for each trajectory arc. Then we are in the special situation where price changes cannot be expected to affect water values. So we can adjust the prices, period by period, using (D-13) and assuming the water values determined by the super-aggregated model rather than those from the previous iteration.

We first consider an alternative to the dual problem DA. We have developed a hierarchial model in which "energy prices" (μ or λ) are manipulated at one level and "water values" (ψ) are manipulated at the next (lower) level. We could, however, manipulate both simultaneously. We will assume that we know the general shape of the optimal trajectories. Experience has shown that, in the price adjustment process, the hydro trajectories change very little after the first or second iteration. So we have, for each reservoir (h), a tentative trajectory (such as that of Figure (5-6)(d)) consisting of a series of free trajectory arcs divided from one another by periods in which the reservoir storage is constrained. Let us index these trajectory arcs by $a=1,\ldots A_h$. Then we can see that, assuming the basic shape of this tentative trajectory is unlikely to change, we have:

$$\sum_{r\varepsilon a} q_h^r = Q_h^a \quad \text{(constant from iteration to iteration)} \quad \text{for all } a = 1,\ldots A_h \qquad \text{(D-26)}$$

That is, for price adjustments which are not too great, the hydro sub-models will react by adjusting the water values so as to re-distribute the release within each arc, but leave the total release in the arc unchanged. In order to cause greater effects than this price changes would have to be so great as to form new arcs, or join old ones.

Let us suppose that we are dealing with a simple model in which we are concerned only with the total releases and generation for each period. We can assign a multiplier ψ_h^a to $(D\text{-}26)_h^a$ and so obtain the Lagrangian:

$$\mathcal{L}'(g,e,\lambda,\psi) = \sum_{t=1}^{T} \sum_{n=1}^{N} \Big[C_n^t(g_{nT}^{t*}) - \lambda_n^t \Big(g_{nT}^{t*} + \sum_{h\varepsilon n} g_h^{t*} + \sum_{m=1}^{N} (f_{mn}^{t*} - e_{nm}^{t*}) - D_n^t \Big) \Big] + \sum_{h\varepsilon n} \sum_{a=1}^{A_h} \psi_h^a \Big(\Big(\sum_{r\varepsilon a} q_h^{r*} \Big) - Q_h^a \Big) \tag{D-27}$$

We could treat this Lagrangian just as we did $\mathcal{L}_c$, producing, as objective for our dual problem:

$$P'(\lambda,\psi) = \underset{g,e}{\mathrm{MIN}}\ \mathcal{L}'(g,e,\lambda,\psi) \tag{D-28}$$

P' is concave and differentiable just as P_c was and we have:

$$\frac{\partial P'}{\partial \lambda_n^t} = -\Big[g_{nT}^{t*} + \sum_{h\varepsilon n} g_h^{t*} + \sum_{m=1}^{N} (f_{mn}^{t*}\ e_{nm}^{t*}) - D_n^t \Big] \tag{D-29}$$

$$\frac{\partial P'}{\partial \psi_h^a} = \sum_{r\varepsilon a} (q_h^{r*} - Q_h^a \tag{D-30}$$

Then:

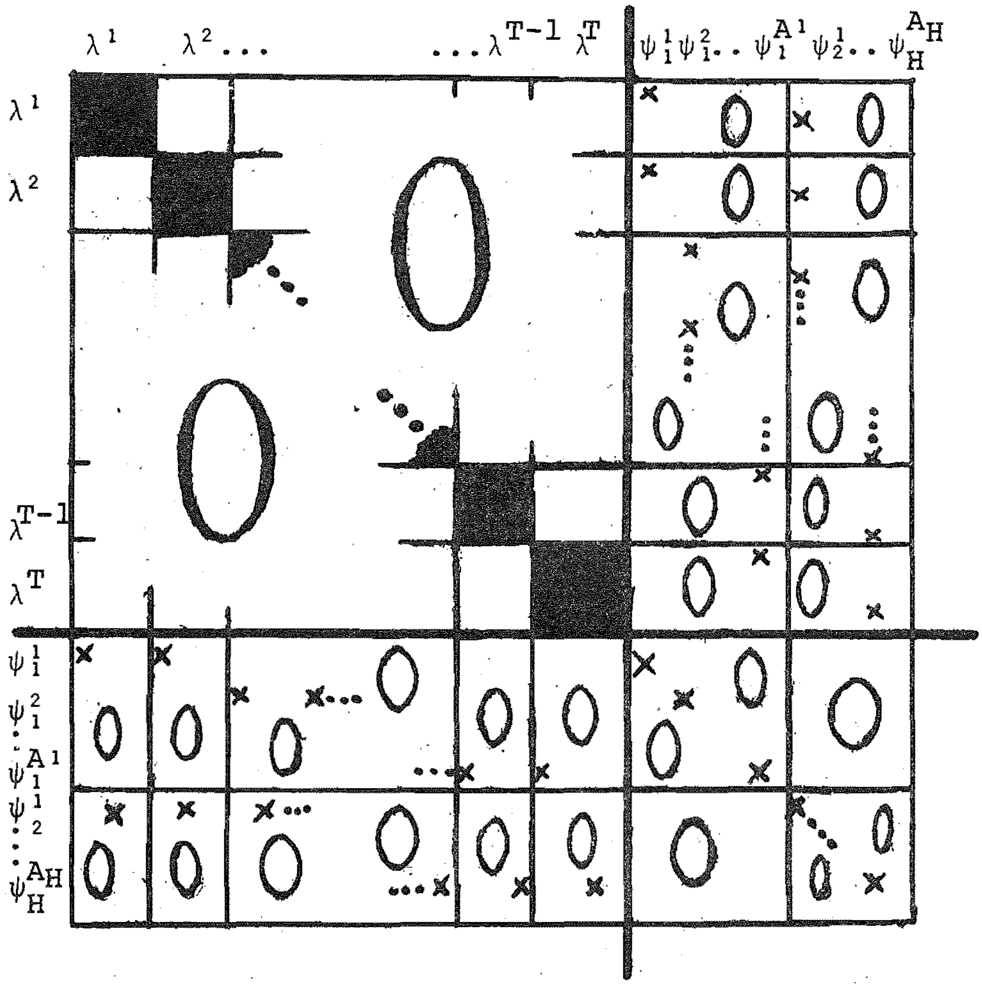
$$\frac{\partial^2 P'}{\partial\lambda_n^t\partial\lambda_n^r} = \begin{cases} -\left[\dfrac{\partial g_{nT}^{t*}}{\partial\lambda_n^t} + \sum\limits_{h\varepsilon n}\dfrac{\partial g_h^{t*}}{\partial\lambda_n^t} + \sum\limits_{m=1}^{N}\left(\dfrac{\partial f_{mn}^{t*}}{\partial\lambda_n^t} - \dfrac{\partial e_{nm}^{*}}{\partial\lambda_n^t}\right)\right] & \text{if } m=n,\ r=t \quad \text{(D-31)} \\ \dfrac{\partial e_{nm}^{t*}}{\partial\lambda_m^t} - \dfrac{\partial f_{mn}^{t*}}{\partial\lambda_m^t} & \text{if } m\neq n,\ r=t \quad \text{(D-32)} \\ 0 & \text{if } r\neq t \quad \text{(D-33)} \end{cases}$$

$$\frac{\partial^2 P'}{\partial\psi_h^a\partial\psi_k^b} = \begin{cases} \sum\limits_{r\varepsilon a}\dfrac{\partial g_h^{r*}}{\partial\psi_h^a} & \text{if } h=k,\ a=b \quad \text{(D-34)} \\ 0 & \text{if } h\neq k \text{ or } a\neq b \quad \text{(D-35)} \end{cases}$$

$$\frac{\partial^2 P'}{\partial\psi_h^a\partial\lambda_n^t} = \frac{\partial^2 P'}{\partial\lambda_n^t\partial\psi_h^a} = \begin{cases} -\dfrac{\partial g_h^{t*}}{\partial\psi_h^a}\left(=\dfrac{\partial g_h^{t*}}{\partial\lambda_n^t}\right) & \text{if } t\varepsilon a,\ h\varepsilon n \quad \text{(D-36)} \\ 0 & \text{if } t\not\varepsilon a \text{ or } h\not\varepsilon n \quad \text{(D-37)} \end{cases}$$

Thus the Hessian, $H'(\lambda,\psi)$, has a structure as in Figure (6-6).

Now we note that the inter-period interaction, expressed by the $\left(\frac{\partial g_h^r}{\partial\lambda_n^t}\right)$ entries dispersed through the matrix $H(\lambda)$, is summarised here (in $H'(\lambda,\psi)$) by the rows and columns corresponding to the arcs. The inversion of this whole



-NxN block whose entries are given by (D-31), (D-32).

-individual entries, given by (D-34) or (D-36).

FIGURE (6-6): Alternative Hessian matrix $H'(\lambda)$

matrix would, however, not be an attractive proposition. Our approach will be in two stages. Firstly we will partition the periods so as to form "super-periods". All of the periods in each such super-period will belong to the same arc of each reservoir's trajectory. Thus each arc will cover one or more super-periods. We will include constrained periods in arcs in such a way as to minimise the number of super-periods involved. Then, aggregating all the appropriate quantities, we may form a model of the type just described, whose dual problem is assigned the task of adjusting the average price in each super-period, along with all the water values, until the average generation and demand are equal in each super-period. The Hessian ($H''(\lambda,\psi)$) involved in this dual problem is of manageable size and so the model may be solved. The super-aggregated model serves to estimate the likely water values for the various arcs.

Secondly, assuming the water values just determined, we may form a hydro response curve as previously described and proceed to adjust the prices for the individual periods (using (D-13)) ignoring the inter-period interaction entirely.

This procedure has been successfully applied to the NZED system. Convergence has been very satisfactory except in the periods where some reservoir has its storage trajectory in constraint. This problem is dealt with in the next section. A sample solution is demonstrated in Section 7.5.

6.3.7 Accounting for Storage Constraints

In all of our previous discussion we have ignored the possibility that the storage in some reservoir may be at either its minimum or maximum level. This profoundly alters the nature of the hydro response curve.

Suppose that, at the optimum, reservoir h is constrained above in periods t-1,...t+2, then, assuming that $\bar{\bar{S}}^r = \bar{\bar{S}}$ (fixed) we have that:

$$q_h^{r*} = F_h^r \quad \text{for } r = t-1,\ t,\ t+1 \tag{D-38}$$

Thus, whatever values we specify for $\tilde{\lambda}^r$, our long-term hydro model will adjust the water values, $\tilde{\psi}^r$, so as to ensure that (D-38) holds. However the relationship between these prices is crucial. There are many special cases according as the trajectory immediately before and after the period in question is constrained, or part of a long arc, or unconstrained but constrained in the previous period. Here we will consider a single case which illustrates the general principles.

Suppose that, at some iteration of the algorithm, the trajectory for reservoir h is constrained above in periods t-1,...t+2. Suppose also that we have already determined $\tilde{\lambda}^{t-1}$ and $\tilde{\lambda}^{t+1}$. Now, experience has shown that the shape of the trajectory changes very little from iteration to iteration. We will assume that the trajectory is likely to remain constrained in periods t and t+2. Then we have that:

$$q^{t*} + q^{t+1*} = F = F^t + F^{t+1} \tag{D-39}$$

Now we wish to determine the response curve for period t. If the price $\tilde{\lambda}^t$ is high enough (relative to $\tilde{\lambda}^{t+1}$) then relase in period t will exceed F^t and so we will have $s^{t+1} < \bar{S}^{t+1}$ as in Figure (6-7). In this case we will have $\psi^t = \psi^{t+1}$ and hence:

$$\left.\frac{\partial\pi^t(q^t,\tilde{\lambda}^t)}{\partial q^t}\right|_{q^{t*}} = \left.\frac{\partial\pi^{t+1}(q^{t+1},\tilde{\lambda}^{t+1})}{\partial q^{t+1}}\right|_{q^{t+1*}} \tag{D-40}$$

Given $\tilde{\lambda}$, we can solve (D-39) and (D-40) simultaneously for q^t and q^{t+1}. The critical price level, $\bar{\tilde{\lambda}}^t$, is then given by:

$$\left.\frac{\partial\pi^t(q^t,\bar{\tilde{\lambda}}^t)}{\partial q^t}\right|_{F^t} = \left.\frac{\partial\pi^{t+1}(q^{t+1},\tilde{\lambda}^{t+1})}{\partial q^{t+1}}\right|_{F^{t+1}} \tag{D-41}$$

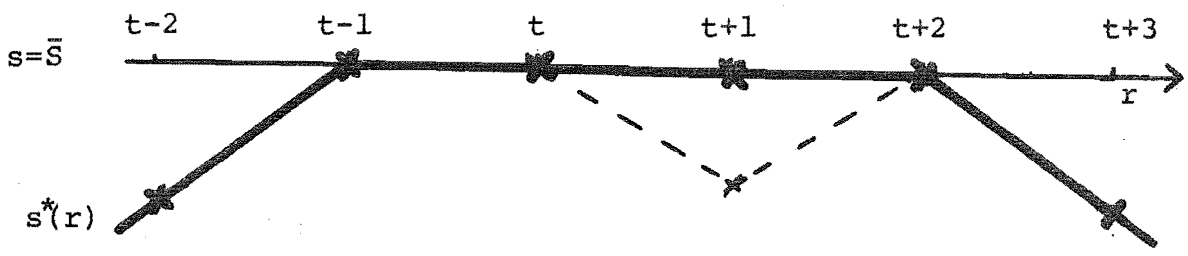
If $\lambda^t \leqslant \bar{\tilde{\lambda}}^t$, then the release in period t will remain at F^t (and so we will have $s^{t+1} = \bar{S}^{t+1}$).

If $\lambda^t > \bar{\tilde{\lambda}}^t$, then the optimal release q^t will be given by the simultaneous solution of (D-39) and (D-40) (and we will have $s^{t+1} < \bar{S}^{t+1}$). In a similar manner we can derive a lower limit, $\hat{\underline{\lambda}}^t$, such that:

$$\left.\frac{\partial\pi^t(q^t,\hat{\underline{\lambda}}^t)}{\partial q^t}\right|_{F^t} = \left.\frac{\partial\pi^{t-1}(q^{t-1},\tilde{\lambda}^{t-1})}{\partial q^{t-1}}\right|_{F^{t-1}} \tag{D-42}$$

Further, we have price limits, $\underline{\lambda}^t, \bar{\lambda}^t$, imposed by the minimum and maximum generation limits. That is:

$$\left.\frac{\partial\pi^t(q^t,\underline{\lambda}^t)}{\partial q^t}\right|_{\underline{Q}^t} = \left.\frac{\partial\pi^{t-1}(q^{t-1},\tilde{\lambda}^{t-1})}{\partial q^{t-1}}\right|_{(F^t+F^{t-1}-\underline{Q}^t)} \tag{D-43}$$



-current storage trajectory

-new trajectory for $\tilde{\lambda}^t$ high enough

FIGURE (6-7): Constrained storage trajectory.

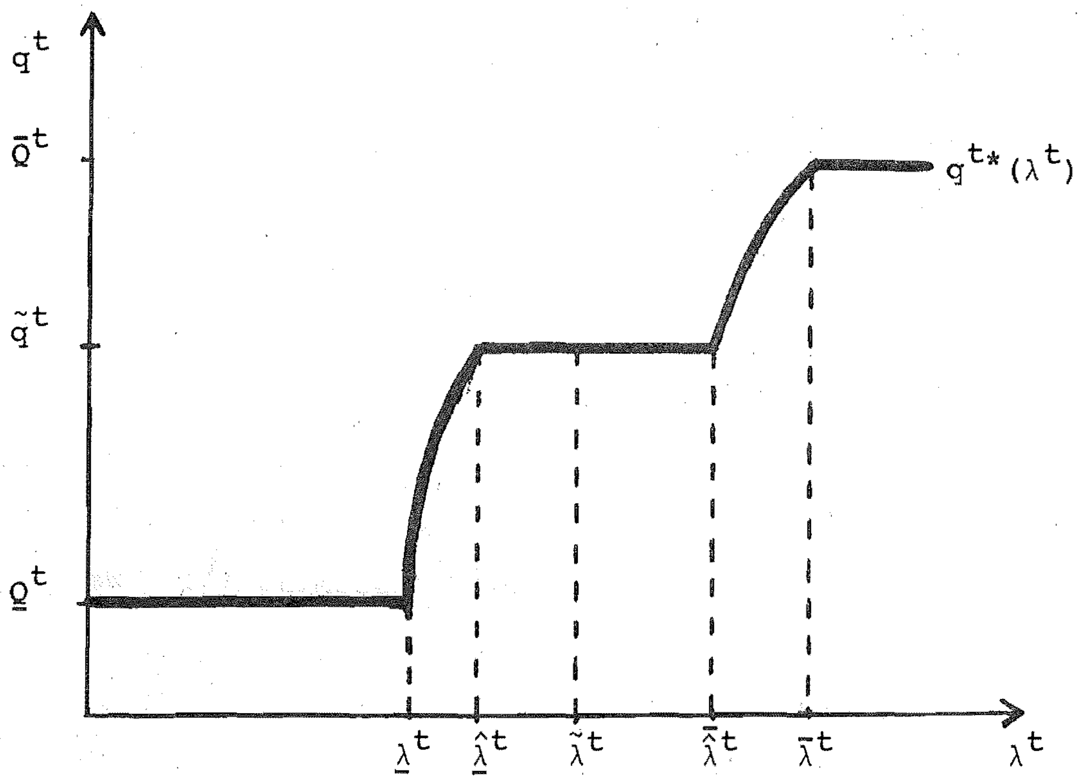


FIGURE (6-8): Release response curve for constrained period.

$$\left.\frac{\partial\pi^t(q^t,\bar{\lambda}^t)}{\partial q^t}\right|_{\bar{Q}^t} = \left.\frac{\partial\pi^{t+1}(q^{t+1},\tilde{\lambda}^{t+1})}{\partial q^{t+1}}\right|_{(F^t+F^{t+1}-\bar{Q}^t)} \tag{D-44}$$

Then we may summarise the optimal release, in period t, from this station as a function of the price, λ^t, by a curve such as that of Figure (6-8) and derive a corresponding generation response curve.

Here, provided that the price λ^t, remains between the bounds $(\hat{\underline{\lambda}}^t, \bar{\bar{\lambda}}^t)$ (determined by $(\tilde{\lambda}^{t-1}, \tilde{\lambda}^{t+1})$, there is no change in the generation level, g^t. Beyond these limits, however, response to price changes is quite marked.

The other cases can be treated in a similar manner, and so we can build up an accurate generation response curve for period t. Then we may apply the procedure of Sections 6.2.3 - 6.2.4 to find the optimal prices $\tilde{\lambda}^t$.

We have assumed in the above that we know both $\tilde{\lambda}^{t-1}$ and $\tilde{\lambda}^{t+1}$, which, in general, is quite unrealistic. We have actually implemented a scheme of this type which, in the first instance, assumes only $\tilde{\lambda}^{t-1}$. Ignoring the possibility of interaction with period t+1 we then find $\tilde{\lambda}^t$. Then, assuming $\tilde{\lambda}^t$, we can find $\tilde{\lambda}^{t+1}$. Now we can test whether these two are in fact compatible. If not we re-evaluate $\tilde{\lambda}^t$, assuming the values $\tilde{\lambda}^{t-1}$ and $\tilde{\lambda}^{t+1}$ just determined, then re-evaluate $\tilde{\lambda}^{t+1}$ assuming $\tilde{\lambda}^t$. We re-iterate in this fashion until the prices converge (two iterations are usually sufficient).

An example of the application of this procedure can be found in Section 7.5. There it can be seen that this method is capable of producing a very close matching of supply and demand in periods where some reservoirs are constrained. It can also be seen that this close match makes very little difference to the first period decision. This is not unexpected since the only interaction between such a period, t, and the first period is via those reservoirs which have unconstrained arcs from the first period to period t. Thus the modification discussed in this section, while greatly aiding the solution of the mathematical program, is largely unnecessary for the purposes of day to day scheduling operations.

6.3.8 Conclusions

We have described a price adjustment scheme which utilises the special properties of the dual objective function $P(\lambda)$ (and the alternative $P'(\lambda)$). We apply Newton's method to solve a super-aggregated model and so determine near optimal water values. We then ignore inter-period interaction and adjust prices in each period separately. In this adjustment we apply Newton's method repeatedly, using a locally accurate Hessian H^t, adjusting prices until the Hessian becomes inaccurate. We then re-evaluate the search direction and continue the process until convergence. Periods in which the storage trajectory of some reservoir is in constraint can be dealt with by the adaptation of Section 6.3.7. This whole approach has been implemented and results in quite rapid convergence (see Section 7.5). Further it appears that, for the purposes of long-term scheduling, we need not concern ourselves with finding accurate prices for periods in which several reservoirs are in storage constraint. Thus the problems discussed in Section 6.3.7 may be safely ignored.

CHAPTER 7

APPLICATION TO THE NZED SYSTEM

7.1 INTRODUCTION.

We have already given a brief account of the NZED system (Section 1.1) and of previous, or parallel, optimisation studies on it (Section 1.4). Here we briefly outline some results from the application to this system of our approach. Our intention, at this stage, is to demonstrate the feasibility of the approach rather than to draw any decisive conclusions about optimal system operation.

The basic aim of this study has been to develop models suitable for application to the NZED system. Thus our eventual goal is the embodiment of a large part of the theory developed here into practical everyday planning tools for long-term scheduling, short-term scheduling and tariff setting. At this stage we are able to present only partial results from the application of the theory in the preceding chapters to simplified models of the system. We have not, as yet, attempted to apply the more general theory of the later chapters (8 to 10).

A test program was written in ALGOL and run on the Burroughs B6718 computer at the University of Canterbury. This program was basically an implementation of the simple model of [47]. In order to be implemented at the NZED this program must be converted into a form suitable for running on the IBM 370/168 machine used there. Unfortunately ALGOL has not been made available on this particular machine, so that the program must be re-written in PL/I. This work is currently in progress, along with appropriate program generalisation.

In the next section we describe our simplified model of the system. Then, after discussion on the implementation of the various sub-models, we give an example of the global optimisation. Finally we describe some preliminary experience with the short-term hydro sub-model.

7.2 THE SIMPLIFIED MODEL

The purpose of the test program described here was to prove the feasibility of our approach. Thus we have worked with a simple model which approximates a representation of the NZED system and for which data was readily available.

We have implemented the simple single-segment aggregate model of [47] (cf. PAI in Section 2.5), using a planning horizon of one year divided into 52 weekly periods. We took no account of detailed short-term requirements and so our model merely ensures that sufficient energy is delivered to each region in each period. This is, of course, quite unrealistic. However there is no conceptual difficulty involved in generalising this model as has been shown in Section 2.4. The only increase in computational requirements for the long-term model would be the need to adjust more prices in the dual algorithm. Apart from this, the short-term requirements may be taken care of in the preparation of input tables as outlined in Chapters 3 to 5. The program currently being developed at NZED will, of course, take full account of short-term requirements. In Section 7.7 we outline some preliminary experience with the application of the short-term hydro scheduling model, PASH, developed for this purpose (see Section 5.5).

We have divided the country into the two obvious regions - the North and South Islands. We ignore the transmission systems within each island, and so our model may be represented as in Figure (7-1). (cf. Figure (1-2)). Here we have not included the Waikaremoana system as being "long-term controllable". This is because, despite its considerable nominal volume, it is subject to very severe leakage. Consequently it is not important as a long-term storage reservoir.

Data on inflows, storage capacities and projected demand levels (including losses) was adapted from input prepared for [5]. All of the inflows were represented in terms of their potential energy content (when utilised under average conditions) and tributary inflows were subtracted directly from loads. This obviously misrepresents the complex interaction between planned releases and tributary inflows. However, since this interaction will eventually be handled by a simple lookup table, this approach does not affect the structure of the model. Further, all data was "normalised" to an hourly basis. Thus, in this chapter, an inflow level of, say, 100 MW, indicates a flow of 100 MW per hour throughout the week. The primary reason for this was so that the characteristics of the various stations

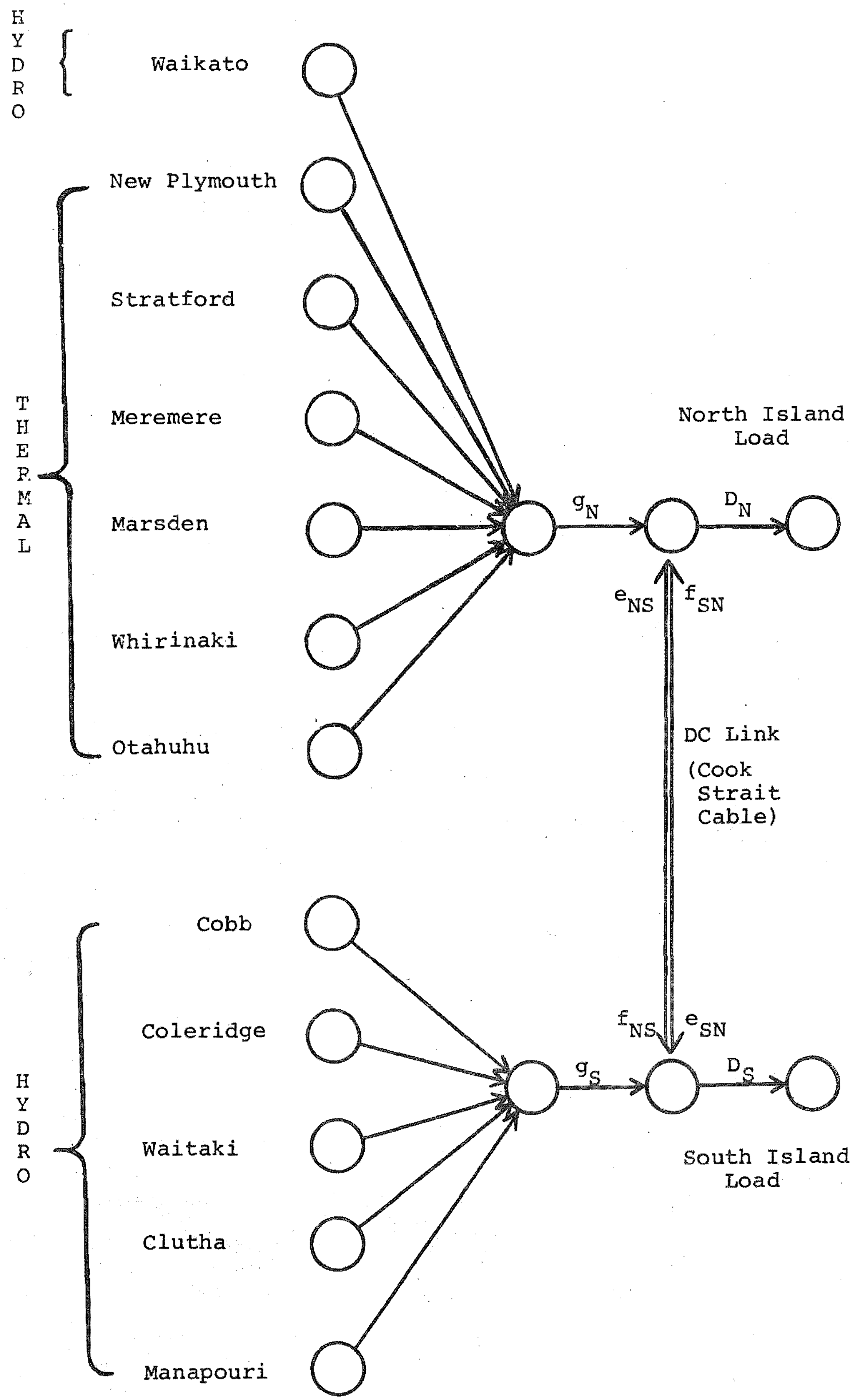


FIGURE (7-1): Simplified model of NZED system.

STATION	FUEL	χ ($/GJ)	α	β	γ	$\bar{G}$	$\hat{\lambda}^{**}$	$\hat{g}^{*}$	$\bar{\lambda}^{***}$	SLOPE
New Plymouth	Gas	0.89	0.00097	1.57	633	600	10.25	808	10.26	****
Stratford	Gas	0.89	$-0.0^{\dagger}$	2.89	116	240	-	-	10.79	****
Meremere	Coal	1.03	0.0081	0.47	253	210	12.37	177	14.37	16.64
Marsden	Heavy Oil	2.83	0.005	0.83	329	240	34.64	257	34.65	****
Whirinaki	Light Oil	3.94	$-0.0^{\dagger}$	2.89	116	200	-	-	49.20	****
Otahuhu††	Light Oil	3.99	$-0.0^{\dagger}$	2.89	224	280	-	-	52.99	****

(a) Physical characteristics of plant

Notes: $C(g) = 3.6\chi(\alpha g^2+\beta g+\gamma)$

factor of 3.6 converts fuel cost χ from $/GJ to $/MWH

† Gas turbines whose marginal cost decreases very slightly with load

†† We have ignored the two types of machine at Otahuhu which is never used in our model (cf. Figure (9-2))

(b) Response Characteristics

Notes: $^{*}\hat{g} = \sqrt{\frac{\gamma}{\alpha}}$

$$^{**}\hat{\lambda} = 3.6\chi(2\alpha\hat{g}+\beta) = \left.\frac{\partial C}{\partial g}\right|_{\hat{g}}$$

$$^{***}\bar{\lambda} = 3.6\chi(2\alpha\bar{G}+\beta) = \left.\frac{\partial C}{\partial g}\right|_{\bar{G}}$$

$^{****}\hat{g} > \bar{G} \Rightarrow$ maximum efficiency at full load

$\Rightarrow$ "step" response function

TABLE (7-1): Thermal station characteristics (from NZED files).

in the system could be expressed in "recognisable" units.

Arbitrary assumptions were made about the initial and final periods and storage levels. Thus the test results described in Section 7.6 do not correspond to any particular historical situation.

In the next two sections we describe in more detail the assumptions made about the thermal, exchange and hydro sectors and describe the implementation of the relevant sub-models. This is followed by a discussion of experience with the price adjustment process illustrated by a major example.

7.3 THE THERMAL AND EXCHANGE SUB-PROBLEMS

In our simple aggregated model we have only needed to apply the simple "instantaneous" models of Chapters 3 and 4. Table (7-1a) summarises relevant physical characteristics of the various thermal stations in the North Island system (as at 1/6/78). From these (and the current fuel prices), we may derive station response curves such as that of Figure (3-3). Table (7-1b) summarises the relevant characteristics of these curves. These station response curves may be aggregated into the system response curve shown in Figure (7-2), which may be summarised by its "corners" as in Table (7-2).

At this point we should perhaps note that the accuracy of some of the data is suspect, so that some caution is appropriate when interpreting these results. It may be seen, however, that, with the exception of Meremere each station is most efficient at its maximum

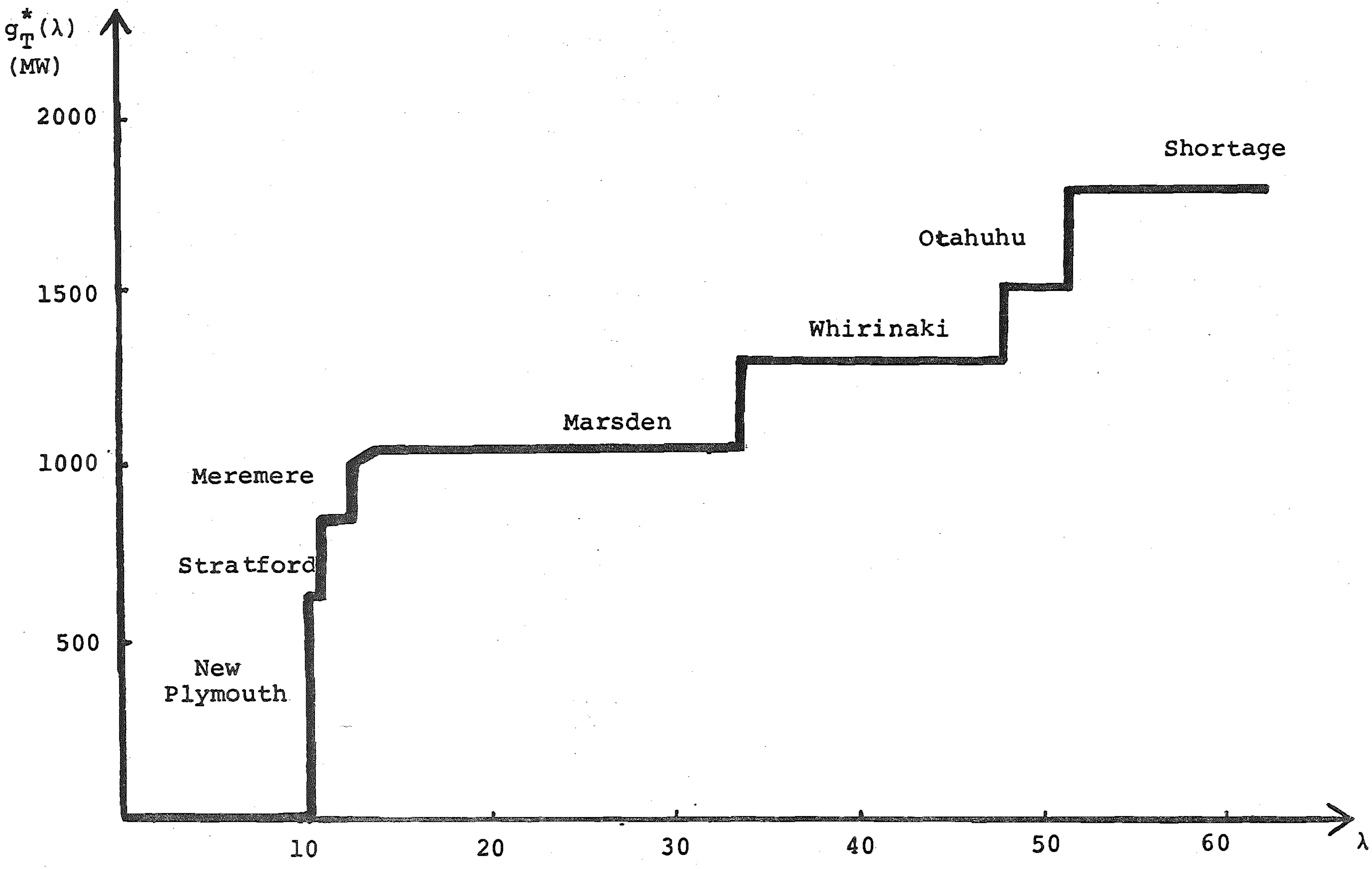


FIGURE (7-2): Thermal response curve

STATION	CORNER	PRICE	STATION OUTPUT	TOTAL OUTPUT	SLOPE BELOW	SLOPE ABOVE
Dummy	0	0	0	0	0	0.7
New Plymouth	1	10.2	0	7	0.7	6000.1
	2	10.3	600	607	6000.1	0.1
Stratford	3	10.7	0	607	0.1	2400.1
	4	10.8	240	847	2400.1	0.1
Meremere	5	12.3	0	848	0.1	1773.0
	6	12.4	177	1025	1773.0	16.6
	7	14.3	209	1056	16.6	0.1
Marsden	8	34.6	0	1058	0.1	2400.1
	9	34.7	240	1298	2400.1	0.1
Whirinaki	10	49.2	0	1300	0.1	2000.1
	11	49.3	201	1500	2000.1	0.1
Otahuhu	12	52.9	0	1500	0.1	2800.1
	13	53.0	281	1780	1800.1	0.1

TABLE (7-2): Corners of thermal response curve.

load (contrary to our expectation, based on earlier data). This "peak efficiency" corresponds to all machines being on maximum load. It is obvious that the same efficiency may be achieved by any number of machines at maximum load for any part of the period. So it is quite realistic to "patch" the discontinuity $((\hat{\mu}^-,0) - (\hat{\mu}^+,\hat{g}))$ assuming that any intermediate level of generation can be achieved at the same level of efficiency.

The only exchange problem faced in our model is that for the inter-island DC link. The loss function of this is in fact non-convex. For simplicity we have assumed a convex function:

$$L(e) = 0.0001e^2 \tag{E-13}$$

This gives marginal losses which are approximately correct over the likely operating range (12% at the (South-North) maximum transfer of 600 MW).

A procedure has been written in Burroughs ALGOL to produce and summarise the system thermal response curve. The results are stored on disk and read into the main program each time it is run. The exchange problem is solved as necessary.

7.4 THE LONG-TERM HYDRO SUB-PROBLEM

7.4.1 Simplified Sub-Models

In our simple model we have assumed, as in [47], that each hydro sub-system consists of one long-term reservoir with a single station immediately downstream from it. This approach is entirely valid when there is, in fact, only one long-term reservoir. (Since we may use the short-term hydro scheduling procedure, PASH, to produce, for any set of circumstances, an accurate aggregate output curve for such a composite station).

The Waitaki river sub-system, including lakes Tekapo and Pukaki, is the only one with more than one long-term controllable reservoir. However we have represented these two as a single aggregate reservoir, Waitaki. This is not likely to seriously mis-represent the system because, as is shown in Figure (1-3)(c), most of this sub-system's generating capacity is downstream from both reservoirs. (At the time the data was collected, before the commissioning of Tekapo B, the proportion was 97%. Currently it is 82%, but this will rise to 90% with the commissioning of the Ohau stations). Thus the marginal value of water stored in the two lakes at any time must be approximately equal, with water in Lake Tekapo always being at least as valuable as that in Lake Pukaki (given the stations currently commissioned). The storage levels in these two reservoirs will, in fact, be "balanced", using [4] so as to equalise the probability of spill. Here we aggregate the two storages, inflating both the capacity of Tekapo and its inflows so as to reflect their greater energy value, and deal with one representative water value for this aggregate reservoir. Thus we assume that the constraints on transfer from Tekapo to Pukaki will not significantly interfere with the "balancing" of the lakes. The advantage of this simplification is that we need only deal with single-reservoir systems in each hydro sub-model.

We may, at some future date, develop a more general long-term hydro sub-model (such as that of the EDF), so as to optimise the Waitaki system exactly. The present approach allows us to cut down on development effort during initial implementation.

The tables derived by the short-term hydro sub-model, PASH, will allow us to treat each (single-reservoir) river system as a single long-term reservoir with one station. Here we have simply assumed that each of these stations has a quadratic generation function:

$$g(q) = q - \alpha q^2 \tag{H-88}$$

(the linear coefficient being unity because the flows are expressed in terms of megawatt hours of potential production). We have not attempted to evaluate the α coefficients exactly because:

(a) the values available are known to be in error (more accurate values are being determined currently).

(b) The effort involved in estimation of valid aggregate values to represent long chains of stations does not seem appropriate. (Because PASH will supersede this whole approach).

(c) The other simplifications we have made, particularly our handling of tributaries, would invalidate any attempt at exactitude in this area.

(d) Accurate coefficients are not required to demonstrate the feasibility of the approach.

In view of these factors we have merely assumed that efficiency at maximum generation levels is about 90% of of the optimum and derived the α coefficients accordingly. Table (7-3) summarises the characteristics of the (equivalent aggregate) hydro plants dealt with.

Finally, we have taken no account of planned or forced outages. These may be accounted for approximately by, respectively, allowing the maximum generation to vary with time and derating the generation function. Eventually they may be dealt with by the methods of Section 5.5.3. In neither case do outages involve any conceptual change to the global optimisation. Accordingly, they have been ignored for reasons very similar to those above ((a) - (d)).

7.4.2 The Program

We have implemented the trajectory method of Section 5.4.3, expanded as outlined below. In our statement of that general algorithm Step (4) involved recognising if the optimal trajectory would be constrained (4(a)) and, if so, adjusting ψ until the initial arc of the trajectory was tangential to the first storage constraint encountered (4(b)). Here we consider that process in more detail.

Firstly, conditions under which it can be recognised that the optimal trajectory is constrained were demonstrated in the expansion of Step (4(a))). In order to use these conditions we must set some ψ, then simulate a trajectory forward until they are seen to hold or until the final period is reached. However, if the trajectory is constrained in one period it is likely to be constrained for some periods thereafter. In this case we could expend considerable unnecessary effort finding the appropriate water value. Instead we test for this possibility first.

RESERVOIR	$\bar{S}^{(1,2)}$	$\underline{Q}^{(1)}$	$\bar{Q}^{(1)}$	$\alpha^{(3)}$	$F^{(1,4)}$	$\bar{G}^{(5)}$	$\left.\frac{\partial g}{\partial q}\right\|_{\underline{Q}}^{(5)}$	$\left.\frac{\partial g}{\partial q}\right\|_{\bar{Q}}^{(5)}$	$S^{o\,(1,6)}$
Taupo	3452	73	427	0.0002	364	391	0.97	0.83	1500
Cobb	179	0	29	0.002	24	27	1.0	0.88	150
Coleridge	232	0	34	0.0025	14	31	1.0	0.83	150
Waitaki	3869	0	483	0.0001	296	460	1.0	0.90	1500
Hawea	1685	2	80	0.0008	26	75	0.99	0.87	1500
Manapouri	2821	310	820	0.0001	584	753	0.94	0.84	1500

Notes: (1) S and Q and F are all expressed in terms of MWH of generation.

(2) $\underline{S}$ has been normalised to zero.

(3) Here $g(q) = q - \alpha q^2$ (H-88). α values have been chosen arbitrarily so as to produce "reasonable" curvatures.

(4) F is average weekly inflow.

(5) $\bar{G} = \bar{Q} - \alpha(\bar{Q})^2$, $\left.\frac{\partial g}{\partial q}\right|_{\underline{Q}} = 1-2\alpha\underline{Q}$, $\left.\frac{\partial g}{\partial q}\right|_{\bar{Q}} = 1-2\alpha\bar{Q}$,

(6) Assumed.

TABLE (7-3): Physical characteristics of hydro reservoirs

So we replace Step (2):

(2) Choose $\psi^{\underline{t}} > 0$

by:

$$(2)' \text{ let: } \psi^{\underline{t}} = \left.\frac{\partial \pi^{\underline{t}}}{\partial q^{\underline{t}}}\right|_{F^{\underline{t}}} \qquad \text{(H-89)} \quad \text{(cf.(D-41))}$$

In other words we first choose the "critical" water value which will leave storage unchanged. (Here we have assumed that $\bar{S}^t = \bar{S}^{t+1}$ or $\underline{S}^t = \underline{S}^{t+1}$ as appropriate. Appropriate generalisations for other cases are obvious). If the trial trajectory corresponding to this value is not constrained beyond $\underline{t} + 1$ or if it next violates the same constraint which is binding in $\underline{t}$, then this value may not be optimal. Then we must find the true value using the iterative technique of Step 4(b) as before. If however, as is more often the case, the trajectory is next constrained at the opposite bound of the storage, then $\psi^{\underline{t}}$ is optimal and we set:

$$\tilde{s}^{\underline{t}+1} = \tilde{s}^t \qquad \text{(H-90)}$$

Then we have:

$$\bar{t} = \underline{t} + 1 \text{ (in Step (4)(c))} \qquad \text{(H-91)}$$

Secondly, in Step (5) and in our expansion of Step (4)(b), we required the adjustment of the water value, $\psi^{\underline{t}}$, so as to bring the corresponding trajectory closer to the target (or tangency to a constraint). In our implementation we use Newton's method to adjust ψ

so long as the trial storage levels all lie on one side of the target. If trial levels have been found on both sides of the target we adopt the "Method of False Positions". (Both methods are well known, see e.g. Chapter 2 of [12]). This latter adaptation was found to be necessary in order to prevent cycling. Cycling occurred because the bounds on releases caused the derivatives (w.r.t. ψ) of the end-point storage, $S^{T*}(\psi)$, to be discontinuous. When this approach is applied to produce a tangential trajectory arc a little more computation is required in order to determine the period in which the peak storage occurs. This does not, however, introduce any practical difficulties.

The sub-program for the long-term hydro problem consists of approximately 700 lines of Burroughs ALGOL. Execution time for this program segment is about 0.6 seconds to determine an optimal trajectory which is not constrained. (e.g. Taupo and Cobb on average inflows). Trajectories which become constrained take rather longer. Trajectories for typical South Island reservoirs (Manapouri, Waitaki and Coleridge on average inflows) constrained above in Autumn and below in Spring, take about 1.6 seconds to determine. Typical trajectories are shown in Figure (7-6).

7.5 THE GLOBAL OPTIMISATION

The solution to our long-term scheduling problem may be found by finding a corresponding set of λ prices. In Chapter 6 we have described an appropriate price adjustment process in some detail. In our simple aggregate model we do not, of course, need to concern ourselves with the derivation of detailed (μ) prices from aggregate (λ) prices. This coarse aggregated approach involves very "erratic" response functions (e.g. Figure (7-2)). Thus we have implemented the solution procedure of Section 6.3 with little modification.

An interactive ALGOL program has been written to test this method (utilising Burroughs CANDE). This program allows the user to select alternative methods for the various sub-problems and also to control output and modify variables. The program will recover from most faults and,if discontinued, leaves a partial solution on disk. This enables the optimisation to be re-started from that point. The program contains about 3500 lines, of which 1200 are concerned with declarations, comments, data handling, table preparation, etc. The hydro sub-model occupies a further 700 lines and the remainder deals with the price adjustment process. Of this a further 500 lines are concerned with the handling of user options and the preparation and manipulation of hydro "response curves" (by various alternative methods). A procedure to "guess" water values, using the procedure of Section 6.3.6, occupies about 300 lines. The remainder of the code (about 850 lines) deals with the actual price adjustment process.

This code, having been designed to test a variety of alternative approaches in an interactive fashion, contains much that will not be necessary for production purposes. On the other hand realistic modelling of the system will require considerable expansion of some sections, particularly data handling. Run-time storage requirements peak at about 25K, but 15K is a more average figure. The program has been developed and tested using CANDE in a multi-user environment on the B6718 machine at the University of Canterbury without causing undue strain on any resources. (Although requiring an extension on the usual 30 second CPU limit imposed to restrain CANDE usage). The remainder of this section discusses the solution of a typical problem. Computation times are quoted as appropriate. Since this experimental code has been designed for ease of programming with little consideration to efficiency it is probable that these could be improved considerably. They seem however to be in the "reasonable" range.

The system model in this example is exactly as discussed in the previous sections. Inflows and (net) demand levels are shown in Table (7-4). The initial period is taken to be the first week of January (mid-summer in New Zealand) and the planning horizon one year. We have assumed the initial storage levels shown in Table (7-3) and require that these same levels be restored in the final period. The prices quoted here are in terms of dollars per megawatt. Division by 10 gives the more common measure of cents per unit.

Week	DEMAND* North Island	DEMAND* South Island	STORABLE INFLOWS** Taupo	Cobb	Cole-ridge	Wai-taki	Hawea	Mana-pouri
1	986	365	318	17	14	491	33	625
2	1201	422	333	15	14	507	29	527
3	1219	440	317	14	14	497	27	540
4	1248	449	306	12	14	504	27	568
5	1266	457	318	13	13	578	31	624
6	1300	483	292	12	12	511	25	541
7	1314	501	301	16	12	503	26	573
8	1322	492	311	16	13	565	27	516
9	1331	509	287	12	12	502	25	492
10	1342	523	274	17	11	430	27	617
11	1348	549	258	15	11	351	22	506
12	1352	564	263	14	11	355	22	554
13	1375	571	249	14	11	348	22	618
14	1316	543	246	17	11	384	23	596
15	1386	562	267	21	12	363	26	573
16	1456	623	286	21	12	322	27	581
17	1463	615	314	23	13	256	25	646
18	1533	682	317	27	13	236	24	623
19	1573	708	323	26	12	216	22	545
20	1604	729	351	30	14	221	27	605
21	1649	754	341	34	13	226	24	570
22	1673	769	372	33	13	212	23	510
23	1670	787	380	31	12	171	18	460
24	1760	836	362	27	11	152	20	551
25	1768	854	438	25	11	140	19	506
26	1794	872	432	27	11	125	18	508
27	1802	871	457	23	11	124	20	512
28	1809	872	438	24	10	132	16	453
29	1802	864	441	32	11	135	19	416
30	1793	854	417	26	11	130	18	386
31	1770	843	416	28	10	127	19	512
32	1743	830	423	32	11	127	19	502
33	1715	812	436	22	11	124	18	437
34	1695	808	424	25	11	119	18	479
35	1683	770	410	28	12	120	21	455
36	1658	753	411	31	12	120	25	598
37	1621	715	432	32	14	150	30	768
38	1587	691	426	29	15	160	26	659
39	1549	666	436	28	14	171	25	666
40	1503	621	447	38	17	199	31	742
41	1471	576	436	30	17	239	30	716
42	1458	559	391	28	17	241	35	723
43	1390	523	415	32	19	315	37	730
44	1400	507	436	28	19	330	37	722
45	1399	490	422	29	20	324	35	720
46	1400	484	389	27	19	333	37	738
47	1395	497	370	24	19	342	36	728
48	1370	469	373	25	18	400	36	701
49	1351	473	362	21	17	426	35	614
50	1310	458	374	20	17	428	34	661
51	1282	455	371	17	16	418	30	590
52	1039	375	320	19	16	499	31	582

* Net demand - average MW load for week.

** Flows in terms of average equivalent MW per hour during week.

TABLE (7-4): Average demand and inflow levels

Our initial estimate of price levels was 12 \$/MW for the Summer and 30 \$/MW for the Winter. This corresponds to base-loading all stations up to and including Meremere in Summer and including Marsden in Winter. Initial setup and calculation of trajectories on the basis of these prices took 14 seconds. (This kind of "step" price structure has consistently required significantly more CPU time for calculation of trajectories). The first iteration using the techniques of Sections 6.3.2 - 6.3.5 to adjust prices on the basis of the water values in the initial solution took 11 seconds. Each such iteration requires the formation of hydro response curves such as that in Figure (7- 3). This curve is typical for the South Island in Winter. Table (7- 5) gives a summary of that curve in terms of its corners. In Table (7- 6), showing two typical blocks from the main diagonal of the Hessian, we may see the relative magnitude of the various derivatives. The result of the first iteration is summarised in Table (7-7)

In the second iteration we applied the technique of Section 6.3.6 to "guess" water values for the various trajectory arcs. The structure of the special Hessian for the first phase of this procedure is shown in Figure (7- 4) (cf. Figure (6-6)). This may be replaced by the two much smaller matrices shown in Figure (7- 4) (b). Typical values are shown in Table (7- 8). This iteration, which took 17 seconds, was followed by an "ordinary" price adjustment iteration taking 11 seconds. Such "ordinary" iterations (that is, applying the techniques of Sections 6.3.2 to 6.3.5 without modification) are frequently helpful in allowing

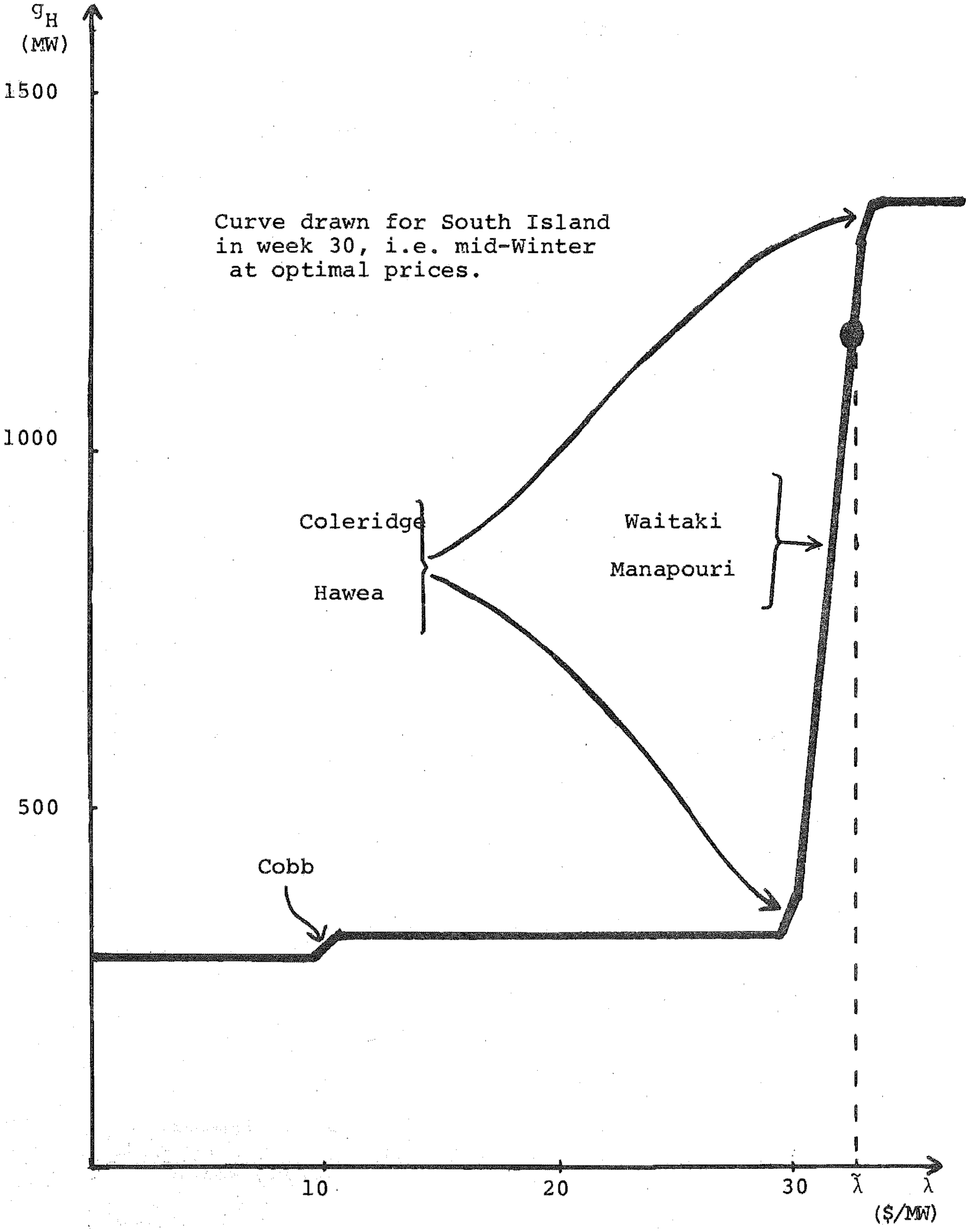


FIGURE (7-3): Typical hydro response curve

CORNER	λ	STATION	GENERATION		SLOPE
			Station	Total	(Above)
0	0	0	-	302	0.1
1	9.348	Cobb	0	302.9	22.11
2	10.575	Cobb	27	330.1	0.1
3	28.316	Coleridge	0	331.8	6.77
4	29.432	Hawea	2	339.4	31.27
5	29.887	Manapouri	300	351.6	35.00
6	29.888	Waitaki	0	352.0	299.74
7	32.58	(*)	-	1,158.9	257.74
8	33.084	Waitaki	460	1,288.8	125.95
9	33.533	Manapouri	753	1,345.4	19.46
10	33.644	Hawea	75	1,347.5	4.00
11	34.116	Coleridge	31	1,349.4	0.1

(* - current price).

TABLE (7-5): Corners of hydro response curve (cf. Figure (7-3))

(a) N.I. (Thermal) response low
S.I. (Hydro) response high

-659	461
461	-1445

(b) N.I. (Thermal) response high
S.I. (Hydro) response high

-6664	464
464	-1447

TABLE (7-6): Typical blocks from leading diagonal of Hessian.

TABLE (7-7): Results from first iteration

	NORTH ISLAND							SOUTH ISLAND						
R	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN
1	11.27	986	-22	993	847	-29	146	11.33	365	111	447	1	29	446
2	11.82	1201	-24	1093	847	84	246	11.62	422	173	680	1	-85	679
3	11.87	1219	-25	1102	847	92	254	11.65	440	174	707	1	-93	706
4	11.94	1248	-26	1114	847	108	266	11.68	449	174	732	1	-109	731
5	11.99	1266	-27	1121	847	118	274	11.70	457	174	750	1	-119	749
6	12.09	1300	-27	1138	848	135	290	11.76	483	174	795	1	-137	794
7	12.14	1314	-26	1146	848	142	298	11.79	501	174	819	1	-145	818
8	12.15	1322	-26	1148	848	149	300	11.78	492	174	817	1	-151	816
9	12.19	1331	-25	1153	848	153	306	11.81	509	173	837	1	-155	836
10	12.22	1342	-25	1159	848	158	312	11.83	523	173	856	1	-161	855
11	12.26	1348	-25	1165	848	159	317	11.86	549	172	882	1	-161	881
12	12.28	1352	-25	1168	848	159	320	11.88	564	171	897	1	-162	896
13	12.31	1375	-25	1187	862	164	324	11.90	571	171	908	1	-167	907
14	12.18	1316	-25	1152	848	139	304	11.83	543	173	857	1	-141	856
15	12.31	1386	-25	1194	869	168	325	11.89	562	171	903	1	-170	902
16	12.35	1456	-25	1273	942	158	331	11.96	623	113	897	1	-161	896
17	12.36	1463	-24	1290	957	149	332	11.99	615	82	848	1	-151	847
18	12.42	1533	-24	1367	1025	142	342	12.07	682	30	856	1	-144	855
19	12.54	1573	-25	1386	1027	162	359	12.13	708	-68	805	1	-165	803
20	12.66	1604	-15	1405	1029	184	375	12.19	729	-84	833	1	-188	831
21	34.65	1649	0	1580	1190	69	391	34.17	754	178	1001	3	-69	998
22	34.67	1673	0	1608	1218	65	391	34.21	769	177	1011	3	-65	1008
23	34.67	1670	0	1611	1220	59	391	34.26	787	176	1022	3	-59	1019
24	34.97	1760	0	1689	1299	71	391	34.48	836	171	1078	3	-71	1075
25	35.12	1768	-1	1689	1299	78	391	34.57	854	169	1102	3	-79	1098
26	35.48	1794	0	1689	1299	105	391	34.73	872	164	1142	3	-106	1138
27	35.57	1802	0	1689	1299	113	391	34.75	871	163	1148	3	-114	1145
28	35.65	1809	0	1689	1299	120	391	34.78	872	162	1155	3	-121	1152
29	35.54	1802	0	1689	1299	113	391	34.73	864	164	1142	3	-114	1138
30	35.40	1793	0	1689	1299	104	391	34.66	854	166	1125	3	-105	1121
31	35.10	1770	-1	1689	1299	80	391	34.54	843	170	1094	3	-81	1090
32	34.77	1743	0	1689	1298	54	391	34.39	830	173	1057	3	-54	1054
33	34.69	1715	0	1663	1272	52	391	34.32	812	175	1039	3	-53	1036
34	34.68	1695	0	1642	1251	53	391	34.31	808	175	1036	3	-53	1033
35	34.67	1683	0	1618	1227	65	391	34.22	770	177	1012	3	-65	1009
36	34.66	1658	0	1588	1198	69	391	34.17	753	178	1001	3	-70	997
37	34.63	1621	0	1531	1141	89	391	34.01	715	236	1041	3	-90	1037
38	34.62	1587	0	1491	1100	96	391	33.95	691	237	1025	3	-97	1021
39	34.57	1549	0	1449	1058	100	391	33.87	666	238	1005	3	-101	1002
40	33.76	1503	0	1449	1058	54	391	33.39	621	300	976	3	-54	973
41	12.36	1471	-25	1283	951	164	332	11.95	576	94	837	1	-167	835
42	12.35	1458	-25	1266	935	167	331	11.93	559	103	832	1	-170	831
43	12.31	1390	-25	1195	870	170	325	11.89	523	147	843	1	-173	842
44	12.31	1400	-25	1199	874	176	325	11.87	507	147	834	1	-179	833
45	12.31	1399	-25	1193	868	181	325	11.86	490	148	822	1	-184	821
46	12.31	1400	-25	1192	868	183	325	11.85	484	148	818	1	-186	817
47	12.31	1395	-25	1192	867	179	325	11.86	497	148	826	1	-182	825
48	12.27	1370	-25	1167	848	179	319	11.83	469	148	799	1	-182	798
49	12.23	1351	-25	1160	848	166	312	11.82	473	149	791	1	-169	790
50	12.11	1310	-26	1142	848	142	294	11.76	458	150	752	1	-144	751
51	12.04	1282	-27	1130	848	125	283	11.74	455	150	731	1	-126	730
52	11.45	1039	-21	1027	847	-9	180	11.47	375	153	519	1	9	518

KEY

R	Period
P	Price
DEM	Demand
XSUP	Excess supply
TOTG	Total generation
TGEN	Thermal generation
NETEX	Net exchange
HGEA	Hydro generation

the prices and water values to "settle" after more major adjustments. A further iteration including the water value guessing procedure took 15 seconds and resulted in the much improved schedule summarised in Table (7-9). This schedule, when modified so as to be feasible in the first period, would, in fact, be quite adequate for operational purposes. The major discrepancies in the South Island in periods 16 to 19 and 36 to 42 are due to reservoirs being in constraint during those periods. In order to remove these discrepancies a special iteration was undertaken, using the technique of Section 6.3.7 and taking 19 seconds. This resulted in some disturbance to the Winter and Spring trajectory arcs which was eliminated by an ordinary iteration (11 seconds) followed by a water value guessing iteration (11 seconds)and finally a further ordinary iteration (11 seconds). The final results are displayed in Table (7-10).

The optimal trajectories,which in fact show no visible change from those formed after the first few iterations, are shown in Figure (7- 6). It may be seen that, with the exception of Lake Hawea, final water values are very similar to initial values. This is the expected consequence of reasonable initial and target levels for storage trajectories. An immediate implication from this is that, unless special circumstances apply, the target storage level for Lake Hawea has been set too high. A lower level would allow much greater flexibility of operation for both this year and the next. Figure (7- 7) shows the convergence of the solution to the optimum in

	λ^1_1	λ^1_2	λ^2_1	λ^2_2	λ^3_1	λ^3_2	ψ_{11}	ψ_{21}	ψ^1_{22}	ψ^2_{22}	ψ^3_{22}	ψ^1_{24}	ψ^2_{24}
λ^1_1	X	X	O	O	O	O	X	O	O	O	O	O	O
λ^1_2	X	X	O	O	O	O	O	X	X	O	O	X	O
λ^2_1	O	O	X	X	O	O	⊗	O	O	O	O	O	O
λ^2_2	O	O	X	X	O	O	O	⊗	O	X	O	O	⊗
λ^3_1	O	O	O	O	X	X	X	O	O	O	O	O	O
λ^3_2	O	O	O	O	X	X	O	X	O	O	X	O	⊗
ψ_{11}	X	O	⊗	O	X	O	X	O	O	O	O	O	O
ψ_{21}	O	X	O	⊗	O	X	O	X	O	O	O	O	O
ψ^1_{21}	O	X	O	O	O	O	O	O	X	O	O	O	O
ψ^2_{21}	O	O	O	X	O	O	O	O	O	X	O	O	O
ψ^2_{21}	O	O	O	O	O	X	O	O	O	O	X	O	O
ψ^1_{24}	O	X	O	O	O	O	O	O	O	O	O	X	O
ψ^2_{24}	O	O	O	⊗	O	⊗	O	O	O	O	O	O	X

Regions: 1 = North Island
2 = South Island

"Super-periods": 1 = current Summer
2 = next Winter
3 = next Summer

Reservoirs: (1,1) = Taupo
(2,1) = Cobb
(2,2) = Coleridge
(2,3) = Waitaki
(2,4) = Hawea
(2,5) = Manapouri

X indicates non-zero entry

O indicates entry will always be 0

⊗ indicates entry which could have been non-zero but which is zero in this example because of release limits.

FIGURE (7-4): Structure of alternative Hessian (cf. Figure (6-6)).

(a) SUMMER

	λ^1_1	λ^1_2	λ^3_1	λ^3_2	ψ_{11}	ψ_{21}
λ^1_1	X	X	O	O	X	O
λ^1_2	X	X	O	O	O	X
λ^3_1	O	O	X	X	X	O
λ^3_2	O	O	X	X	O	X
ψ_{11}	X	O	X	O	X	O
ψ_{21}	O	X	O	X	O	X

(b) WINTER

	λ^2_1
λ^2_1	X

Note: All other relevant prices and values may be derived from these.

FIGURE (7-5): Structure of alternative Hessian sub-matrices.

(a) SUMMER

-33,283	7,843	0	0	3,674	0
7,843	-8,808	0	0	0	394
0	0	-20,750	4,750	2,309	0
0	0	4,750	-5,476	0	249
3,674	0	2,309	0	-6,841	0
0	394	0	249	0	-694

(b) WINTER

19,205

TABLE (7-8): Values for sub-matrices.

	NORTH ISLAND							SOUTH ISLAND						
R	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN
1	10.29	966	-9	766	559	210	208	9.85	365	-12	568	1	-215	567
2	10.69	1201	-5	891	607	304	284	10.02	422	-3	733	1	-314	732
3	10.71	1219	-4	911	625	303	287	10.04	440	-3	750	1	-313	749
4	10.72	1248	-4	940	651	303	289	10.05	449	-2	760	1	-313	759
5	10.73	1266	-4	959	669	303	290	10.06	457	-2	767	1	-312	766
6	10.74	1300	-4	997	704	298	293	10.08	483	-1	790	1	-308	789
7	10.75	1314	-4	1015	721	295	294	10.09	501	-1	804	1	-304	803
8	10.75	1322	-4	1020	725	298	294	10.09	492	-1	798	1	-307	797
9	10.75	1331	-4	1033	737	294	295	10.10	509	-1	811	1	-303	810
10	10.76	1342	-4	1047	750	291	296	10.11	523	0	823	1	-300	822
11	10.76	1346	-4	1060	762	284	297	10.13	549	0	842	1	-293	841
12	10.77	1352	-4	1068	770	280	298	10.15	564	0	853	1	-289	851
13	10.78	1375	-4	1090	791	280	299	10.15	571	0	860	1	-289	859
14	10.75	1316	-4	1029	734	283	295	10.13	543	0	834	1	-291	833
15	10.78	1386	-4	1093	798	284	300	10.15	562	0	855	1	-293	854
16	10.91	1456	-4	1169	847	283	322	10.27	623	-11	903	1	-291	902
17	10.98	1463	-5	1182	847	276	334	10.36	615	-22	877	1	-284	876
18	11.33	1533	-1	1237	847	295	390	10.64	682	-107	879	1	-304	878
19	11.76	1573	0	1238	847	335	391	10.96	708	-251	804	1	-347	803
20	11.94	1604	0	1238	847	360	391	11.03	729	-272	837	1	-360	836
21	33.11	1649	0	1449	1058	200	391	31.76	754	2	960	3	-204	957
22	33.44	1673	0	1449	1058	224	391	31.91	769	4	1002	3	-229	999
23	33.43	1670	0	1449	1058	221	391	31.96	787	4	1018	3	-226	1014
24	34.60	1760	0	1460	1069	300	391	32.46	836	6	1152	3	-310	1149
25	34.61	1768	0	1473	1083	295	391	32.50	854	6	1164	3	-304	1161
26	34.62	1794	0	1504	1113	290	391	32.55	872	6	1177	3	-299	1173
27	34.63	1802	0	1511	1121	291	391	32.55	871	6	1176	3	-300	1173
28	34.63	1809	0	1519	1128	290	391	32.55	872	6	1177	3	-299	1174
29	34.63	1802	0	1509	1119	293	391	32.53	864	6	1172	3	-302	1169
30	34.62	1793	0	1497	1107	296	391	32.51	854	6	1165	3	-305	1162
31	34.61	1770	0	1472	1081	298	391	32.48	843	6	1157	3	-308	1154
32	34.50	1743	0	1449	1058	294	391	32.41	830	6	1139	3	-303	1136
33	34.10	1715	0	1449	1058	266	391	32.23	812	6	1092	3	-274	1088
34	33.65	1695	0	1449	1058	246	391	32.14	808	6	1066	3	-252	1063
35	33.56	1683	0	1449	1058	234	391	31.95	770	4	1014	3	-240	1011
36	33.21	1658	0	1449	1058	209	391	31.79	753	2	968	3	-214	965
37	31.98	1621	0	1449	1058	172	391	30.86	715	31	922	3	-175	918
38	31.55	1587	0	1449	1058	138	391	30.67	691	51	882	3	-140	879
39	30.98	1549	0	1449	1058	100	391	30.35	666	88	855	3	-101	852
40	29.33	1503	0	1443	1058	55	391	29.01	621	255	930	3	-55	928
41	12.09	1471	0	1238	846	233	391	11.51	576	130	945	1	-239	944
42	10.79	1458	-4	1131	829	323	302	10.07	559	-82	811	1	-334	810
43	10.72	1390	-4	951	662	434	290	9.75	523	-433	545	1	-455	544
44	10.77	1400	-4	1077	778	319	298	10.06	507	-5	832	1	-330	831
45	10.77	1399	-4	1071	773	324	298	10.05	490	-5	820	1	-335	819
46	10.77	1400	-4	1070	772	326	298	10.04	484	-4	817	1	-337	816
47	10.77	1395	-4	1069	771	321	298	10.05	497	-5	825	1	-333	824
48	10.76	1370	-4	1038	743	327	296	10.03	469	-5	803	1	-339	802
49	10.75	1351	-4	1022	728	325	295	10.03	473	-5	804	1	-336	803
50	10.73	1310	-4	981	689	325	292	10.01	458	-5	789	1	-336	788
51	10.72	1282	-4	954	665	323	290	10.01	455	-5	785	1	-334	784
52	10.30	1039	-9	799	590	230	209	9.81	375	-5	606	1	-236	605

KEY

R	Period
P	Price
DEM	Demand
XSUP	Excess supply
TOTG	Total generation
TGEN	Thermal generation
NETEX	Net exchange
HGEN	Hydro generation

TABLE (7-7): Results from fourth iteration

TABLE (7-8): Results from final (eighth) iteration.

	NORTH ISLAND							SOUTH ISLAND						
R	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN
1	10.30	986	-1	785	578	200	207	9.87	365	-5	565	1	-204	564
2	10.71	1201	6	909	623	298	286	10.05	422	4	733	1	-307	732
3	10.71	1219	6	930	643	295	287	10.06	440	4	748	1	-304	747
4	10.73	1248	6	959	670	295	289	10.07	449	5	758	1	-304	757
5	10.73	1266	6	978	687	294	291	10.08	457	5	765	1	-303	764
6	10.75	1300	6	1016	723	290	293	10.11	483	6	788	1	-299	786
7	10.76	1314	6	1034	740	286	294	10.12	501	6	802	1	-295	801
8	10.76	1322	6	1039	744	289	295	10.11	492	6	796	1	-298	795
9	10.76	1331	6	1052	756	285	296	10.13	509	6	809	1	-294	808
10	10.77	1342	6	1066	769	282	297	10.14	523	6	820	1	-291	819
11	10.77	1348	6	1079	781	276	297	10.16	549	7	839	1	-284	838
12	10.78	1352	6	1086	780	272	298	10.17	564	7	850	1	-279	849
13	10.78	1375	6	1109	810	272	300	10.18	571	7	858	1	-280	857
14	10.76	1316	6	1048	752	274	295	10.15	543	7	832	1	-282	831
15	10.79	1386	6	1117	817	275	300	10.18	562	7	852	1	-284	851
16	10.96	1456	6	1177	847	284	330	10.32	623	-6	910	1	-293	909
17	11.05	1463	5	1193	847	275	345	10.43	615	-6	892	1	-283	891
18	12.30	1533	0	1340	949	193	391	11.87	682	0	879	1	-197	878
19	32.17	1573	0	1449	1058	124	391	31.36	708	14	848	3	-126	845
20	32.57	1604	0	1449	1058	155	391	31.55	729	15	901	3	-158	898
21	33.22	1649	0	1449	1058	200	391	31.86	754	5	964	3	-204	960
22	33.55	1673	0	1449	1058	224	391	32.01	769	6	1005	3	-229	1002
23	33.58	1670	0	1449	1058	221	391	32.06	787	7	1020	3	-226	1017
24	34.61	1760	0	1469	1078	291	391	32.53	836	9	1146	3	-300	1142
25	34.61	1768	0	1482	1092	286	391	32.57	854	9	1158	3	-295	1154
26	34.63	1794	0	1513	1122	281	391	32.62	872	9	1170	3	-289	1167
27	34.63	1802	0	1520	1130	282	391	32.62	871	9	1170	3	-290	1167
28	34.63	1809	0	1527	1137	282	391	32.62	872	9	1171	3	-290	1168
29	34.63	1802	0	1518	1128	284	391	32.60	864	9	1165	3	-292	1162
30	34.62	1793	0	1506	1116	287	391	32.58	854	9	1159	3	-295	1155
31	34.61	1770	0	1480	1090	290	391	32.55	843	9	1151	3	-298	1147
32	34.60	1743	0	1450	1060	293	391	32.51	830	9	1141	3	-302	1138
33	34.21	1715	0	1449	1058	266	391	32.34	812	9	1094	3	-274	1091
34	33.96	1695	0	1449	1058	246	391	32.24	808	8	1069	3	-252	1066
35	33.67	1683	0	1449	1058	234	391	32.05	770	7	1017	3	-240	1014
36	33.31	1658	0	1449	1058	209	391	31.89	753	5	972	3	-214	968
37	31.39	1621	0	1449	1058	172	391	30.29	715	11	901	3	-175	898
38	30.70	1587	0	1449	1058	138	391	29.84	691	16	847	3	-140	844
39	12.38	1549	0	1384	994	165	391	11.97	666	0	834	1	-168	833
40	11.18	1503	3	1214	847	292	366	10.51	621	-13	909	1	-301	908
41	10.99	1471	5	1182	847	295	334	10.32	576	-8	871	1	-304	870
42	10.95	1458	6	1175	847	289	327	10.29	559	6	862	1	-298	861
43	10.78	1390	6	1109	809	287	300	10.15	523	-3	816	1	-296	815
44	10.79	1400	6	1113	813	293	300	10.13	507	-3	806	1	-302	805
45	10.76	1399	6	1107	808	298	299	10.12	490	-2	795	1	-307	794
46	10.78	1400	6	1107	807	300	299	10.12	484	-2	791	1	-309	790
47	10.78	1395	6	1106	806	295	299	10.13	497	-2	799	1	-305	798
48	10.77	1370	6	1075	778	301	297	10.10	469	-2	778	1	-311	777
49	10.76	1351	6	1059	763	298	296	10.10	473	-2	779	1	-307	778
50	10.75	1310	6	1018	724	298	293	10.09	458	-1	765	1	-308	764
51	10.74	1282	6	991	700	297	291	10.08	455	-1	760	1	-306	759
52	10.35	1039	1	826	607	214	218	9.90	375	-1	593	1	-219	592

KEY
(see opposite)

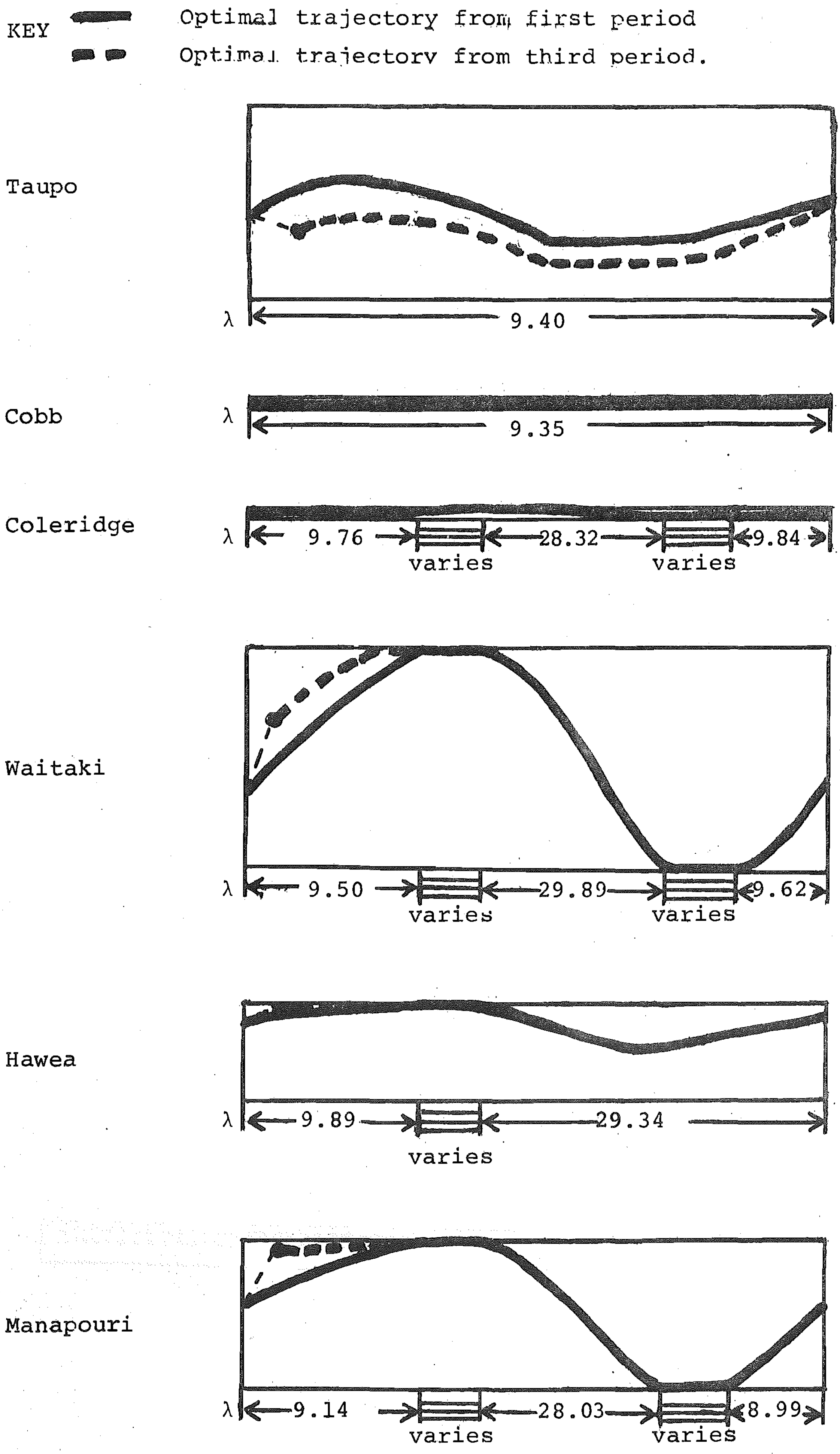


FIGURE (7-6): Storage trajectories.

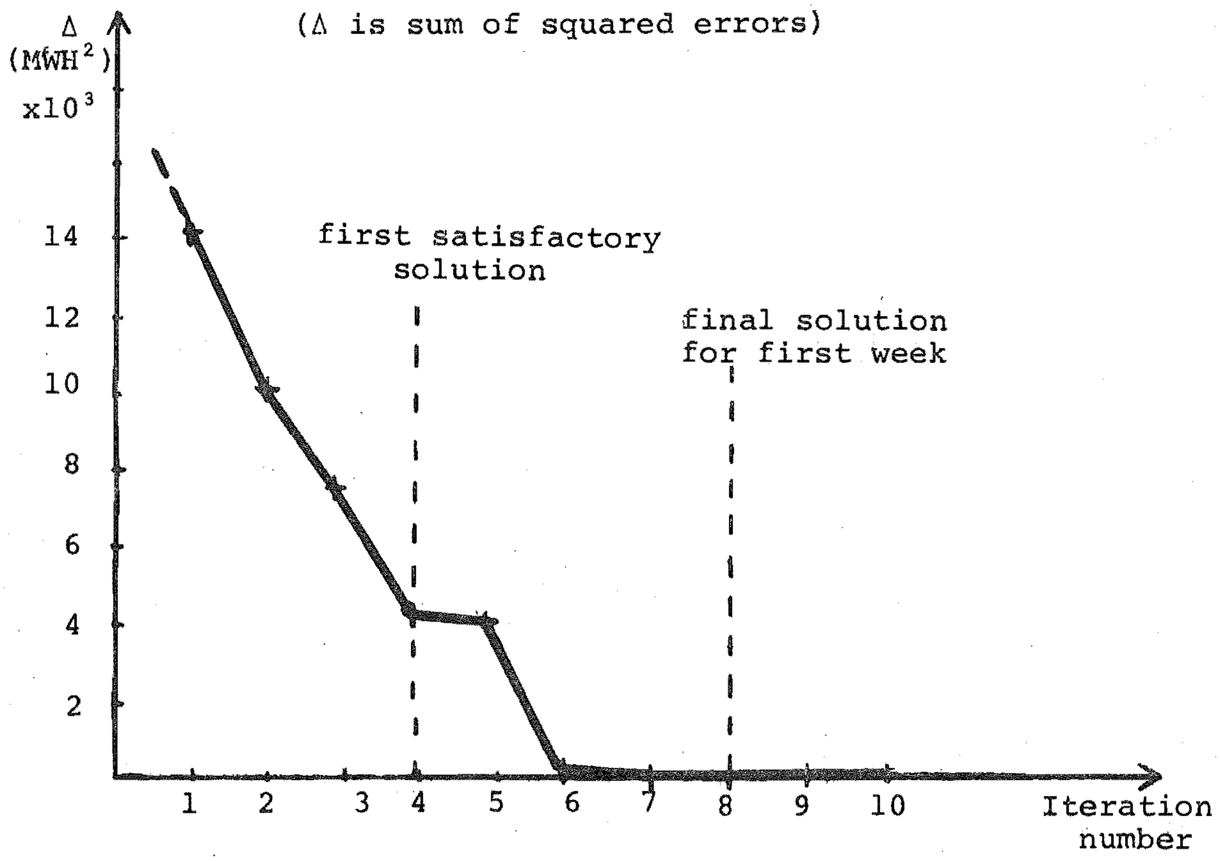


FIGURE (7-7): Convergence of sum of squared errors.

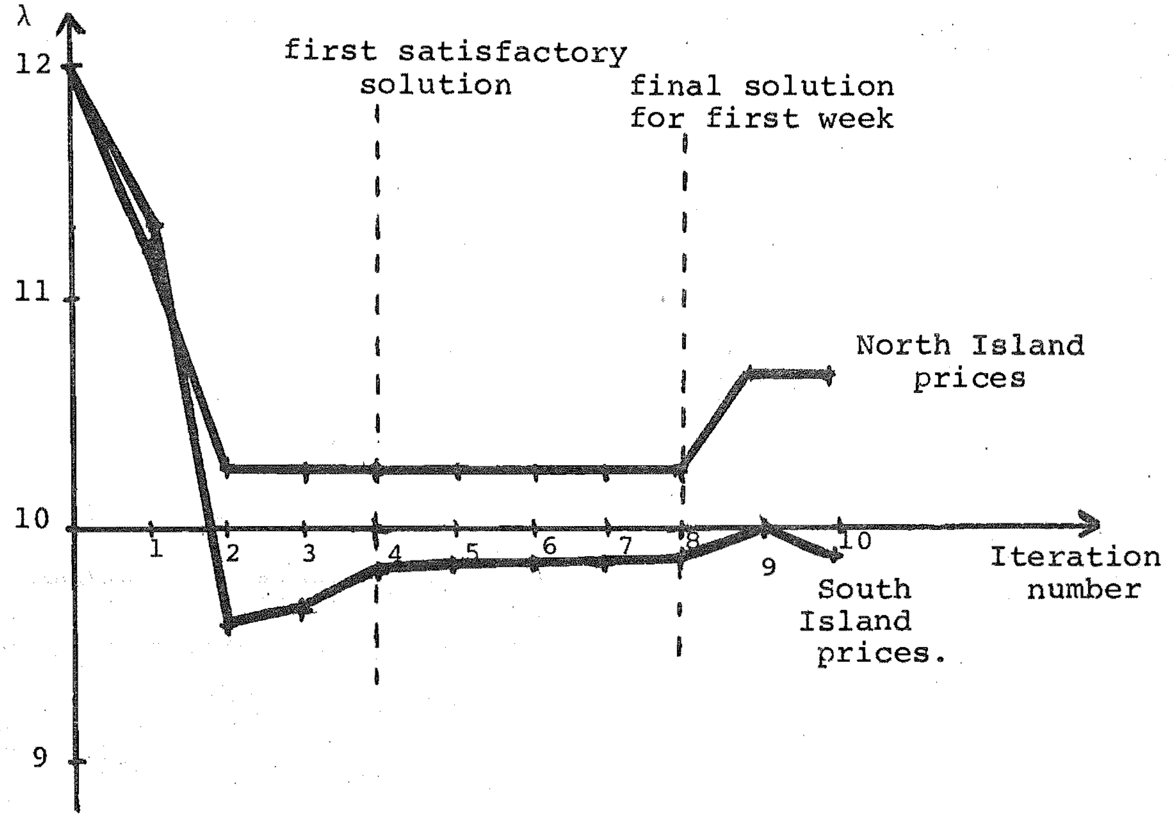


FIGURE (7-8): Convergence of prices to optimum.

terms of the sum of squared deviations. Figure (7-8) shows the changes in prices and production levels for the first period.

Figure (7-9) shows the pattern of generation and transmission levels over the year. It may be seen that, apart from the Christmas/New Year period, New Plymouth has been base-loaded. As demand levels build up in late Summer, South-North exchange and Waikato generation are held constant,while Stratford generation is increased. After Stratford has reached its maximum output Waikato generation is increased to its maximum. At the same time Meremere is brought in at its maximum output while South-North transfer is lowered so as to conserve water in South Island reservoirs (no longer in danger of spilling) for later use. As the Winter progresses the build up in demand levels is absorbed by increased South-North transfer up to a certain level after which a small amount of generation is required at Marsden to cover the mid-Winter peak load period. During the remainder of the year, as demand levels decline the process just described is reversed.

This pattern appears quite reasonable. However it should be noted that our simple model is concerned only with energy without any regard to power requirements. Also, because it is deterministic, it recommends very risky policies (holding reservoirs empty, for instance). Further changes to the system since the data were collected mean that the high thermal loadings shown would not now be necessary. Thus, we cannot at this stage draw any firm conclusions for system operation. Figure (7-10) shows the optimal price pattern for the year.

It can be seen that, so far as the initial period is concerned, the final solution is not significantly different to that in Table (7-9). Thus our procedure took 4 iterations and a total of 68 seconds to reach a satisfactory solution from an initial guess. A further 4 iterations and 52 seconds were required to remove the discrepancies due to storage constraints in the hydro system. This performance is, however, not particularly relevant to the use of this model as an operational tool. In that situation it will normally be possible to re-start the solution procedure using the previous week's solution. Our program allows us to simulate this situation - inserting an imaginary set of first period inflows and adjusting storage levels accordingly. We show two iterations of this simulation. The inflows are shown in Table (7-11) and the results in Tables (7-12) and (7-13). Final trajectories are shown in Figure (7- 6). Each week's optimisation took one iteration of the water value guessing procedure (each taking about 15 seconds). Although extra iterations, involving, say, the technique of Section 6.3.7 to deal with problems due to storage constraints, may be required from time to time, it would appear that the computational burden involved in using this deterministic model for weekly optimisation would not be great. Thus the introduction of some kind of stochastic model on the IBM 370/168 (which is considerably faster than the Burroughs) should be feasible.

It should be noted that the extremely erratic thermal response curve (see Figure (7- 2)) can lead to problems in the optimisation. This effect is particularly strong in our example as may be seen (Table (7-10)) from the

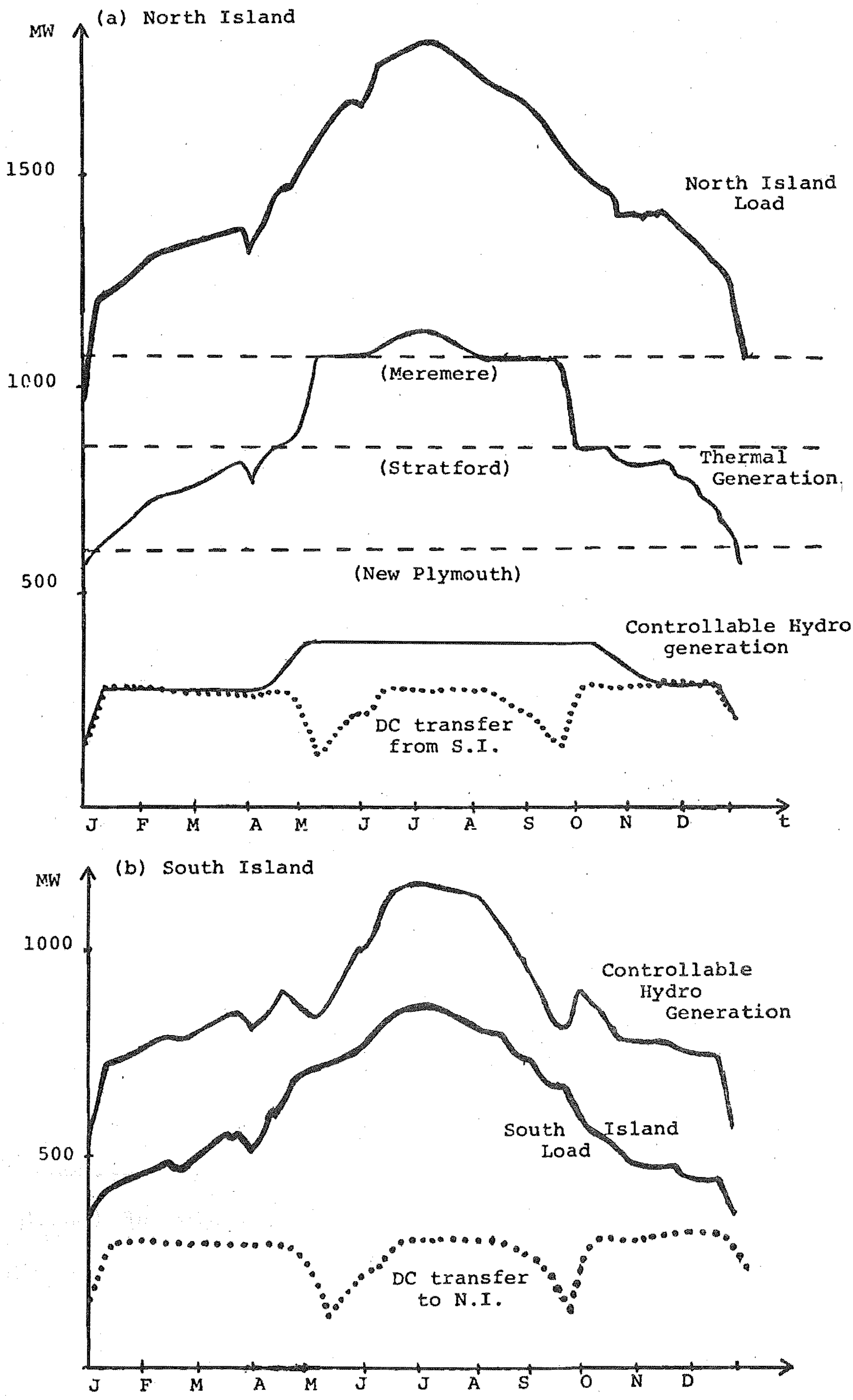


FIGURE (7-9): Generation and transfer patterns

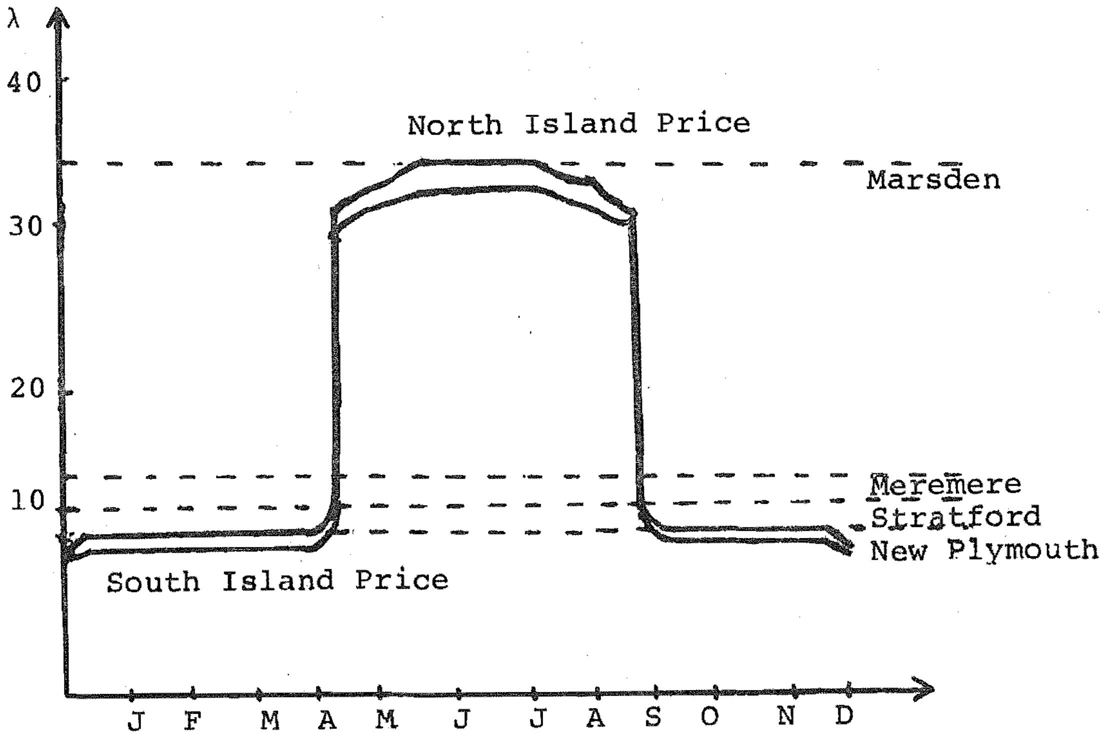


FIGURE (7-10): Price pattern

Reservoir	Period 1 Expected	Period 1 Actual	Period 2 Expected	Period 2 Actual
Taupo	318	200	333	200
Cobb	17	30	15	30
Coleridge	14	30	14	30
Waitaki	491	600	507	750
Hawea	33	45	29	50
Manapouri	625	800	527	750

TABLE (7-11): Actual and expected inflows for simulation.

TABLE (7-12): Results from second week's optimisation

R	NORTH ISLAND							SOUTH ISLAND						
	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN
2	10.69	1201	1	887	607	315	279	9.99	422	0	748	1	-326	747
3	10.71	1219	0	905	622	315	283	10.01	440	1	766	1	-325	765
4	10.72	1248	1	934	649	315	285	10.02	449	1	776	1	-326	775
5	10.72	1266	1	952	666	314	286	10.03	457	2	784	1	-325	783
6	10.74	1300	1	991	702	310	289	10.05	483	3	806	1	-320	805
7	10.75	1314	1	1009	719	306	290	10.07	501	4	821	1	-316	820
8	10.75	1322	1	1014	723	309	291	10.06	492	3	815	1	-320	814
9	10.75	1331	1	1027	735	305	292	10.07	509	4	828	1	-315	827
10	10.76	1342	1	1041	748	302	292	10.09	523	4	840	1	-312	839
11	10.76	1348	1	1054	760	295	293	10.11	549	5	859	1	-305	858
12	10.77	1352	1	1062	768	291	294	10.12	564	5	870	1	-301	869
13	10.78	1375	1	1085	789	292	296	10.13	571	6	877	1	-301	876
14	10.75	1316	1	1023	732	294	291	10.10	543	5	851	1	-303	850
15	10.78	1386	1	1092	796	295	296	10.12	562	5	872	1	-305	871
16	10.92	1456	2	1168	847	291	320	10.26	623	-1	922	1	-300	921
17	11.02	1463	3	1184	847	282	337	10.38	615	-2	903	1	-290	902
18	12.36	1533	0	1340	949	193	391	11.87	682	0	879	1	-197	878
19	32.13	1573	0	1449	1058	124	391	31.32	708	9	843	3	-126	840
20	32.53	1604	0	1449	1058	155	391	31.51	729	11	898	3	-158	895
21	33.20	1649	0	1449	1058	200	391	31.84	754	5	963	3	-204	960
22	33.53	1673	0	1449	1058	224	391	31.99	769	6	1004	3	-229	1001
23	33.57	1670	0	1449	1058	221	391	32.05	787	7	1020	3	-226	1017
24	34.61	1760	0	1467	1077	293	391	32.52	836	9	1147	3	-302	1144
25	34.61	1768	0	1481	1090	287	391	32.56	854	9	1159	3	-296	1156
26	34.63	1794	0	1511	1121	282	391	32.61	872	9	1171	3	-291	1168
27	34.63	1802	0	1519	1128	283	391	32.61	871	9	1171	3	-292	1168
28	34.63	1809	0	1526	1135	283	391	32.61	872	9	1172	3	-291	1169
29	34.63	1802	0	1517	1126	285	391	32.59	864	9	1167	3	-294	1163
30	34.62	1793	0	1505	1114	288	391	32.57	854	9	1160	3	-297	1156
31	34.61	1770	0	1479	1088	291	391	32.54	843	9	1152	3	-300	1148
32	34.60	1743	0	1449	1058	294	391	32.50	830	10	1143	3	-303	1140
33	34.19	1715	0	1449	1058	266	391	32.32	812	9	1094	3	-274	1091
34	33.94	1695	0	1449	1058	246	391	32.23	808	8	1069	3	-252	1065
35	33.65	1683	0	1449	1058	234	391	32.04	770	7	1016	3	-240	1013
36	33.30	1658	0	1449	1058	209	391	31.87	753	5	971	3	-214	968
37	31.36	1621	0	1449	1058	172	391	30.26	715	12	902	3	-175	899
38	30.65	1587	0	1449	1058	138	391	29.79	691	14	846	3	-140	843
39	12.38	1549	0	1384	994	165	391	11.97	666	0	834	1	-168	833
40	11.20	1503	2	1214	847	291	366	10.53	621	-9	913	1	-300	912
41	10.99	1471	3	1180	847	293	333	10.33	576	-5	874	1	-302	873
42	10.95	1458	3	1173	847	287	326	10.30	559	10	865	1	-296	864
43	10.78	1390	1	1106	809	286	297	10.15	523	-1	816	1	-294	815
44	10.79	1400	1	1110	813	291	297	10.14	507	-1	806	1	-300	805
45	10.78	1399	1	1105	808	296	297	10.13	490	0	795	1	-305	794
46	10.78	1400	1	1104	807	298	297	10.12	484	0	791	1	-307	790
47	10.78	1395	1	1103	806	293	297	10.13	497	-1	799	1	-303	798
48	10.77	1370	1	1072	777	299	295	10.11	469	0	778	1	-309	777
49	10.76	1351	1	1056	762	296	294	10.11	473	0	779	1	-306	778
50	10.75	1310	1	1014	724	297	291	10.09	458	1	764	1	-306	763
51	10.74	1282	1	988	699	295	289	10.09	455	1	760	1	-304	759
52	10.37	1039	7	827	607	219	220	9.91	375	1	599	1	-224	598

KEY

R	Period
P	Price
DEM	Demand
XSUP	Excess supply
TOTG	Total generation
TGEN	Thermal generation
NETEX	Net exchange
HGEN	Hydro generation

TABLE (7-13): Results from third week's optimisation

	NORTH ISLAND							SOUTH ISLAND						
R	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN	P	DEM	XSUP	TOTG	TGEN	NETEX	HGEN
3	10.67	1219	1	881	607	339	274	9.92	440	3	794	1	-352	793
4	10.71	1248	1	903	622	346	280	9.94	449	3	811	1	-359	810
5	10.71	1266	1	921	640	346	282	9.95	457	3	819	1	-358	818
6	10.73	1300	1	960	675	341	284	9.97	483	4	841	1	-354	840
7	10.74	1314	1	978	692	337	286	9.98	501	5	856	1	-350	855
8	10.74	1322	1	982	696	341	286	9.98	492	5	850	1	-353	849
9	10.74	1331	1	995	709	337	287	9.99	509	5	863	1	-349	862
10	10.75	1342	1	1010	722	334	288	10.00	523	6	874	1	-346	873
11	10.75	1348	1	1022	734	327	289	10.03	549	6	894	1	-338	893
12	10.76	1352	1	1030	741	323	289	10.04	564	7	905	1	-334	904
13	10.76	1375	2	1053	763	323	291	10.04	571	7	912	1	-334	911
14	10.74	1316	1	992	705	326	287	10.02	543	6	886	1	-337	885
15	10.77	1386	2	1061	770	327	291	10.04	562	7	907	1	-338	906
16	10.87	1456	2	1156	847	302	309	10.19	623	-7	928	1	-312	927
17	10.97	1463	3	1173	847	293	326	10.30	615	-8	909	1	-302	908
18	12.36	1533	0	1340	949	193	391	11.87	682	0	879	1	-197	878
19	32.10	1573	0	1449	1058	124	391	31.29	708	7	840	3	-126	837
20	32.50	1604	0	1449	1058	155	391	31.48	729	9	896	3	-158	893
21	33.18	1649	0	1449	1058	200	391	31.83	754	4	963	3	-204	960
22	33.51	1673	0	1449	1058	224	391	31.98	769	6	1004	3	-229	1001
23	33.55	1670	0	1449	1058	221	391	32.03	787	6	1020	3	-220	1016
24	34.61	1760	0	1466	1075	294	391	32.51	836	8	1148	3	-303	1145
25	34.61	1768	0	1479	1089	289	391	32.55	854	8	1160	3	-298	1157
26	34.63	1794	0	1510	1120	284	391	32.60	872	8	1173	3	-292	1169
27	34.63	1802	0	1518	1127	284	391	32.60	871	8	1172	3	-293	1169
28	34.63	1809	0	1525	1134	284	391	32.60	872	8	1173	3	-293	1170
29	34.63	1802	0	1515	1125	287	391	32.58	864	8	1168	3	-295	1164
30	34.62	1793	0	1503	1113	290	391	32.56	854	8	1161	3	-298	1158
31	34.61	1770	0	1478	1087	292	391	32.53	843	8	1153	3	-301	1150
32	34.50	1743	0	1449	1058	294	391	32.48	830	8	1142	3	-303	1138
33	34.17	1715	0	1449	1058	266	391	32.30	812	8	1094	3	-274	1091
34	33.92	1695	0	1449	1058	246	391	32.21	808	8	1068	3	-252	1065
35	33.63	1683	0	1449	1058	234	391	32.02	770	6	1016	3	-240	1013
36	33.28	1658	0	1449	1058	209	391	31.86	753	4	971	3	-214	968
37	31.32	1621	0	1449	1058	172	391	30.22	715	12	902	3	-175	899
38	30.60	1587	0	1449	1058	138	391	29.74	691	13	844	3	-140	841
39	12.38	1549	0	1384	994	165	391	11.97	666	0	834	1	-168	833
40	11.21	1503	2	1213	847	292	366	10.54	621	-5	917	1	-301	916
41	11.00	1471	3	1179	847	295	332	10.33	576	-3	877	1	-304	876
42	10.96	1458	3	1172	847	289	325	10.31	559	12	868	1	-298	867
43	10.78	1390	2	1106	811	286	294	10.15	523	-1	816	1	-295	815
44	10.79	1400	2	1110	815	292	295	10.14	507	-1	807	1	-301	806
45	10.78	1399	2	1104	810	297	294	10.12	490	-1	795	1	-306	794
46	10.78	1400	2	1103	809	298	294	10.12	484	0	791	1	-308	790
47	10.78	1395	2	1103	808	294	294	10.13	497	-1	800	1	-303	799
48	10.77	1370	2	1072	780	300	292	10.11	469	0	778	1	-309	777
49	10.77	1351	2	1056	765	297	291	10.11	473	0	779	1	-306	778
50	10.75	1310	1	1014	726	297	288	10.09	458	0	765	1	-307	764
51	10.74	1282	1	988	702	295	286	10.08	455	1	760	1	-305	759
52	10.38	1039	1	825	607	221	216	9.91	375	1	601	1	-226	600

KEY

(See opposite)

fact that the optimum involves much production very near the "corners" of the curve (e.g., an average of just 20MW per hour from Marsden during Winter). This can lead to rather unstable behaviour particularly when "guessing" water values where the response of the thermal system to average prices depends crucially on the distribution of the prices about the average. In order to account for this we had to re-evaluate derivatives frequently during the "guessing" phase of each such iteration (hence the rather large computation times for these iterations). Experiments with slightly more regular (although still quite erratic) thermal response curves have yielded faster convergence (2-3 iterations from an initial guess) without incurring this additional computational burden. As we model the short-term behaviour of the system more accurately we shall be able to deal with much smoother response curves and so expect a considerable improvement in performance.

7.6 THE SHORT-TERM HYDRO SUB-PROBLEM

In the previous sections we have discussed the implementation of a simple aggregated model in ALGOL on the B6718. That model took no account of short-term requirements. In particular we assumed simple quadratic generation functions for each equivalent aggregate hydro station. The model currently being implemented at the NZED will take account of short-term variations using the price-derivation approach of Sections 2.4 and 6.2. In Section 5.5 we developed a short-term hydro scheduling procedure, PASH, which could be used to build up tables summarising the optimal output from a river system for any combination of aggregate prices. Here we describe the implementation of that procedure in PL/I on the IBM 370/168 used by the NZED.

The program is interactive and contains about 350 lines, of which the main algorithm takes about 200 lines. Head effects are dealt with as outlined in Section 5.5.2 (point (a) on p.138). The difficulties encountered in giving a precise definition to the extent of each "period" and "delay time" are handled by the device suggested in Section 5.4.1 (point (b) p.140). Tributary inflows are scheduled first (as in Figure (5-9)). Minimum flow limits and differing initial and final levels are dealt with by the methods of [49].

In Section 5.5.2 we showed the application of PASH to a very simple example. Here we consider a realistic example with 24 hourly "instants", 8 stations, tributary inflows, quadratic generation functions and head effects. This example corresponds to the Waikato River sub-system - the longest chain in the NZED system. The relevant plant characteristics are shown in Table (7-14) (where station 0 corresponds to release from Lake Taupo, the long-term reservoir, which has no generating equipment immediately downstream from it). It should be noted that the α and β coefficients shown here are not to be interpreted as coefficients for a quadratic generation function:

$$g(q) = \alpha q^2 + \beta q \qquad \text{(H-92)}$$

but rather, in accordance with NZED practice, for a "water consumption" function:

$$q(g) = \alpha g^2 + \beta g \qquad \text{(H-93)}$$

$$\text{(H-93)} \Rightarrow \quad g(q) = \frac{-\beta + \sqrt{\beta^2+4\alpha q}}{2\alpha} \qquad \text{(H-94)}$$

(These equations must be modified to account for the head effect yielding:

$$q(g) = (\alpha g^2 + \beta g)\,\frac{\bar{H}}{h} \qquad \text{(H-95)}$$

$$\text{(H-95)} \Rightarrow g(q) = \frac{1}{2\alpha}\left(-\beta + \sqrt{\frac{\bar{H}\beta^2+4\alpha hq}{\bar{H}}}\right) \qquad \text{(H-96)}$$

where h is the head and $\bar{H}$ is the maximum head of the station).

We have assumed that tributary inflows, upper and lower storage limits and release limits are constant throughout the day. The assumed price distribution for this example is shown in Figure (7-14). Using increments of 5 cumecs the tributary inflows were scheduling in approximately 5 seconds (including all setup times). The resultant schedule is summarised in Figure (7-11), the total value of production from tributary streamflows being $4,538. Then increments of 10 cumecs were scheduled successively up to the Taupo release limit of 200 cumecs per hour. The resultant curves for "profit" and marginal "profit" as a function of total release are shown in Figures (7-10) and (7-11). Figures (7-12) and (7-13) show the trajectories for each reservoir at moderate (2,000 cumecs total) and high (4,800 cumecs total) release levels. Figure (7-14) shows the corresponding total generation patterns.

NO.	STATION NAME	DELAY w^{i-1}	d^{i} (*)	STORAGE (CMD) (**)	HEAD (***) Minimum	$\frac{dH}{dS}$	$\hat{Q}$ (+) (CM)	(++) α	β	$\bar{G}$	INFLOW RATE F (CM)	S^{o} (+++)
0	TAUPO	0	0	9,905	-	-	200	-	-	-	-	5,000
1	ARATIATIA	1.5	1.5	7	34	0.167	281	0.00130	3	90	0	5
2	OHAKURI	7	8.5	61	35	0.0074	361	0.00200	3	112	10	50
3	ATIAMURI	0.1	8.6	31	25	0.0470	384	0.00680	4	84	0	150
4	WHAKAMARU	0.4	9	15	38	0.0135	203	0.00030	2	100	0	6
5	MARAETAI	0.1	9.1	34	61	0.0178	391	0.00024	1	360	0	20
6	WAIPAPA	0.3	9.4	11	16	0.0546	332	0.0101	6	51	0	5
7	ARAPUNI	2.5	11.9	63	53	0.0096	347	0.00123	2	158	10	30
8	KARAPIRO	2	13.9	159	30	0.0117	298	0.00350	3	90	0	60

(*) Total delay from Taupo to station, $d^{i} = \sum_{j=1}^{i-1} w^{i}$

(**) 1 CMD = 1 cumec-day ≈ Flow of 1 CM for 1 day

(***) Head in metres, $\frac{dH}{dS}$ in m/CMD.

(+) Maximum utilisable release in cumecs.

(++) $q(g) = \alpha g^{2} + \beta g$

(+++) Initial storage (i.e. at time w^{i}) in CMD.
(= Final storage except for Taupo).

TABLE (7-14): Waikato station characteristics.

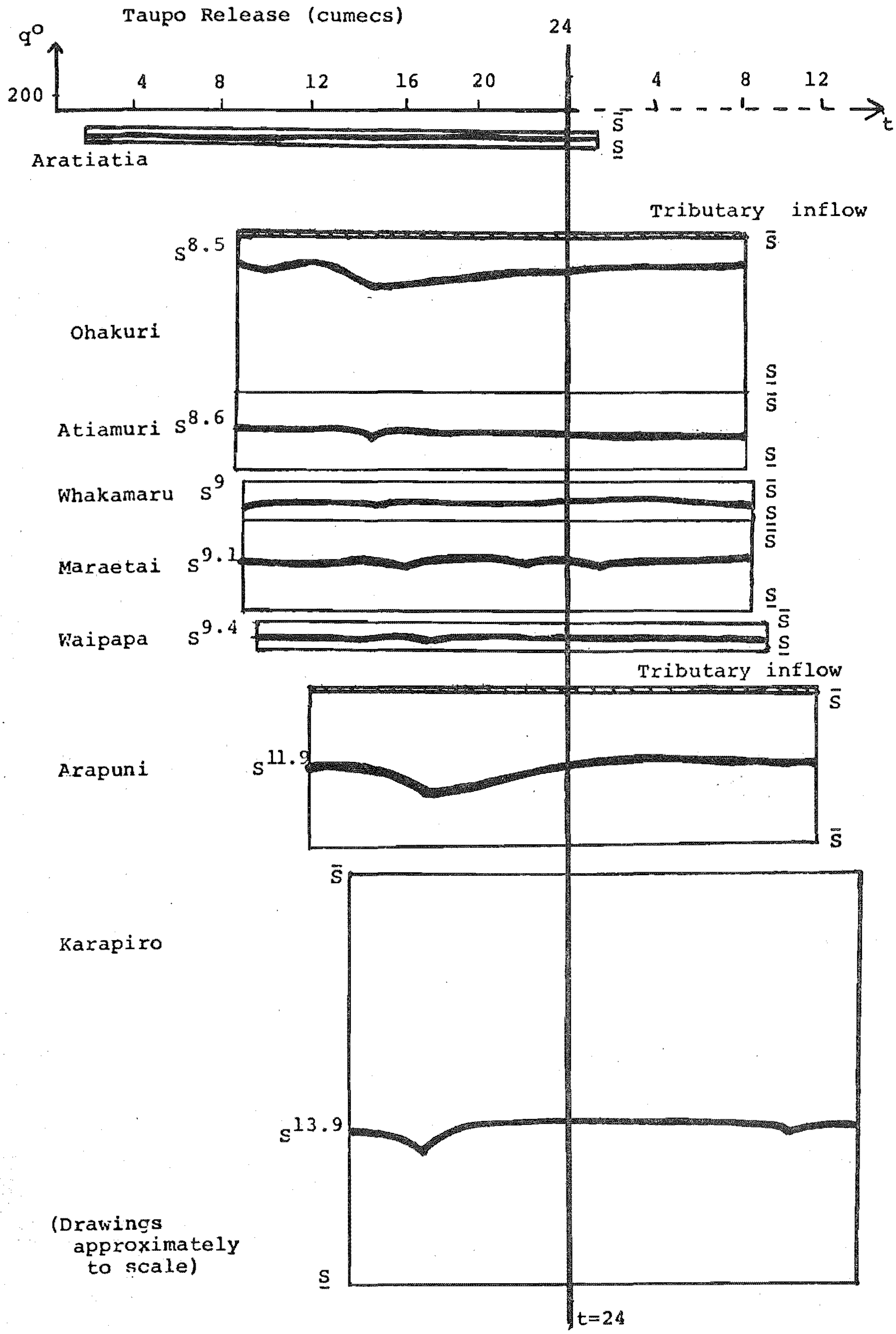


FIGURE (7-11): Waikato trajectories-tributary flows only

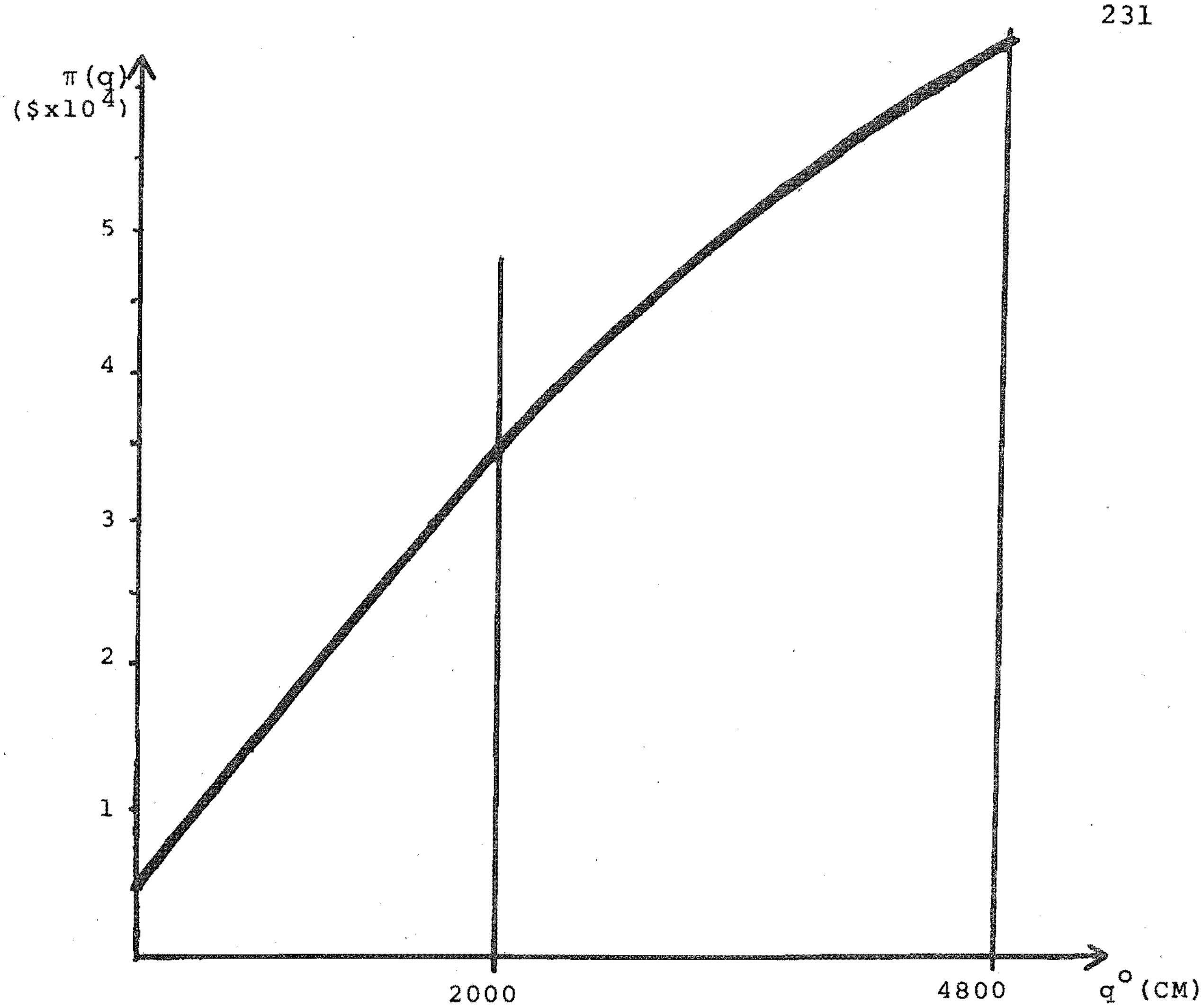


FIGURE (7-12): Value of release.

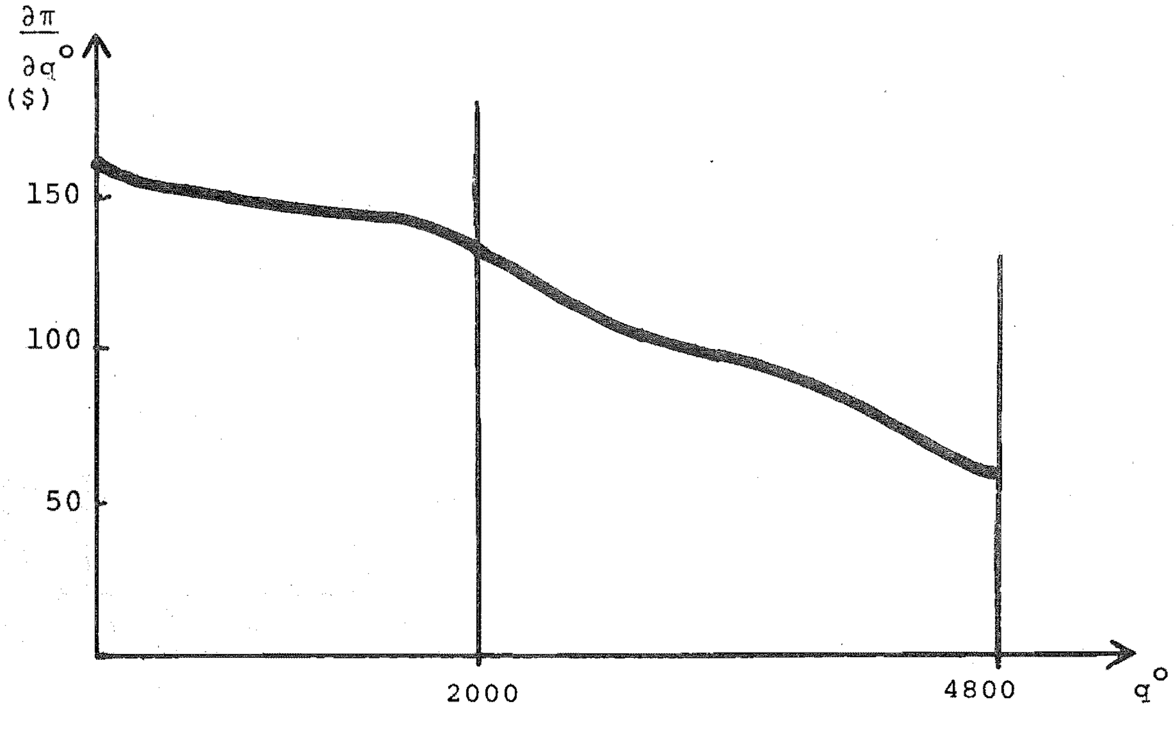


FIGURE (7-13): Marginal value of release.

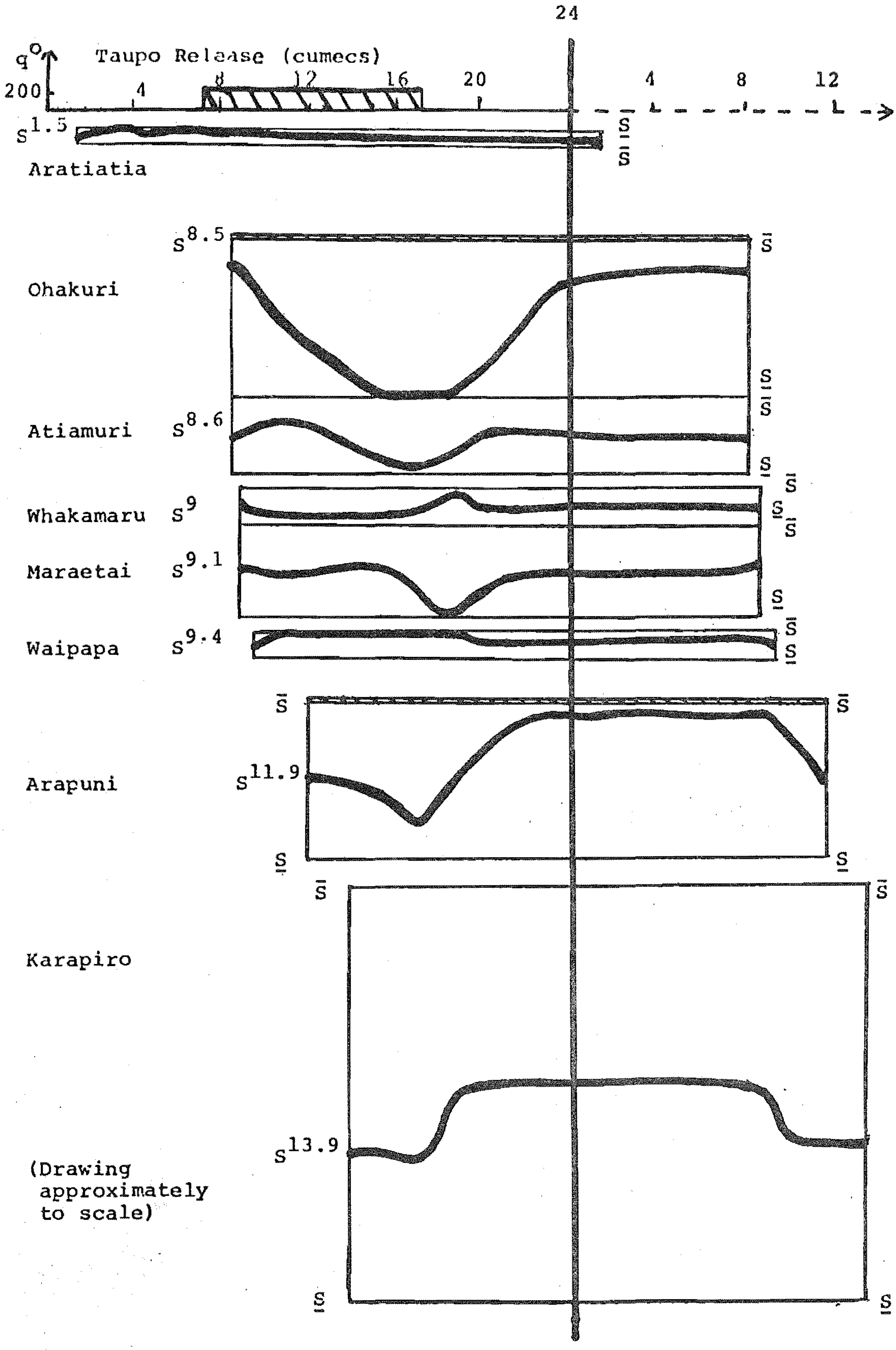


FIGURE (7-14): Waikato trajectories - moderate release

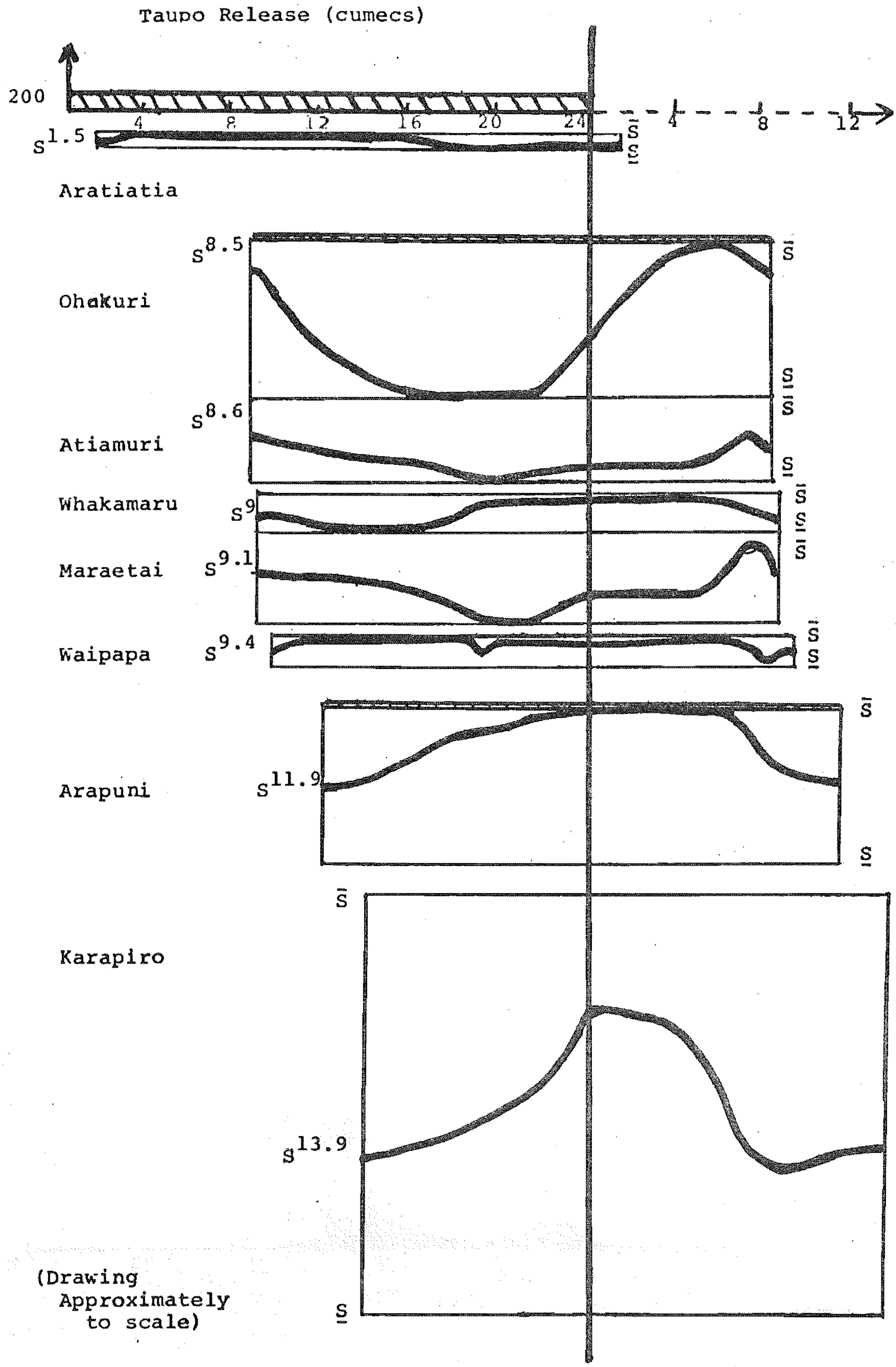


FIGURE (7-15): Waikato trajectories - high release

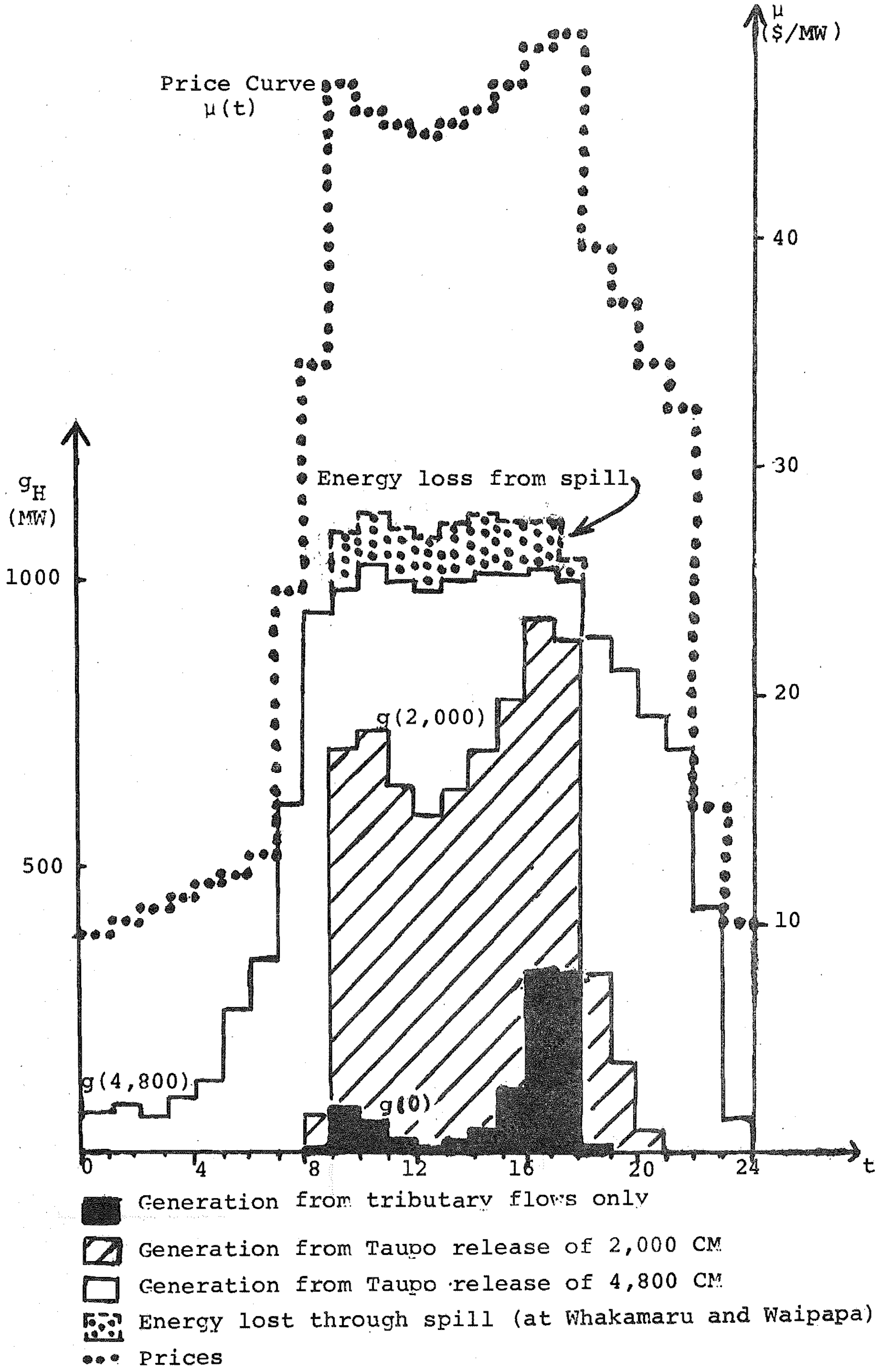


FIGURE (7-16): Price and generation patterns.

Total CPU times required for this example were 5 seconds to set up the problem and schedule the tributary inflows (48 increments), and a further 43 seconds to schedule the maximum total release (480 increments). An alternative procedure in which the water consumption function is converted into an (approximate) generation function (cf. (H-92)) requires slightly less time - 4 seconds to schedule the inflows and 40 seconds for the remainder. (All times very approximate).

It can be seen from Figure (7-16) that the generation patterns determined by the procedure are clearly consistent with the assumed price distribution. Further, the "profit" function ($\pi(q)$ in Figures (7-12) and (7-13)) is well behaved. Thus our short-term scheduling procedure appears to be capable of producing suitable "profit" and output curves for use in long-term scheduling without undue computational requirements.

Further, this procedure is considerably more accurate than that currently used for short-term scheduling in the NZED, in terms of both detailed system modelling and optimisation. Thus it could prove useful when applied directly to that problem. In [49] we outline a model of this type. This could be used, if not to replace the present heuristic approach, at least to provide insight into the form of optimal river schedules. The feasibility of such an approach is currently being investigated.

7.6 CURRENT RESEARCH

Our model has not yet reached the stage of being used to make real scheduling decisions. At present these decisions are made using the traditional "security guideline" approach (e.g., [63]). However a single-reservoir "stochastic" optimisation model ([67]) which is intended to supplement this approach has been developed at NZED. This program incorporates a number of alternative optimisation procedures, including adaptations of the methods described in this thesis and of the "Swedish" approach ([31] and [58]). Thus this model will enable us to run comparative tests on a variety of optimisation methods. Such a single-reservoir model cannot take into account, however, the additional constraints imposed on the individual reservoirs (let alone river chains) or the inevitable imbalance between reservoirs caused by regional variations in their inflows. Nor can it explicitly take into account the effect of losses and limitations in the transmission network - an important factor in the NZED system. Thus it will tend to be over optimistic in its policies. Further, it will not shed any light on the question of how the national hydro release should be apportioned among the various hydro reservoirs.

Thus the multi-load multi-reservoir model developed here can be expected to provide significant benefits for the long-term scheduling of the NZED system. The ALGOL test program described in this chapter has demonstrated that the solution of single-load-segment deterministic problems does not incur undue computational burdens on a Burroughs B6700. This program is currently being converted to PL/I on the IBM 370/168 used at the NZED. No difficulty is anticipated in solving multi-segment deterministic problems with this program. At the same time we have developed a PL/I program which can be used to prepare the input tables required to provide an accurate modelling of the short-term aspects of hydro-electric scheduling. This program has also been shown to perform satisfactorily on the largest river system in the NZED system. The use of these tables, along with their thermal counterparts, will provide much smoother system response curves. Experience has shown that we can expect our global optimisation procedure to converge considerably faster in this situation. Thus the practical application of all of the techniques discussed in Chapters 2 to 6 should be realised without great difficulty.

The remaining chapters of this thesis deal with two extensions to our model. The first, and by far the most important, is the stochastic model developed in Chapters 8 and 9. Chapter 8 develops a general theory of long-term scheduling in a stochastic environment, while Chapter 9 discusses practicable approaches to the problem. We have not as yet tested these techniques in a multi-reservoir model. However we are currently using the single-reservoir model of [6] to evaluate the various proposed approaches prior to selection of the best algorithm for the multi-reservoir problem. Computational experience with the deterministic algorithm leads us to expect that the implementation of a stochastic multi-reservoir algorithm should be computationally feasible.

It may prove desirable, however, to limit our model to optimising on a restricted set of "representative" inflow sequences rather than the full set of 42 annual inflow sequences available.

The final chapter deals with ways in which we could utilise the "prices" developed by our model. Apart from their use to evaluate development proposals, they could be used to indicate appropriate tariffs. The adaptations required to solve this "Optimal Tariff Problem" are of a comparatively minor nature and would not incur any significant extra computational burden. We have not, as yet, implemented this aspect of our model. However, when the new version of the model is running on data which is sufficiently accurate to give real significance to the results, this extension will be incorporated. Thus these "optimal tariffs" will be a free "bonus" from our scheduling model.

In summary our test program has shown the feasibility of the approach which is now in the process of being implemented. Also, in addition to the research being undertaken on the long-term problem, the short-term hydro sub-model is being evaluated for application directly to short-term scheduling problems ([49]).

CHAPTER 8

A STOCHASTIC MODEL

8.1 INTRODUCTION

The models discussed in [47] and Chapter 2 are obviously inadequate as a representation of reality because they ignore the essential difficulty facing any power system manager - his uncertainty about the future. There are three major areas of uncertainty:

(1) The future streamflows (both reservoir inflows and tributary flows).

(2) The future demand.

(3) The future availability of plant.

The models described here are designed to cope with the first kind of uncertainty which, in a predominately hydro system such as that of the NZED, is of paramount importance. Even on a national basis maximum monthly flows are approximately 2.6 times greater than minimum flows and the variations are obviously greater for particular rivers over shorter periods. Furthermore, the presence of storage constraints means that not only the level but the pattern of inflows is important. This presents a formidable forecasting exercise with results which are inevitably inconclusive. Inflow forecasting for New Zealand rivers is beyond the scope of our work.

The future demand levels depend on a number of factors including the economic climate, the pricing of electricity relative to alternative energy sources, government controls and the weather. The overall variability of demand is however very much lower than that of the hydro inflows. The NZED forecasts for daily demand have a standard deviation ranging from 2% in the short-term up to 4% a year ahead (when compensated for weather dependent fluctuations). Chapter 10 considers a model where electricity demand depends on its pricing. Our present model can only allow for demand variability to the extent (about 1% for NZED ([15])) that this is dependent on the weather and can be correlated with streamflows. (However demand as another stochastic variable could also be incorporated in the same theoretical framework).

Plant availability is an important factor, particularly with thermal plant, some types of which are subject to frequent and extensive 'forced outage' due to breakdowns. This will be completely ignored in this model. Modelling this aspect presents great difficulties and limited benefits due to the following factors:

(a) The future availability of a thermal plant has no direct impact on its optimal current operating level.

(b) Thermal generation is relatively unimportant in New Zealand (by comparison with many overseas systems).

(c) No one station is so large that a temporary breakdown would be likely to unduly affect the optimal hydro-thermal mix in the long term. In particular there are no nuclear plants.

A scheme to account for the availability of hydro plants is outlined in Section 5.5.3.

Schemes designed to incorporate stochastic elements into the decision process have been implemented or proposed by utilities in other parts of the world. These have been discussed briefly in Section 1.3. In Section 8.2 we summarise some results from the general theory of stochastic convex programming. In Section 8.3 we extend our deterministic model within this framework.

In the next chapter we first analyse a number of possible approaches in this general framework then propose a new method which appears to be suitable for our problem.

8.2 THE OPTIMAL RECOURSE PROBLEM

8.2.1 Introduction

In Section 8.3 we will develop an extension of PAI, an aggregated multi-regional model which ignores short-term requirements (see Sections 2.4, 2.5), largely for reasons of notational sanity.

We wish to make extensive use of some recent results due to Rockafeller and Wets which we summarise in this section. The results on which we will base our model appear in their most complete and appropriate form for application to our problem in [53]. Several papers which have appeared deal with similar problems from different points of view and with a variety of notation. We intend to adopt the approach and notation of [53] supplemented by results from [52] and modified where necessary to conform to the notation of the deterministic model of Chapter 2. We will first state the general problem and then introduce the necessary assumptions before proceeding to the results. Appendix C contains a comparative summary of the notation used in the two key papers ([52] and [53]) and in this thesis.

Consider the following general model. We divide time into T discrete stages (indexed by $t = 1, \ldots T$). At each stage, t, we observe a particular vector, $\xi^t \varepsilon R^{\nu^t}$, of the random variables. Based on this information we choose a response $u^t \varepsilon R^{n^t}$.

Our choices are subject to constraints:

$$u(\xi)\varepsilon\mathcal{U}(\xi) \tag{S-1}$$

$$f_i(\xi,u) \leqslant 0 \quad \text{for } i=1, \ldots .m. \tag{S-2}$$

where:

$$\xi=(\xi^t)_{t=1}^T \; \varepsilon R^\nu = R^{\nu^1} x R^{\nu^2} x \ldots x R^{\nu^T} \tag{S-3}$$

$$u=(u^t)_{t=1}^T \; \varepsilon R^n = R^{n^1} x R^{n^2} x \ldots x R^{n^T} \tag{S-4}$$

Here some of the f_i functions may depend on only a few of the components of ξ and u. (Note that the subscripts n and i employed here have different connotations from those used in Chapter 2.)

The cost of the decision process u is given by $f_o(\xi,u)$. We assume that the distribution of ξ is given by a regular Borel probability measure, σ, on R^ν with support Ξ and let F denote the σ-field generated by Ξ. We obviously wish to determine decision rules so as to minimize the expected cost given that the decision at each stage r can depend only on observations up to that stage $(\hat{\xi}^r=(\xi^1 \ldots \xi^r))$ but not on future observations. We say that such a decision process is nonanticipative.

8.2.2 Assumptions for the Model

We assume henceforth that:

(a) For each $\xi\varepsilon\Xi$: $\mathcal{U}(\xi)$, the feasible set of decisions for that ξ, is closed, convex and with non-empty interior.

(b) For each $\xi\varepsilon\Xi$: the functions $f_i(\xi,u)$ $(i=0,\ldots m)$ are defined for all $u\varepsilon\mathcal{U}(\xi)$, (finite i.e., real valued), convex and lower semi-continuous. Further, it is assumed that, for each $u \varepsilon \mathcal{R}^n$:

$$\mathcal{U}^{-1}(u) = \{\xi \varepsilon \Xi \mid u \varepsilon \mathcal{U}(\xi)\} \qquad \text{(S-5)}$$

is Borel measurable (i.e., $\mathcal{U}^{-1}(u) \varepsilon \mathcal{F}$), and that the functions $f_i(\xi,u)$ are all Borel measurable relative to $\mathcal{U}^{-1}(u)$.

(c) Let $\mathcal{D}(\xi)=\{u\varepsilon\mathcal{U}(\xi) \mid f_i(\xi,u) \leqslant 0$ for $i=1,\ldots m\}$.

We further assume that the sets $\mathcal{D}(\xi)$ are uniformly bounded (i.e., $\bigcup_{\xi\varepsilon\Xi} \mathcal{D}(\xi)$ is a bounded subset of $\mathcal{R}^n$) and that for each bounded set, $K \subset \mathcal{R}^n$, there is a corresponding summable function, $\alpha:\Xi \to \mathcal{R}^n$, and a constant, $\beta\varepsilon\mathcal{R}$, such that, for all $\xi\varepsilon\Xi$:

$$|f_o(\xi,u)| \leqslant \alpha(\xi) \qquad \text{for all } u\varepsilon\mathcal{U}(\xi)\cap K \qquad \text{(S-6)}$$

$$|f_i(\xi,u)| \leqslant \beta \qquad \text{for all } u\varepsilon\mathcal{U}(\xi)\cap K,\ i=1,\ldots.m \qquad \text{(S-7)}$$

Then we can state our problem as:

(PS) Find $\underset{u\varepsilon N_\infty}{\text{MIN}} \quad E\{f_o(\xi,u(\xi))\}$ (S-8)

Such that: $f_i(\xi,u(\xi)) \leqslant 0$ for all $i=1,\ldots m$ a.s.(S-2)

Where:

$$N_\infty = \{u \mid (u^1,\ldots u^T)\ \varepsilon L_n^\infty, u^r \text{ is } \mathcal{F}^r \text{ measurable for all } r=1,\ldots.T\} \qquad \text{(S-9)}$$

Thus N_∞ is the set of all nonanticipative recourse functions.

We require that the problem PS be strictly feasible. That is, that there be some $u\varepsilon N_\infty$ and $\varepsilon>0$ such that:

$$f_i(\xi,u(\xi)) \leqslant -\varepsilon \text{ a.s for } i=1,\ldots m. \qquad \text{(S-10)}$$

and:

$$u(\xi)+\varepsilon B \subset D(\xi) \quad \text{a.s.} \tag{S-11}$$

Where B is a closed unit ball in R^n.

We also require the property of "essentially complete recourse". Formally, we require that for all t = 1,...T, the multifunction:

$$\mathcal{D}^t : \xi \to \mathcal{D}^t(\xi) = \{(u^1, \ldots u^t) \mid u \varepsilon \mathcal{D}(\xi)\} \text{ is } F^t \text{ measurable} \tag{S-12}$$

We say then that the constraint multifunction is essentially nonanticipative. The purpose of this condition is to ensure that we have zero probability of getting into a "blind alley" situation, at any stage t, by having made a sequence of decisions, $\hat{u}^t$, which, in combination with some adverse random series, ξ^t, have brought the system to a state from which further feasible progress is impossible.

8.2.3 The Value Function and Multipliers on the Nonanticipativity Restriction.

In [52] two major results about this model are developed. The first is a kind of stochastic dynamic programming and the second a partial result on the existence of multipliers on the nonanticipativity restriction. This section summarises the results of that paper. Here we ignore the structure of the constraints (S-1) and (S-2) replacing them by re-defining the objective function so that:

$$f_o(\xi,u) = +\infty \text{ if } u \notin \mathcal{D}(\xi) \text{ or } f_i(\xi,u) > 0 \text{ for some } i=1,..m \tag{S-13}$$

The following results have been obtained about this modified problem, PSM.

Firstly, we can define a value function, V^t, for each stage t recursively, by:

$$V^T(\hat{\xi}^T,\hat{u}^T) = V^T(\xi,u) = f_o(\xi,u) \tag{S-14}$$

$$V^{t-1}(\hat{\xi}^{t-1},\hat{u}^{t-1}) = E_{\hat{\xi}^t}\left\{\inf_{u^t} V^t(\hat{\xi}^t,\hat{u}^t)\,|\,\hat{\xi}^{t-1}\right\} \tag{S-15}$$

Where this conditional expectation is assumed to be a regular conditional expectation (the existence of which can be guaranteed in this case).

Then we have the following theorem: (Theorem 1 of [52]).

<u>Theorem (S-1)</u> Given the assumptions made about problem PSM, then:

(a) For all t = 1,..T, the problems $P\hat{S}M^t$:

$$\text{Find} \inf_{\hat{u}^t \in \hat{N}^t_\infty} E\{V^t(\hat{\xi}^t,\hat{u}^t)\}. \tag{S-16}$$

are well defined.

(Where $\hat{N}^t_\infty$ consists of all nonanticipative decision functions for the first t periods).

The value function may be expressed by:

$$V^t(\hat{\xi}^t,\hat{u}^t) = E\left\{\inf_{\hat{w}^{t+1}} V^{t+1}(\eta^{t+1},\hat{w}^{t+1})\,\middle|\,\hat{\eta}^t=\hat{\xi}^t,\hat{w}^t = \hat{u}^t)\right\} \tag{S-17}$$

It is a normal convex integrand on $\hat{\Xi}^t \times R^{n^t}$, with $\hat{\xi}^t \to \hat{\mathcal{D}}^t(\hat{\xi}^t) = \text{dom } V^t(\hat{\xi}^t,\cdot)$ a closed valued, uniformly bounded, measurable multifunction and $E\{V^t(\hat{\xi}^t,\hat{u}^t)\}$

is a proper, convex, lower semi-continuous (relative to the weak topology $w-(L^{\infty}_{n^t}, L^{1}_{n^t})$) functional on $L^{\infty}_{n^t}$.

(c) PSM is solvable and, for all $t = 1, \dots . T$, the programs $\hat{PSM}^t$ are solvable. Moreover, if u^* is an optimal solution of PSM, then $\hat{u}^{*t}$ is an optimal solution of PSM^t, and, if $\hat{u}^{*t}$ is an optimal solution of $\hat{PSM}^t$, then it can be extended to an optimal solution, w^*, of PSM such that: $\hat{w}^{*t} = \hat{u}^{*t}$. $\blacksquare$

We now turn our attention to duality results. Let M be the closed linear subspace of L^1_n consisting of those functions ρ which satisfy the martingale property:

$$E\{\rho^t(\eta) \,|\, \hat{\eta}^t = \hat{\xi}^t\} = 0 \text{ a.s. for all } t = 1, \dots T \qquad \text{(S-18)}$$
$$(\text{Here } \rho^t(\xi) \in R^{n^t})$$

Then it can be seen that M is orthogonal to N_∞. 'Multipliers' on the nonanticipativity constraint are chosen to be elements of this space.

If we let $f^*(\xi,\cdot)$ be the conjugate of $f(\xi,\cdot)$ i.e.,

$$f^*(\xi,x^*) = \sup\{\langle u,x^*\rangle - f(\xi,u) \,|\, u \in R^n\} . \qquad \text{(S-19)}$$

Then the dual problem:

$$\text{(DSM)} \quad \text{Find } \sup_{\rho \in M} -E\{f^*(\xi,\rho)\} \qquad \text{(S-20)}$$

can be introduced. The result of Theorem (S-1) is used to prove the following result, (Theorem 2 of [57]).

<u>Theorem (S-2)</u> Under the assumptions we have made about PSM, both PSM and the associated dual problem,

DSM, are solvable and:

$$\text{MIN (PSM)} = \text{MAX (DSM)} \qquad \text{(S-21)}$$

From this is obtained the following:

Corollary (S-2.1) Under these hypotheses, a recourse function $\tilde{u}$ is optimal for the multi-stage stochastic program PSM if and only if there exists a $\tilde{\rho}\varepsilon M$ such that the pair $(\tilde{u},\tilde{\rho})$ is a saddle point of the Lagrangian function ($\mathcal{R}^-$) defined on $L_n^\infty x(L_n^\infty)^*$ by:

$$\mathcal{R}^-(u,\rho) = \begin{cases} E\{f_o(u,\xi)\} - \langle u,\rho\rangle & \text{if } \rho\varepsilon M \\ -\infty & \text{if } \rho\not\varepsilon M \end{cases} \qquad \text{(S-22)}$$

In the terminology of [53] this Lagrangian is known as the reduced Lagrangian and is given the alternative definition:

$$\mathcal{R}^-(u,\rho) = E\{f_o(\xi,u(\xi)) - u(\xi)\cdot\rho(\xi)\} \text{ for all } \rho\varepsilon M \qquad \text{(S-23)}$$

8.2.4 Multipliers on the Inequality Constraints

We turn our attention here to the results of [53] and introduce the (Rockafellar-Wets) Lagrangian for our problem. First define:

$$h:\Xi x R^n x R_+^m x R^n \to R$$

by: $$h(\xi,u,\lambda,\rho) = f_o(\xi,u) + \sum_{i=1}^{m} \lambda_i f_i(\xi,u) - u\cdot\rho \qquad \text{(S-24)}$$

Then the Lagrangian $\mathcal{R}$ is:

$$\mathcal{R}(u,\lambda,\rho) = E\{h(\xi,u(\xi),\lambda(\xi),\rho(\xi))\} \qquad \text{(S-25)}$$

for all $(u,\lambda,\rho)\varepsilon U x \Lambda x M$.

Where:

$$\Lambda = \{\lambda=(\lambda_1,\ldots\lambda_m)\varepsilon L_m^1 \mid \lambda_i(\xi) \geqslant 0 \quad \text{a.s. for } i=1,\ldots m\} \qquad \text{(S-26)}$$

$$M = \{\rho=(\rho,\ldots\rho^T)\varepsilon L_n^1 \mid E^t\{\rho^t(\xi)\} = 0 \quad \text{a.s. for } t=1,\ldots T\} \qquad \text{(S-27)}$$

Where E^t is the conditional expectation given F^t.

Note that this definition of the space M differs slightly from that of [52]. However the difference is immaterial since it is confined to sets of measure zero.

A saddle point of $\mathcal{R}$ w.r.t. minimization in u and maximization in (λ,ρ) is an element, $(\tilde{u},\tilde{\lambda},\tilde{\rho})$, of $U x \Lambda x M$ satisfying:

$$\mathcal{R}(\tilde{u},\lambda,\rho) \leqslant \mathcal{R}(\tilde{u},\tilde{\lambda},\tilde{\rho}) \leqslant \mathcal{R}(u,\tilde{\lambda},\tilde{\rho}) \quad \text{for all } (u,\lambda,\rho)\varepsilon U x \Lambda x M \,. \qquad \text{(S-28)}$$

With these definitions the main results can be stated.

Theorem (S-3) (Theorem 1 of [53]). The Lagrangian $\mathcal{R}$ has at least one saddle point, $(\tilde{u},\tilde{\lambda},\tilde{\rho})$, relative to $U x \Lambda x M$, and the $\tilde{u}$ components of any such saddle point are the optimal recourse functions of PS.

$\square$

<u>Corollary (S-3.1)</u> The restricted Lagrangian:

$$\mathcal{R}'(u,\lambda)=E\{f_o(\xi,u(\xi))+\sum_{i=1}^{m}\lambda_i(\xi)f_i(\xi,u(\xi))\} \tag{S-29}$$

$$\text{for } (u,\lambda)\varepsilon(\mathcal{U}\cap N_\infty) \times \Lambda$$

has at least one saddle point, $(\tilde{u},\tilde{\lambda})$, relative to $(\mathcal{U}\cap N_\infty) \times \Lambda$ and the $\tilde{u}$ components of any such saddle point are the optimal recourse functions of PS. ∎

<u>Theorem (S-4)</u> (Theorem 2 of [53]). An element $(\tilde{u},\tilde{\lambda},\tilde{\rho})$ is a saddle point of $\mathcal{R}$ relative to $\mathcal{U}\times\Lambda\times M$ if and only if the following "Khun-Tucker" conditions are satisfied.

(a) $\tilde{u}\varepsilon N_\infty$ and $\tilde{u}(\xi)\varepsilon\mathcal{U}(\xi)$ a.s. (S-1)

and $f_i(\xi,\tilde{u}(\xi))\leqslant 0$ for $i=1,\ldots m$ (a.s) (S-2)

(b) $\tilde{\lambda} = (\tilde{\lambda},\ldots\tilde{\lambda}_m)\varepsilon L^1_m$

and $\tilde{\lambda}_i(\xi) \geqslant 0$ for $i=1,\ldots m$ a.s. (S-30)

$\tilde{\lambda}_i(\xi)f_i(\xi,\tilde{u}(\xi)) = 0$ for $i=1,\ldots m$ a.s. (S-31)

(c) $\tilde{\rho}\varepsilon M$

$$\text{and } h(\xi,\tilde{u}(\xi),\tilde{\lambda}(\xi),\tilde{\rho}(\xi)) = \underset{u\varepsilon\mathcal{U}(\xi)}{\text{MIN}}\; h(\xi,u,\tilde{\lambda}(\xi),\tilde{\rho}(\xi)) \text{ a.s.} \tag{S-32}$$ ∎

We now define a dual problem (DS). First define:

$$g(\xi,\lambda,\rho) = \begin{cases} \inf\limits_{u\varepsilon\mathcal{U}(\xi)} h(\xi,u,\lambda,\rho) & \text{if } \lambda\varepsilon\mathcal{R}^m_+ \\ -\infty \text{ if } & \lambda\notin\mathcal{R}^m_+ \end{cases} \tag{S-33}$$

<u>Theorem (S-5)</u> (Theorem 3 of [53]) The functional:

$$P(\lambda,\rho) = E\{g(\xi,\lambda(\xi),\rho(\xi))\} \tag{S-34}$$

is well defined and concave with:

$$\mathcal{P}(\lambda,\rho) = \inf_{u\varepsilon\mathcal{U}} \mathcal{R}(u,\lambda,\rho) \quad \text{for all } (\lambda,\rho)\varepsilon\Lambda x\mathcal{M} . \tag{S-35}$$

Thus optimal solutions, $(\tilde{\lambda},\tilde{\rho})$, to the problem:

$$\text{(DS)} \quad \text{Find} \quad \underset{(\lambda,\rho)\varepsilon\Lambda x\mathcal{M}}{\text{MAX}} \quad \mathcal{P}(\lambda,\rho) \tag{S-36}$$

exist and they are the (λ,ρ) components of the saddle points, $(\tilde{u},\tilde{\lambda},\tilde{\rho})$, of the Lagrangian $\mathcal{R}$. So:

$$\text{MIN(PS)} = \text{MAX(DS)} \tag{S-37}$$

$\square$

Further, let us restrict our attention to the separable case. Then we may assume:

$$\text{(i)} \quad \mathcal{U}(\xi) = \underset{t=1}{\overset{T}{X}} \mathcal{U}^t(\xi^t) \qquad \text{for all } \xi\varepsilon\Xi \tag{S-38}$$

$$\text{(ii)} \quad f_i(\xi,u) = \sum_{t=1}^{T} f_i^t(\xi,u^t) \quad \text{for all } \xi\varepsilon\Xi, u\varepsilon\mathcal{U}(\xi),\ i=0,\ldots m. \tag{S-39}$$

(iii) The multifunctions: $\mathcal{U}^t=\xi\to\mathcal{U}^t(\xi)\subset R^{n^t}$

are F measurable.

The functions: $f_i^t(\xi,u^t)$

are F^t measurable relative to the set:

$$(\mathcal{U}^t)^{-1}(u^t) = \{\xi\varepsilon\Xi \,|\, u^t\varepsilon\mathcal{U}^t(\xi)\} \ \varepsilon F^t \tag{S-40}$$

Then we can define our (separable) primal (SPS) as before. If we let:

$$\ell^t(\xi,u^t,\lambda) = f_o^t(\xi,u^t) + \sum_{i=1}^{m} \lambda_i f_i^t(\xi,u^t) \tag{S-41}$$

then we can restate our definition of $h(\xi,u,\lambda,\rho)$ as:

$$h(\xi,u,\lambda,\rho) = \sum_{t=1}^{T} [\ell^t(\xi,u^t,\lambda) - u^t \cdot \rho^t] \qquad \text{(S-42)}$$

and obtain a restatement of Theorem (S-4) as:

Theorem (S-6) (Theorem 4 of [53]) A function $\tilde{u}$ solves the separable optimal recourse problem (SPS) if and only if there is a multiplier function, $\tilde{\lambda}$, such that $(\tilde{u},\tilde{\lambda})$ satisfies (a) and (b) of the Khun-Tucker conditions in Theorem (S-4) and a modified version of condition (c), namely:

(c') for all $t=1,\ldots T$:

$$\ell^t(\xi,\tilde{u}^t(\xi),E^t\{\tilde{\lambda}(\xi)\}) = \underset{u^t \varepsilon \mathcal{U}^t(\xi)}{\text{MIN}} \ \ell^t(\xi,u^t,E^t\{\tilde{\lambda}(\xi)\}) \text{ a.s.} \mathcal{F}^t \qquad \text{(S-43)}$$

where $E^t\{\tilde{\lambda}(\xi)\}$ is the conditional expectation of $\tilde{\lambda}(\xi)$ given $\mathcal{F}^t$.

$\blacksquare$

This theorem assures us that, in this case, the optimal recourse function for stage t can be decided entirely on the basis of information relevant to stage t and independent of the future. The decision taken at stage t depends only on the vector $\hat{\xi}^t$ of past information and the expected prices, $E^t\{\tilde{\lambda}(\hat{\xi}^t)\}$.

We can also define a function:

$$g^t(\xi,\lambda) = \inf_{u^t \varepsilon \mathcal{U}^t(\xi)} \ell^t(\xi,u^t,\lambda) \quad \text{if } \lambda \varepsilon R^m_+ \qquad \text{(S-44)}$$

and introduce a special dual problem:

(SDS) Find $\underset{\lambda \varepsilon \Lambda}{\mathrm{MAX}} \sum_{t=1}^{T} E\{q^t(\xi, E^t\{\lambda(\xi)\})\}$. (S-45)

Then the following theorem is obtained.

<u>Theorem (S-7)</u> (Theorem 5 of [53]) The dual problem (SDS) has optimal solutions and they are precisely the $\tilde{\lambda}$ components of the pairs, $(\tilde{u},\tilde{\lambda})$, satisfying the Khun-Tucker conditions (a),(b),(c') of the previous theorem. Further:

$$\mathrm{MIN(SPS)} = \mathrm{MAX(SDS)} \tag{S-46}$$

□

We note that in the differentiable case where $\mathcal{U}(\xi)=\mathcal{R}^n$ and the f_i functions are each differentiable with gradient $\nabla f_i(\xi,u)$, then condition (c) of Theorem (S-4) can be restated as:

$$E^t\{\nabla f_o(\xi,\tilde{u}(\xi)) + \sum_{i=1}^{m} \tilde{\lambda}_i(\xi)\nabla f_i(\xi,\tilde{u}(\xi))\}= 0 \text{ a.s. for } t=1,\ldots T \tag{S-47}$$

(observing that (S-32) is equivalent to:

$$\nabla f_o(\xi,\tilde{u}(\xi)) + \sum_{i=1}^{m} \tilde{\lambda}_i(\xi)\nabla f_i(\xi,\tilde{u}(\xi)) = \tilde{\rho}(\xi) \quad \text{a.s.} \qquad \text{(S-48))}$$

Further, in the separable case, condition (c') of Theorem (S-6) can be stated as:

$$\nabla f_o^t(\xi,\tilde{u}^t)) + \sum_{i=1}^{m} E^t\{\tilde{\lambda}_i(\xi)\}\nabla f_i^t(\xi,\tilde{u}^t(\xi)) = 0 \text{ a.s.} \mathcal{F}^t \text{ for all } t=1,\ldots T. \tag{S-49}$$

(see [53] Section 3).

Paper [53] then goes on to specialize the above models to the case where the f_i $(i=0,\ldots m)$ functions are linear and to point out the fact that many optimal control problems can (as we shall later see) be modified to fit this optimal recourse framework. This completes our survey of the general theory and we now turn to models for our own particular problem.

8.3 A GENERAL STOCHASTIC MODEL OF A POWER SYSTEM

8.3.1 Introduction

The purpose of this section is to show how the theory just described can be applied to the optimisation of a power system. The result is a model which can be decomposed in a manner analogous to the decomposition of the deterministic model. We first describe the model then consider the theoretical solution of each of the sub-models.

8.3.2 The Model and its Dual

This model is a direct extension, into the stochastic framework discussed, of the deterministic multi-regional aggregated model of [47] (ignoring short-term requirements). (Or, equivalently, PAI, the model of Section 2.4 with K=1 and without the restriction, (A-8), ensuring that detailed demand patterns be met.) We let $\xi \varepsilon \Xi$ denote the inflows.

Here $\xi = (\xi^t)^{t=1,\ldots T}$ (SA-1)

$$\xi^t = (\xi^t_n)_{n=1} \qquad \text{for all } t=1,\ldots T \quad n=1,\ldots N \qquad \text{(SA-2)}$$

$$\xi_n^t = (\xi_h^t)_{h\varepsilon n} \qquad \text{for all } n=1,\ldots N \qquad \text{(SA-3)}$$

This convention is followed for the remainder of this section.

Hence:

$$\nu^t = \sum_{n=1}^{N} H_n \qquad \text{for all } t=1,\ldots T \qquad \text{(SA-4)}$$

(where H_n is the number of hydro systems in region n.)

We let σ be a probability measure on R^ν (with support Ξ) describing the probability distribution of ξ. We let σ be discrete with finite support (Ξ) and so σ and Ξ satisfy all the properties required by Theorems (S-1) - (S-5). Further we may replace the condition "for almost all $\xi\varepsilon\Xi$" by "for all $\xi\varepsilon\Xi$" wherever it appears.

We assume that demand in each region for each period is a function of the inflows only, thus:

$$D_n^{\xi t} = D_n^t(\xi) \qquad \text{for all } \xi\varepsilon\Xi, T=1,\ldots T$$
$$n=1,\ldots N. \qquad \text{(SA-5)}$$

The thermal cost functions are assumed, as before, to be dependent only on the pattern of generation within the week. (For convenience we group all thermal stations in each region).

Thus: $$C_n^{\xi t} = C_n^t(g_{nT}^{\xi t}) \qquad \text{for all } \xi\varepsilon\Xi, n=1,\ldots N$$
$$t=1,\ldots T \qquad \text{(SA-6)}$$

We require that C_n^t be a convex function, increasing in all of its arguments.

Similarly the loss functions are dependent only on the transmission levels so that:

$$L_{nm}^{\xi t} = L_{nm}^t(e_{nm}^{\xi t}) \qquad \text{(SA-7)}$$

and:

$$f_{mn}^{\xi t} = e_{mn}^{\xi t} - L_{mn}^{\xi t}(e_{mn}^{\xi t}) \qquad \text{for all } \xi\varepsilon\Xi, n=1,\ldots N$$
$$m=1,\ldots N$$
$$t=1,\ldots T. \qquad \text{(SA-8)}$$

We require that L_{nm}^{t} be a convex increasing function of e_{nm}^{t}.

The hydro output functions may depend, not only on the release pattern and the head, but also on the tributary inflows. Hence:

$$g_h^{\xi t} = g_h^t\,(q_h^{\xi t}, s_h^{\xi t}, \xi_h^t) \qquad \text{for all } h\varepsilon n \quad t=1,\ldots T \quad \xi\varepsilon\Xi. \tag{SA-9}$$

These functions are assumed to be strictly concave in q and convex in s. We assume for the sake of notational simplicity that the tributary inflows can be determined from the reservoir inflow. We also have all the constraints on generation, release, storage and exchange, as in the deterministic model.

Our problem is to find a nonanticipative decision rule:

$$z : \Xi \rightarrow R^n \text{ (where } z(\xi) = (g_T(\xi), q(\xi), e(\xi)) \tag{SA-10}$$

so as to minimise the total expected fuel costs, while meeting the demand, in each region, for each period, for all possible flow sequences, ξ, and also maintaining feasibility for all $\xi\varepsilon\Xi$. Thus we have the general problem:

$$\text{(PSA) Find} \quad \underset{z\varepsilon N_\infty}{\text{MIN}} \int_\Xi \sum_{n=1}^{N} \sum_{t=1}^{T} C_n^t(g_{nT}^{\xi t})\,\sigma(d\xi) \tag{SA-11}$$

Such that, for all $t = 1, \ldots T$, $n = 1, \ldots N$, $\xi\varepsilon\Xi$:

$$g_{nT}^{\xi t} + g_{nH}^{\xi t} + \sum_{m=1}^{N} (f_{mn}^{\xi t} - e_{nm}^{\xi t}) - D_n^{\xi t} \geq 0 \qquad \text{(SA-12)}$$

$$\text{(where } g_{nH}^{\xi t} = \sum_{h\varepsilon n} g_h^{\xi t}\text{)} \qquad \text{(SA-13)}$$

$$\underline{E}_{nm}^{t} \leq e_{nm}^{\xi t} \leq \bar{E}_{nm}^{t} \qquad \text{for all } m = 1, \ldots N \qquad \text{(SA-14)}$$

$$\underline{G}_{nT}^{t} \leq g_{nT}^{t} \leq \bar{G}_{nT}^{t} \qquad \text{(SA-15)}$$

$$s_h^{\xi t} = s_h^{\xi t-1} + \xi_h^{t} - q_h^{\xi t} \qquad \text{for all } h\varepsilon n \qquad \text{(SA-16)}$$

$$\underline{S}_h^{t} \leq s_h^{\xi t} \leq \bar{S}_h^{t} \qquad \text{for all } h\varepsilon n \qquad \text{(SA-17)}$$

$$\underline{Q}_h^{t} \leq q_h^{\xi t} \leq \bar{Q}_h^{t} \qquad \text{for all } h\varepsilon n \qquad \text{(SA-18)}$$

And, for all $n = 1, \ldots N$, $h\varepsilon n$, $\xi\varepsilon\Xi$:

$$s_h^{\xi o} = S_h^{o} \qquad \text{(SA-19)}$$

$$s_h^{\xi T} = S_h^{T} \qquad \text{(SA-20)}$$

This model is the obvious stochastic generalisation of PAI. It is, in fact, a problem of stochastic optimal control, but can obviously ([53]-Section 5) be recast in the framework of a discrete time optimal recourse problem by substituting the water balance equations, (SA-16), into the state-space constraints, (SA-17) and (SA-20). Here we outline the relationship between this model and the general model (PS) of the previous section. We show that it can be adapted so as to satisfy the conditions required by Theorems (S-1) - (S-5).

Firstly, we take as our recourse functions, u:

$$z(\xi) = (g_T(\xi), q(\xi), e(\xi)) \qquad \text{(SA-21)}$$

Defining the feasible region for recourse functions, $\mathcal{Z}(\xi)$, by the constraints (SA-13) -(SA-19). $\mathcal{Z}(\xi)$ is clearly closed and convex and has non-empty interior for any resonable power system. Also:

$$\mathcal{Z}^{-1}(z) = \{\xi \varepsilon \Xi : z \varepsilon \mathcal{Z}(\xi)\} \qquad \text{(SA-22)}$$

is Borel measurable.

Secondly, we take as the explicit constraints, $f_i(\xi,z)$:

$$f_n^t(\xi,z) = D_n^t(\xi) - g_{nH}^t(\xi_n^t, q_n^t, s_n^t) - g_{nT}^t - \sum_{m=1}^{N} (f_{mn}^t - e_{nm}^t)$$

$$\text{for } i = 1,\dots m \qquad \text{(SA-12')}$$

and for objective:

$$f_o(\xi,z) = \sum_{t=1}^{T} \sum_{n=1}^{N} C_n^t(g_{nT}^{\xi t}) \qquad \text{(SA-11')}$$

As for the deterministic problem, we can modify the cost curve, and the generation and loss curves, so as to ensure that $f_i(\xi,z)$ $(i=1,\dots m)$ are defined, finite, convex and lower semi-continuous for all $z \varepsilon \mathcal{Z}(\xi)$. With our assumptions about Ξ the functions $f_i(\xi,z)$ are clearly measurable relative to $\mathcal{Z}^{-1}(z)$.

It is also clear that the set:

$$\mathcal{D}(\xi) = \{z \varepsilon \mathcal{Z}(\xi) : f_i(\xi,z) \leqslant 0 \quad i = 1,\dots m\} \qquad \text{(SA-23)}$$

is uniformly bounded and that $f_i(\xi,z)$ is itself bounded and summable, for all $i = 0, 1,\ldots m$, and, hence, that (a) and (b) hold.

Strict feasibility can also be guaranteed (extending the cost curve where necessary to allow for the possibility of a shortfall in supply as in the deterministic model). However relatively complete recourse (non-anticipative feasibility) cannot be guaranteed. This is because the artificial constraints imposed on reservoir storage and outflow introduce the very real possibility of reaching a dead-end with no feasible recourse for some ξ. For example, if, in period t, we have storage s_h^t in reservoir h, the inflows $(\xi_h^r)^{r=t,\ldots T}$ from period t on, may be insufficient to raise storage to the target storage, S_h^T. On the other hand they may be too great to allow us to lower storage to S_h^T without spilling. In order to avoid this problem we modify the constraint structure as follows.

(i) As in the deterministic case we increase $\bar{Q}_h^t$ to the maximum total release allowable, setting $g_{ho}^t(q_h^t)$, the generation from the station immediately below reservoir h, to $\bar{G}_{ho}^t$ for all $q_h^t > \hat{Q}_h^t$ (where $\hat{Q}_h^t$ is the maximum usable release and:

$$\bar{G}_{ho}^t = g_{ho}^t(\hat{Q}_h^t)). \qquad \text{(SA-24)}$$

(ii) We relax (SA-20) to become a minimum constraint:

$$s_h^T(\xi) \geqslant \underline{S}_h^T \qquad \text{for almost all } \xi\varepsilon\Xi \qquad \text{(SA-25)}$$

relying on the cost minimising algorithm to ensure that in fact $s_h^T(\xi)$ will be as close to S_h^T as possible. We will also have a maximum storage constraint at T, in order to avoid excessive spill in the next planning horizon.

(iii) Assume that the distribution of inflows is such that we can define, for each $r,t \in (0,T)$, $r < t$, a maximum inflow between r and t:

$$\bar{\xi}_h(t,r) = \underset{\xi \in \Xi}{\text{MAX}} \left\{ \sum_{k=t+1}^{r} \xi_h^k \right\} \qquad \text{(SA-26)}$$

and a minimum total inflow:

$$\underline{\xi}_h(t,r) = \underset{\xi \in \Xi}{\text{MIN}} \left\{ \sum_{k=t+1}^{r} \xi_h^k \right\} \qquad \text{(SA-27)}$$

letting:

$$\underline{\xi}_h(r,r) = \bar{\xi}_h(r,r) = 0 \qquad \text{(SA-28)}$$

Then we can define new storage constraints, recursively backwards from T, by first letting:

$$\bar{S}'^{t}_h = \bar{S}^t_h, \qquad \underline{S}'^{t}_h = \underline{S}^t_h \qquad \text{for all } t=1,\ldots T \qquad \text{(SA-29)}$$

Then for all $t = T-1,\ldots 1$:

$$\bar{S}'^{t}_h = \underset{r \geq t}{\text{MIN}} \left\{ \bar{S}'^{r}_h - (\bar{\xi}_h(t,r) - \sum_{k=t+1}^{r} \tilde{Q}^k_h) \right\} \qquad \text{(SA-30)}$$

and:
$$\underline{S}'^{r}_h = \underset{r \geq t}{\text{MAX}} \left\{ \underline{S}'^{r}_h (\underline{\xi}_h(t,r) - \sum_{k=t+1}^{r} \underline{Q}^k_h) \right\} \qquad \text{(SA-31)}$$

An example of this kind of modification to the constraints is shown in Figure (8-1). Care must be taken to ensure that the problem is still feasible, particularly when choosing the limits $\bar{S}^T_h$, $\underline{S}^T_h$. It is easy

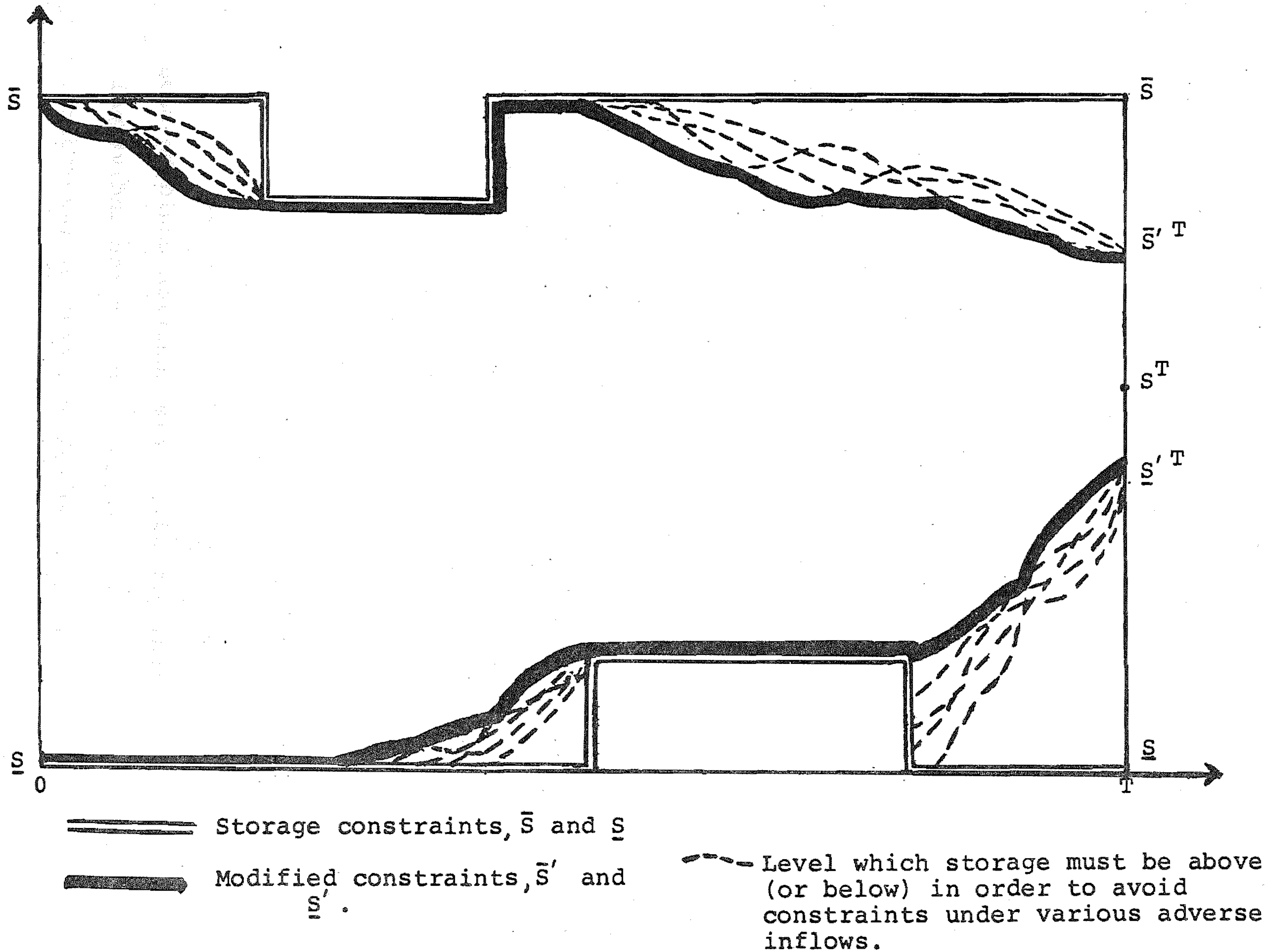


FIGURE (8-1): Modified storage constraints to see that, particularly if $\bar{\xi}_h^k > \bar{Q}_h^k$ and $\underline{\xi}_h^k < \underline{Q}_h^k$ (r<k<T), we could have $\bar{S}_h^{\prime r} < \underline{S}_h^{\prime r}$, indicating that there is no storage level at period r from which we can guarantee to be able to reach the target set $[\underline{S}_h^T, \bar{S}_h^T]$. This problem is unlikely to have significant practical effects in view of our intended policy of abandoning the target S_h^T at some stage in the optimisation and substituting a new target, S_h^{T+R}, probably a year further in the future. (See Section 9.2.5).

Our assumption that we can place bounds on the inflow distributions is obviously only true in a probabilistic sense and corresponds to the requirement that the constraints be met "almost surely". Practically, the utility must decide how "sure" they have to be that these constraints are met. Typically, we require that the probability of shortfall be less that say 5% in which case the sequences $\bar{\xi}, \underline{\xi}$ would correspond to 95% and 5% Design Dry Years. Finally, it will be seen that this modified constraint structure is very similar to the original Basic Rule Curve/No Spill Curve approach used by the TVA in [9]. We deal henceforth with these modified constraints dropping the $'$.

We note that this is only one of several modifications we could incorporate. For instance, we could specify a (possibly non-linear) value function for water in storage at the final period. This function could be made to depend merely upon the storage level in the individual reservoir or on storage levels in a number of reservoirs. This last scheme, while more realistic, would destroy the spatial separability of the problem. In order to be useful this approach would probably require a non-linear value function which would destroy the temporal separability of the objective function and thus bar us from applying Theorems (S-6) and (S-7). For this reason we prefer the approach previously outlined although, in practice, especially if the head effect destroys temporal separability anyway, we may wish to employ the latter scheme.

With this former modification our problem satisfies the conditions of Theorem (S-3) and we introduce the restricted Lagrangian $\mathcal{R}'$:

$$\mathcal{R}'(z,\lambda) = E\{\ell(\xi,z(\xi),\lambda(\xi))\} \quad \text{for } (z,\lambda)\varepsilon(Z\cap N_\infty) \times \Lambda \qquad \text{(SA-32)}$$

Where:

$$\ell(\xi,z,\lambda) = \sum_{t=1}^{T}\sum_{n=1}^{N}\Big[C_n^t(g_{nT}^{\xi t}) - \lambda_n^t\Big(g_{nT}^t + g_{nH}^t(\hat{q}_n^t,\hat{\xi}_n^t) + \sum_{m=1}^{N}(f_{mn}^t - e_{nm}^t) - D_n^t(\xi)\Big)\Big] \qquad \text{(SA-33)}$$

Corollary (S-3.1) assures us that this Lagrangian has at least one saddle point, $(\tilde{z},\tilde{\lambda})$, relative to $(Z\cap N_\infty)\times\Lambda$. Moreover, the $\tilde{z}$ components of any such saddle point are precisely the optimal recourse functions of PSA. Similarly, it is clear that if the saddle point of the Lagrangian $\mathcal{R}'$ is almost surely unique then the $\tilde{\lambda}$ components are the optimal λ multipliers of the restricted dual problem DSA. This will in fact be the case in our problem because the objective function, when modified as outlined in Chapters 3 to 5.

is almost surely strictly convex (see [53]p20).

We propose a Uzawa type iterative scheme to solve the problem of finding a saddle point of $\mathcal{R}'$. At each iteration of this we will face the Lagrangian problem for a particular set of multipliers, λ:

$$\text{(PSA}'\text{)} \quad \text{Find } \underset{z \in Z \cap N_\infty}{\text{MIN}} \; E\left\{ \sum_{t=1}^{T} \sum_{n=1}^{N} \left[C_n^t(g_{nT}^{\xi t}) - \lambda_n^t(\xi) g_{nT}^{\xi t} \right.\right.$$

$$-\lambda_n^t(\xi) \left(\sum_{h \in n} g_h^t(\hat{q}_h^{\xi t}, \hat{\xi}_h^t) \right)$$

$$\left.\left. -\lambda_n^t(\xi) \left(\sum_{m=1}^{N} (f_{mn}^{\xi t} - e_{nm}^{\xi t}) \right) + \lambda_n^t(\xi) D_n^t(\xi) \right] \right\} \qquad \text{(SA-34)}$$

This can obviously be decomposed, as in the deterministic case, into thermal, exchange and hydro problems. As before we can ignore the "demand problem" since $D_n^t(\xi)$ is considered to be constant and $\lambda_n^t(\xi)$ is fixed for each iteration. The next three sections outline the theoretical solution of each of these sub-models. The next chapter considers some practicable approaches to the solution of the general model.

8.3.3 The Thermal Sub-Problem

The thermal sub-problem, (PST), is:

$$\text{Find } \underset{g_T \in N_\infty}{\text{MIN}} \; E\left\{ \sum_{t=1}^{T} \sum_{n=1}^{N} \left[(C_n^t(g_{nT}^{\xi t}) - \lambda_n^t(\xi) g_{nT}^{\xi t} \right] \right\} \qquad \text{(SA-34)}_T$$

Such that:

$$\underline{G}^{t}_{nT} \leqslant g^{\xi t}_{nT} \leqslant \bar{G}^{t}_{nT} \qquad \text{for all } \xi\varepsilon\Xi \qquad \text{(SA-15)}$$

$$t = 1,\ldots T$$

$$n = 1,\ldots N.$$

This is a separable optimal recourse problem with no inter-period interaction either in the constraints or in the objective. So the restriction that $g_{nT}\varepsilon N_{\infty}$ is redundant and the problem can be reduced to a set of NxT x card (Ξ) one-period one-region thermal problems, $(PST)^{\xi t}_{n}$. These can be solved exactly as in the deterministic case (see Chapter 3), yielding:

$$g^{\xi t*}_{nT}(\lambda) = g^{\xi t*}_{nT}(\lambda^{t}_{n}(\xi)) \qquad \text{(SA-35)}$$

8.3.4 The Exchange Sub-Problem

The exchange sub-problem, (PSE), is:

$$\text{Find } \underset{\xi\varepsilon N_{\infty}}{\text{MAX}} \quad E\left\{\sum_{t=1}^{T} \sum_{n=1}^{N}\left[\lambda^{t}_{n}(\xi)\left(\sum_{m=1}^{N}(f^{\xi t}_{mn} - e^{\xi t}_{nm})\right)\right]\right\} \qquad \text{(SA-34)}_{E}$$

Such that:

$$\underline{E}^{t}_{nm} \leqslant e^{t}_{nm} \leqslant \bar{E}^{t}_{nm} \qquad \text{for all } \xi\varepsilon\Xi$$

$$T = 1,\ldots T$$

$$n = 1,\ldots N \qquad \text{(SA-14)}$$

This problem likewise results in Nx(N-1)xTx card (Ξ) one-period inter-regional exchange problems, $(PSE)^{\xi t}_{mn}$:

$$\text{Find } \underset{e^{\xi t}_{mn}}{\text{MAX}} \quad \left[e^{\xi t}_{mn}\left(\lambda^{t}_{n}(\xi) - \lambda^{t}_{m}(\xi)\right) - L^{t}_{mn}(e^{\xi t}_{mn})\left(\lambda^{t}_{n}(\xi)\right)\right] \qquad \text{(SA-34)}^{\xi t}_{E_{mn}}$$

Such that:

$$\underline{E}^{t}_{mn} \leqslant e^{\xi t}_{mn} \leqslant \bar{E}^{t}_{mn} \qquad \text{(SA-14)}^{\xi t}$$

These can be solved, exactly as in the deterministic case (see Chapter 4), yielding:

$$e^{\xi t*}_{mn} = e^{\xi t*}_{mn}\left(\lambda^{t}_{n},(\xi),\lambda^{t}_{m}(\xi)\right) \qquad \text{(SA-36)}$$

8.3.5 The Hydro Sub-Problem

The hydro sub-problem (PSH) is:

$$\text{Find} \max_{q\in N_\infty} \; E\left\{\sum_{t=1}^{T}\sum_{n=1}^{N}\lambda^{\xi t}_{n}\left[\sum_{h\varepsilon n} g^{t}_{h}(\hat{q}^{\xi t}_{h},\hat{\xi}^{t}_{h})\right]\right\} \qquad \text{(SA-34)}_{H}$$

Such that, for all $n = 1,\ldots N$, $h\varepsilon n$, $t = 1,\ldots T$, $\xi\varepsilon\Xi$:

$$S^{o}_{h} + \sum_{r=1}^{t} \xi^{r}_{h} - \bar{S}^{t}_{h} \leqslant \sum_{r=1}^{t} q^{\xi r}_{h} \leqslant S^{o}_{h} + \sum_{r=1}^{t} \xi^{r}_{h} - \underline{S}_{h} \qquad \text{(SA-16)}$$

(substituting (SA-15) and (SA-18) into (SA-16)).

$$\underline{Q}^{t}_{h} \leqslant q^{\xi t}_{h} \leqslant \bar{Q}^{t}_{h} \qquad \text{(SA-18)}$$

We will henceforth assume there is no head effect so that:

$$g^{t}_{h}(\hat{q}^{\xi}_{h},\hat{\xi}^{t}_{h}) = g^{t}_{h}(q^{\xi t}_{h},\xi^{t}_{h}) \qquad \text{(SH-1)}$$

and the problem (SPSH) is temporally separable (otherwise objective function is not separable (temporally)). In any case the problem is obviously spatially separable so that, just as in the deterministic case, we can deal with H separate local hydro problems, PSH. Henceforth we will deal with such a problem and omit the subscripts h and n. We can then state our problem as:

$$\text{(PSH)} \quad \text{Find} \quad \underset{q \varepsilon N_{\infty}}{\text{MAX}} \quad E \left\{ \sum_{t=1}^{T} \lambda^{\xi t} g^{t} (\hat{q}^{\xi t}, \hat{\xi}^{t}) \right\} \tag{SH-2}$$

Such that, for all $\xi \varepsilon \Xi$, $t = 1 \ldots T$:

$$S^{o} + \sum_{r=1}^{t} \xi^{r} - \bar{S}^{t} \leqslant \sum_{r=1}^{t} q^{\xi r} \leqslant S^{o} + \sum_{r=1}^{t} \xi^{r} - \underline{S}^{t} \tag{SH-3}$$

$$\underline{Q}^{t} \leqslant q^{\xi t} \leqslant \bar{Q}^{t} \tag{SH-4}$$

This problem (or its predecessor) is itself in the form of a T-stage stochastic optimal recourse problem, just as was the original problem. Here the underlying stochastic process is observed via two variables ξ_n, the local inflows, and $\lambda_n^t(\xi)$, the regional energy price. These energy prices summarise all the information about the past, present and likely future states of the entire system which could be required by the local manager in order to determine a complete set of optimal release decisions for his own reservoir. Thus we can apply the theory previously developed to this problem. Note, however, that we are concerned here with the maximisation of a concave functional rather than the minimisation of a convex one.

The theory developed in Section 8.2 produced two types of results. The first, developed in [52] and described in Section 8.2.3, concerned a dynamic programming type "value function". In the next section we apply these results to our local hydro problem. The following section deals with the application of the second set of results, developed in [52] and [53] and summarised in

Sections 8.2.3 and 8.2.4.

8.3.6 A Water Value Function

Here we ignore the structure of the constraints, (SH-3) and (SH-4), and, given the assumptions we have already made, apply Theorem (S-1) to problem PSH. This "dynamic programming" type result assures us that, if we define a value function recursively, for all $\xi\varepsilon\Xi$, by:

$$V^T(\hat{\xi}^T,\hat{q}^T) = V^T(\xi,q) = \begin{cases} \sum_{t=1}^{T} \lambda^{\xi t} g^t(\hat{q}^t,\hat{\xi}^t) & q\varepsilon Q \\ -\infty & q \notin Q \end{cases} \tag{SH-5}$$

$$V^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) = E^t_{\hat{\xi}^t}\left\{\sup_{q^t} V^t(\hat{\xi}^t,\hat{q}^t)\,|\,\hat{\xi}^{t-1}\right\} \tag{SH-6}$$

Where $E^t_{\hat{\xi}^t}\{\cdot|\hat{\xi}^{t-1}\}$ is (a version of) the conditional expectation for the random vector $\hat{\xi}^t$ given the random vector $\hat{\xi}^{t-1}$. (Q is defined by (SH-3) and (SH-4)).

Then we can define, for each $t = 1,\ldots T$, a problem of the same type as PSH:

$$(\hat{PSH}^t) \quad \text{Find} \sup_{\hat{q}^t\varepsilon\hat{Q}(\hat{\xi}^t)\cap N^t_\infty} E\{V^t(\hat{\xi}^t,\hat{q}^t)\} \tag{SH-7}$$

which is solvable. (Theorem (S-2)). Where $V^t(\hat{\xi}^t,\hat{q}^t)$ is defined by (SH-5) - (SH-6).

We now utilise the temporal separability of our objective function to decompose the water value function into two parts - a past value function and an expected future value function. (Note here that the manager is assumed to observe inflows then release. In fact the observation and decision continue during the week. This does not materially affect the analysis). First we define:

$$\hat{V}^t(\hat{\xi}^t,\hat{q}^t) = \sum_{r=1}^{t} \lambda^{\xi r} g^r(\hat{q}^r,\hat{\xi}^r) \qquad \text{(SH-8)}$$

$$\text{for all } t = 1,\ldots T \quad \xi\varepsilon\Xi, q\varepsilon Q$$

Here $\hat{V}^t(\hat{\xi}^t,\hat{q}^t)$, the past value function, gives the total profit already realised if, at time t, the inflows $\hat{\xi}^t$ have been experienced and decisions $\hat{q}^t$ made.

We can also define an expected future value function, $\check{V}^t(\hat{\xi}^t,\hat{q}^t)$ for all $\xi\varepsilon\Xi, q\varepsilon Q$, recursively, by letting:

$$\check{V}^T(\hat{\xi}^T,\hat{q}^T) = 0 \qquad \text{(SH-9)}$$

$$\check{V}^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) = E\left\{\sup_{p^t}\left[\lambda^{\xi t} g^t(\hat{p}^t,\hat{\eta}^t)+\check{V}^t(\hat{\eta}^t,\hat{p}^t)\right] \middle| \hat{\eta}^{t-1}=\hat{\xi}^{t-1}, \hat{p}^{t-1}=\hat{q}^{t-1}\right\} \qquad \text{(SH-10)}$$

The function $\check{V}^t(\hat{\xi}^t,\hat{q}^t)$ gives the total expected value of water in storage at the end of period t if inflows $\hat{\xi}^t$ have occurred and decisions $\hat{q}^t$ have been made. We will refer to the problem of determining $\check{V}^{t-1}$ via (SH-10) as problem PSH^t. We can now derive several useful properties of $\hat{V}$ and $\check{V}$.

<u>Proposition (S-8)</u> Given the above definitions of V, $\hat{V}$ and $\check{V}$, and the assumptions we have made about the model and also that $g^t(\hat{q}^t)$ is differentiable and the partial derivatives, $\frac{\partial g^t}{\partial q^r}$ are continuous, for all $1 \leqslant r \leqslant t$, then we have, for all $t = 1,\ldots T$:

(i) $$V^t(\hat{\xi}^t,\hat{q}^t) = \hat{V}^t(\hat{\xi}^t,\hat{q}^t) + \check{V}^t(\hat{\xi}^t,\hat{q}^t)$$

$$\text{for all } \xi\varepsilon\Xi \quad q\varepsilon Q(\xi) \qquad \text{(SH-11)}$$

(ii) $\hat{V}^t$ and $\check{V}^t$ are both concave in $\hat{q}^t$

(iii) The one sided directional derivatives $D_{\hat{V}}(\hat{\xi}^t,\hat{q}^t;d)$

and $D_{\check{V}}(\hat{\xi}^t,\hat{q}^t;d)$, both exist, for all $\hat{q}^t \varepsilon \hat{\mathcal{Q}}^t(\hat{\xi}^t)$

$d = (d^1,\ldots d^t) \varepsilon \mathcal{R}^t$

Where:

$$D_F(\hat{\xi}^t,\hat{q}^t;d) = \lim_{\alpha \to 0^+} \frac{F^t(\hat{\xi}^t,\hat{q}^t+\alpha d) - F^t(\hat{\xi}^t,\hat{q}^t)}{\alpha} \qquad \text{(SH-12)}$$

(iv) $\hat{V}^t(\hat{\xi}^t,\hat{q}^t)$ and $\check{V}^t(\hat{\xi}^t,\hat{q}^t)$ are both differentiable for all $\hat{q}^t \varepsilon \operatorname{int} \hat{\mathcal{Q}}^t(\hat{\xi}^t)$, and the partial derivatives, $\dfrac{\partial \hat{V}^t(\hat{\xi}^t,\hat{q}^t)}{\partial q^r}$ and $\dfrac{\partial \check{V}^t(\hat{\xi}^t,\hat{q}^t)}{\partial q^r}$, are continuous for all $r \leqslant t$ (and so $\hat{V}^t$ and $\check{V}^t$ are both continuous at all interior points of $\mathcal{Q}$). $\square$

<u>Proof</u>

(1). This may be proved by induction. Firstly, for all $\xi \varepsilon \Xi, q \varepsilon \mathcal{Q}(\xi)$:

$$V^T(\hat{\xi}^T,\hat{q}^T) = \sum_{t=1}^{T} \lambda^{\xi t} g(\hat{q}^{\xi t},\hat{\xi}^t) + 0 \qquad \text{(SH-7)}$$

$$= \hat{V}^T(\hat{\xi}^T,\hat{q}^T) + \check{V}^T(\hat{\xi}^T,\hat{q}^T) \qquad \text{(SH-11)}^T$$

Secondly, if, for some $1 < t \leqslant T$:

$$V^t(\hat{\xi}^t,\hat{q}^t) = \hat{V}^t(\hat{\xi}^t,\hat{q}^t) + \check{V}^t(\hat{\xi}^t,\hat{q}^t) \quad \text{for all } \xi \varepsilon \Xi, \; q \varepsilon \mathcal{Q}(\xi) \qquad \text{(SH-11)}^t$$

Then, for any $\xi \varepsilon \Xi$, $q \varepsilon \mathcal{Q}(\xi)$:

$$V^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) = E\left\{\sup_{q^t} V^t(\hat{\eta}^t,\hat{q}^t) \mid \hat{\eta}^{t-1}=\hat{\xi}^{t-1}\right\}$$

$$= E\left\{\sup_{q^t}\left[\hat{V}^t(\hat{\eta}^t,\hat{q}^t) + \check{V}^t(\hat{\eta}^t,\hat{q}^t)\right|\hat{\eta}^{t-1}=\hat{\xi}^{t-1}\right\}$$

$$= E\left\{\sup_{q^t}\left[\sum_{r=1}^{t-1}\left(\lambda^{\xi r}g(\hat{q}^{\xi r},\hat{\xi}^r)\right) + \lambda^{\eta t}g(\hat{g}^t,\hat{\eta}^t)\right.\right.$$

$$\left.\left. + \check{V}^t(\hat{\eta}^t,\hat{q}^t)\right]\right|\hat{\eta}^{t-1}=\hat{\xi}^{t-1}\Bigg\}$$

$$= \hat{V}^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) + E\left\{\sup_{q^t}\left[\lambda^{\eta t}g(\hat{q}^t,\hat{\eta}^t)+\check{V}^t(\hat{\eta}^t,q^t)\right|\hat{\eta}^{t-1}=\hat{\xi}^{t-1}\right\}$$

$$= \hat{V}^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) + \check{V}^t(\hat{\xi}^{t-1},\hat{q}^{t-1}) \qquad \text{(SH-11)}^{t-1}$$

So (i) holds for t-1, and hence, by induction, for all t.
(ii). $\hat{V}^t$ is obviously concave in $\hat{q}^t$. The concavity of $\check{V}^t$ may be proved, as was the convexity of V^t in Theorem (S-1), by repeated applications of Lemmas 1 and 2 of [52]. (Although, in our case, simpler arguments would suffice).

(iii) and (iv). The assertions about $\hat{V}^t$ are obvious. We may prove the assertions about $\check{V}^t$ by induction. Firstly, for all $\xi\varepsilon\Xi, q\varepsilon\mathcal{Q}(\xi)$:

$$\check{V}^T(\hat{\xi}^T,\hat{q}^T) = 0 \qquad \text{(SH-9)}$$

clearly satisfies both conditions.

Secondly, suppose that, for some $1 < t \leqslant T$, (iii) and (iv) hold. Then, for all $\xi\varepsilon\Xi, q\varepsilon\mathcal{Q}(\xi)$ the partial derivatives:

$\frac{\partial \overset{v}{V}^t}{\partial q^t}(\hat{\xi}^t,\hat{q}^t)$ exist and are continuous for $1 \leqslant r < t$. Also $\overset{v}{V}^t(\hat{\xi}^t,\hat{q}^t)$ is continuous.

$$\text{Let: } \phi(\hat{\xi}^t,\hat{q}^{t-1}) = \underset{q^t \varepsilon \mathcal{Q}^t(\xi)}{\text{MIN}} \left[\overset{v}{V}^t(\hat{\xi}^t,\hat{q}^t) + \lambda^{\xi t} g^t(\hat{q}^t,\hat{\xi}^t)\right] \quad \text{(SH-13)}$$

By a theorem due to Danskin (Theorem 8.5.1 in [30]) we have that $D_\phi(\hat{\xi}^t,\hat{q}^{t-1};d)$ exists and:

$$D_\phi(\hat{\xi}^t,\hat{q}^{t-1};d) = \underset{q^t \varepsilon \mathcal{Q}^t(\xi)}{\text{MIN}} \sum_{r=1}^{t-1} d^r \left(\frac{\partial \overset{v}{V}^t(\hat{\xi}^t,\hat{q}^t)}{\partial q^r} + \lambda^{\xi r}\frac{\partial g^t}{\partial q^r}(\hat{q}^t,\hat{\xi}^t)\right)$$

$$\text{for any } d \varepsilon R^{t-1}. \quad \text{(SH-14)}$$

Now:

$$\overset{v}{V}^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1}) = E\left\{\underset{q^t \varepsilon \mathcal{Q}^t(\eta)}{\sup} \left[\lambda^{\eta t} g^t(\hat{q}^t,\hat{\eta}^t) + \overset{v}{V}^t(\hat{\eta}^t,\hat{q}^t)\right] \,\middle|\, \hat{\eta}^{t-1} = \hat{\xi}^{t-1}\right\}$$

$$= E\left\{\phi(\hat{\eta}^t,\hat{q}^{t-1}) \,\middle|\, \hat{\eta}^{t-1} = \hat{\xi}^{t-1}\right\} \quad \text{(SH-15)}$$

(Since $\mathcal{Q}^t(\xi) \subset R$ is compact).

In our case, since σ is a discrete measure with finite support (Ξ), this expectation can be found by a finite summation so we have that:

$$D_{\overset{v}{V}}(\hat{\xi}^{t-1},\hat{q}^{t-1};d) = E\left\{D_\phi(\hat{\eta}^t,\hat{q}^{t-1};d) \,\middle|\, \hat{\eta}^{t-1} = \hat{\xi}^{t-1}\right\}$$

exists for all $d \varepsilon R^{t-1}$. (SH-16)

So we have proved that assertion (iii) holds (w.r.t.$\check{V}$) for t-1.

Also, if $q \varepsilon$ int $\mathcal{Q}(\xi)$, then we can see from (SH- 14) that all of the partial derivatives of ϕ exist at q and are given by:

$$\frac{\partial\phi}{\partial q^r}(\hat{\xi}^t,\hat{q}^{t-1}) = \underset{q^t \varepsilon \mathcal{Q}^t(\xi)}{\mathrm{MIN}} \left[\frac{\partial\check{V}^t(\hat{\xi}^t,\hat{q}^t)}{\partial q^r} + \lambda^{\xi r}\frac{\partial g^t(\hat{q}^t,\hat{\xi}^t)}{\partial q^r}\right]$$

$$\text{for all } 1 \leqslant r < t \qquad \text{(SH-17)}$$

(using e.g. Lemma 8.5.1 of [30] due to Fenchel).
Now, since we have assumed that the expression in square brackets is continuous at q and $\mathcal{Q}^t(\xi)$ is compact, then $\frac{\partial\phi}{\partial q^r}(\hat{\xi}^t,\hat{q}^{t-1})$ is continuous for all $1 \leqslant r \leqslant t$. (e.g. Theorem 7.2 in [67]).

So $\frac{\partial\check{V}^{t-1}(\hat{\xi}^{t-1},\hat{q}^{t-1})}{\partial q^r}$ is also continuous at q (given our assumptions about σ and Ξ).

This last condition implies the continuity of $\check{V}^{t-1}$ at all interior points of $\mathcal{Q}(\xi)$.

Now we have shown that both conditions (iii) and (iv) hold (w.r.t.$\check{V}$) for t-1 and hence, by induction, for all t.

QED

Now, letting t = 1 in (SH-11), we have that:

$$V^1(\hat{\xi}^1,\hat{q}^1) = V^1(\xi^1,q^1) = \lambda^1 g^1(q^1,\xi^1) + \check{V}^1(\xi^1,q^1) \qquad \text{(SH-18)}$$

We can find the optimal release, q^{1*}, by solving the problem $\hat{PSH}^1$:

$$\text{Find} \sup_{q^1 \varepsilon Q^1(\xi^1) \cap N_\infty} E\{V^1(\hat{\xi}^1,\hat{q}^1)\} \qquad \text{(SH-19)}$$

$$\text{i.e. Find} \sup_{q^1 \varepsilon Q^1(\xi^1)} \lambda^1 g^1(q^1,\xi^1) + \check{V}^1(\xi^1,q^1) \qquad \text{(SH-20)}$$

In order to solve this problem we must know something about the expected future value function $\check{V}^1(\xi^1,q^1)$. According to (SH-14) we should determine this by finding the future benefits to be derived, under the assumption of optimal future nonanticipative management, for each possible sequence, η, such that $\eta^1 = \xi^1$ (and decision, q^1, resulting in storage level, $s^1 = S^0 + \xi^1 - q^1$). This can, theoretically, be determined by dynamic programming style backwards recursion as in the original definition of V. However we will show later (Chapter 9) that this is impracticable and will take a simulation approach.

Note that, since $\check{V}$ is concave and differentiable, we can solve $P\hat{S}H^1$ by finding $\tilde{q}$ such that:

$$\lambda^1 \frac{\partial g(q^1,\xi^1)}{\partial q^1}\bigg|_{\tilde{q}^1} + \frac{\partial \check{V}(\xi^1,q^1)}{\partial q^1}\bigg|_{\tilde{q}^1} = 0 \qquad \text{(SH-21)}$$

for each ξ^1.

(Setting $\tilde{q}^1 = \bar{Q}^1$ or $\underline{Q}^1$, as appropriate, if there is no such feasible $\tilde{q}^1$).

In general, for all $t = 1,\ldots T$, if we have solved the problem $P\hat{S}H^{t-1}$ to get $\hat{q}^{t-1*}$, then we know that there is a solution, $\hat{p}^t$, to $P\hat{S}H^t$, with $\hat{p}^{t-1} = \hat{q}^{t-1*}$. So we can solve problem $P\hat{S}H^t$ by finding $\tilde{p}$ such that:

$$\tilde{p}^{\xi t-1} = \hat{q}^{\xi t-1*} \tag{SH-22}$$

$$\lambda^{\xi t} \frac{\partial g^t}{\partial p^{\xi t}}(\hat{p}^{\xi t}, \hat{\xi}^t)\bigg|_{\tilde{p}} + \frac{\partial \check{V}^t(\hat{\xi}^t, \hat{p}^{\xi t})}{\partial p^{\xi t}}\bigg|_{\tilde{p}} = 0 \tag{SH-23}$$

for each $\xi\varepsilon\Xi$. (If no such $\tilde{p}$ exists set $\tilde{p}^{\xi t} = \bar{Q}^{\xi t}$ or $\underline{Q}^{\xi t}$ as appropriate.)

Thus, if at each stage t we can estimate $\frac{\partial \check{V}^t}{\partial p^{\xi t}}(\hat{\xi}^t, \hat{p}^{\xi t})$ for all $\xi\varepsilon\Xi$, we can solve PSH by solving the sequence of problems $P\hat{S}H^t$ (or equivalently PSH^t).

8.3.7 Multipliers for Storage Constraints

Alternatively to the above analysis, we can assign multipliers to the storage constraints (SH-1) and (SH-2) and introduce the function:

$$\begin{aligned} h(\xi,\lambda(\xi),q,\delta,\gamma,\rho) = \sum_{t=1}^{T}\Bigg[& \lambda^{\xi t} g^t(\hat{q}^t, \hat{\xi}^t) \\ & - \delta^{\xi t}\left(S^o + \sum_{r=1}^{t} \xi^r - \bar{S}^t - \sum_{r=1}^{t} q^r\right) \\ & + \gamma^{\xi t}\left(S^o + \sum_{r=1}^{t} \xi^r - \underline{S}^t - \sum_{r=1}^{t} q^r\right) \\ & - q^t \rho^{\xi t}\Bigg] \end{aligned} \tag{SH-24}$$

As in the deterministic case, we can rewrite this as:

$$h(\xi,\lambda(\xi),q,\delta,\gamma,\rho) = \sum_{t=1}^{T}\Big[\lambda^{\xi t}g^{t}(\hat{q}^{t},\hat{\xi}^{t}) - \psi^{\xi t}q^{t} + K^{\xi t}(\delta,\gamma) - q^{t}\rho^{\xi t}\Big] \qquad \text{(SH-25)}$$

Where: $$\psi^{\xi t} = \sum_{r=t}^{T}\Big[\gamma^{\xi r} - \delta^{\xi r}\Big] \qquad \text{(SH-26)}$$

and: $$K^{\xi t}(\delta,\gamma) = -\delta^{\xi t}\Big[S^{o} + \sum_{r=1}^{t}\xi^{r} - \underline{S}^{t}\Big] + \gamma^{\xi t}\Big[S^{o} + \sum_{r=1}^{t}\xi^{r} - \bar{S}^{t}\Big] \qquad \text{(SH-27)}$$

If we let:

$$g(\xi,\lambda,\delta,\gamma,\rho) = \begin{cases} \sup\limits_{q\in\mathcal{Q}(\xi)} \; h(\xi,\lambda,q,\delta,\gamma,\rho) & \text{if } (\delta,\gamma)\in\mathcal{R}_{+}^{2T} \\ -\infty & \text{if } (\delta,\gamma)\notin\mathcal{R}_{+}^{2T} \end{cases} \qquad \text{(SH-28)}$$

Then we get as our dual problem, DSH:

$$\text{Find} \min_{(\delta,\gamma,\rho)\in\mathcal{R}_{+}^{2T}\times\mathcal{M}} E\{g(\xi,\lambda(\xi),\delta(\xi),\gamma(\xi),\rho(\xi)\} \qquad \text{(SH-29)}$$

If we were to solve this by an Uzawa type iterative scheme we would face at each iteration the problem of finding, for a particular $(\tilde{\lambda},\tilde{\delta},\tilde{\gamma},\tilde{\rho})$ and for each $\xi\in\Xi$, $q(\xi,\tilde{\lambda},\tilde{\delta},\tilde{\gamma},\tilde{\rho})$ so as to maximise, over $q\in\mathcal{Q}(\xi)$, the expression:

$$h'(\tilde{\xi},\tilde{\lambda},q,\tilde{\psi},\tilde{\rho}) = \sum_{t=1}^{T}\Big[\tilde{\lambda}^{\xi t}g^{t}(\hat{q}^{t},\hat{\xi}^{t}) - \tilde{\psi}^{\xi t}q^{t} - q^{t}\tilde{\rho}^{t}\Big] \qquad \text{(SH-30)}$$

The remainder of h being obviously constant for any particular $(\tilde{\delta},\tilde{\gamma},\tilde{\rho})$. (This conclusion is derived from condition (c) of Theorem (S-4)).

Further, if the problem PSH is temporally separable, then we can define:

$$\ell^t(\xi,\lambda,q^t,\delta,\gamma) = \lambda^{\xi t} g^t(\xi^t,q^t) - \psi^{\xi t} q^t \tag{SH-31}$$

Now we can effectively ignore ρ, defining:

$$d^t(\xi,\delta,\gamma) = \begin{cases} \sup\limits_{q^t \in \mathcal{Q}^t(\xi)} \ell^t(\xi,\lambda,q^t,\delta,\gamma) & \text{if } (\delta,\gamma) \in \mathcal{R}_+^{2T} \\ +\infty & \text{if } (\delta,\gamma) \notin \mathcal{R}_+^{2T} \end{cases} \tag{SH-32}$$

So we can introduce the separable dual hydro problem:

$$\text{(SDSH) Find } \underset{(\delta,\gamma)\in\mathcal{R}_+^{2T}}{\text{MIN}} \sum_{t=1}^{T} E\left\{d^t(\xi,E^t\{\delta\},E^t\{\gamma\})\right\} \tag{SH-33}$$

Then we know that the (a.s. unique) optimal solutions $(\tilde{q},\tilde{\delta},\tilde{\gamma})$ are characterised by the Kuhn-Tucker conditions:

(a) $q(\xi) \in \mathcal{Q}(\xi)$ a.s. (SH-34)

and

(c') for all $t=1,\ldots T$

$$\tilde{q}^t \text{ maximises } \ell^t(\hat{\xi}^t,\hat{\lambda}^t,q^t,E^t\{\tilde{\gamma}\},E^t\{\tilde{\gamma}\}). \tag{SH-35}$$

$$\text{i.e. } \tilde{q}^t \text{ maximises } \lambda^{\xi t} g^t(\xi^t,q^t) - q^t\bar{\psi}^{\xi t} \tag{SH-36}$$

$$\text{where: } \bar{\psi}^{\xi t} = E\{\psi^{\eta t}|\hat{\eta}^t = \hat{\xi}^t\} = \sum_{r=t}^{T} \Big[E\{\delta^{\eta r}|\hat{\eta}^t=\hat{\xi}^t\} - E\{\gamma^{\eta r}|\hat{\eta}^t=\hat{\xi}^t\}\Big] \tag{SH-37}$$

Hence, for any particular $\hat{\xi}^t$ (and hence $\hat{\xi}_h^t(\xi)$, $\hat{\lambda}_h^t(\xi)$, and the series of decisions, $\hat{q}_h^{\xi t}$, resulting in storage level

$s^{\xi t}$), we can make an optimal decision at stage t on the basis of the expected values of the multipliers γ and δ. Hence ψ^t can be thought of, just as in the deterministic case, as the marginal value of water held in storage at the end of period t.

Obviously, since $g^t(\xi^t,q^t)$ is concave and differentiable, we can solve (SH-36) by finding $\tilde{q}^{\xi t}$ such that:

$$\left.\frac{\partial g^t(\xi^t,q^t)}{\partial q^{\xi t}}\right|_{\tilde{q}^{\xi t}} = \bar{\psi}^{\xi t} \tag{SH-38}$$

(i.e. the marginal value of water released equals the marginal value of water stored) then setting:

$$q^{\xi t*} = \mathrm{MAX}\left\{\mathrm{MIN}\left\{\tilde{q}^{\xi t},\ \bar{Q}^t\right\},\ \underline{Q}^t\right\} \tag{SH-39}$$

Comparing (SH-38) with (SH-23) we conclude that, if $\underline{Q}^t < q^{\xi t} < \bar{Q}^t$, then:

$$\bar{\psi}^{\xi t} = -\frac{\partial \check{V}^t(\hat{\xi}^t,\hat{q}^{\xi t})}{\partial q^{\xi t}} \tag{SH-40}$$

This result, for the separable case, assures us that the marginal value of water in storage at the end of period t (as defined by (SH-23)) should, at the optimum, equal that of water released in t (as defined by (SH-38)). The negative sign arises from the fact that q is released, not stored, having a negative effect on storage and hence on future production.

More generally we could adopt (SH-40) as our definition of $\bar{\psi}$.

We will deal later (in Chapter 9) with the estimation of $\bar{\psi}$.

Note that, if each local hydro sub-model is separable, then so is the national problem, PSA (including the constraints (SA-15) and (SA-16) among the explicit inequality constraints (f_i) rather than as part of the definition of $\mathcal{Q}(\xi)$). Thus we can optimise our decision, $q^t(\hat{\xi}^t)$ at time t, using the expected prices, $E\{\lambda^r(\eta)\,|\,\hat{\eta}^t = \hat{\xi}^t\}$, for future periods $t \leqslant r \leqslant T$.

8.3.8 The Multiplier on the Nonanticipativity Restriction

Before leaving this theoretical discussion we should consider the significance of the multiplier ρ, which has no counterpart in the deterministic case and which, at least in the separable case, seems to mysteriously disappear from our calculations. Consider the optimal $\tilde{\rho}$, ρ component of the saddle point solution $(\tilde{z},\tilde{\lambda},\tilde{\rho})$.

We know that (Theorem S-4):

$$\text{(a)}\quad E^t\{\check{\rho}^t(\xi)\} = 0 \quad \text{a.s. for all } t = 1,\ldots T \tag{SH-41}$$

and that:

$$\text{(c)}'\quad \tilde{q}(\xi) \text{ maximises } h(\xi,\tilde{\lambda}(\xi),q,\tilde{\delta}(\xi),\tilde{\gamma}(\xi),\tilde{\rho}(\xi)) \quad \text{over } q\varepsilon\mathcal{Q}(\xi) \tag{SH-42}$$

If we consider the temporally separable case, then (Theorem (S-6)):

$$\tilde{q}^t(\xi) \quad \text{a.s.} \quad (F^t) \quad \text{maximises}$$

$$\ell^t(\xi^t,\tilde{\lambda}^t,q^t,E^t\{\tilde{\lambda}(\xi)\},\tilde{\bar{\psi}}^t(\hat{\xi}^t)) \quad \text{over } q^t\varepsilon\mathcal{Q}^t(\xi) \tag{SH-43}$$

Now, consider the multiplier ρ' defined by:

$$\rho'^t(\xi) = \bar{\psi}^t(\hat{\xi}^t) - \tilde{\psi}^t(\xi) \tag{SH-44}$$

Certainly:

$$\begin{aligned} E^t\{\rho'^r\} &= E^t\{\tilde{\bar{\psi}}^r(\hat{\xi}^r)\} - E^t\{\tilde{\psi}^r(\xi)\} \\ &= E\{E\{\tilde{\psi}^r(\eta)\,|\,\hat{\eta}^r=\hat{\xi}^r\}\,|\,\hat{\eta}^t=\hat{\xi}^t\}-E\{\tilde{\psi}^r(\eta)\,|\,\hat{\eta}^t=\hat{\xi}^t\} \\ &= 0 \qquad \text{for all } t \leqslant r \leqslant T. \end{aligned} \tag{SH-45}$$

So $\rho' \varepsilon M$.

Also (in this separable case) :

$$h(\xi,\tilde{\lambda}(\xi),q,\tilde{\delta}(\xi),\tilde{\gamma}(\xi),\rho'(\xi))$$

$$= \sum_{t=1}^{T}\left[\tilde{\lambda}^{t}(\xi)g^{t}(q^{\xi t},\xi^{t}) - \tilde{\psi}^{\xi t}q^{t} - q^{t}\rho'^{\,\xi t}\right]$$

$$= \sum_{t=1}^{T}\left[\tilde{\lambda}^{t}(\xi)g^{t}(q^{\xi t},\xi^{t}) -q^{t}(\tilde{\tilde{\psi}}^{t}(\hat{\xi}^{t}) -\psi^{t}(\xi) +\psi^{t}(\xi))\right]$$

$$= \sum_{t=1}^{T}\left[\tilde{\lambda}^{t}(\xi)g^{t}(q^{\xi t},\xi^{t}) -q^{t}(\tilde{\tilde{\psi}}(\hat{\xi}^{t}))\right]$$

$$= \sum_{t=1}^{T} \ell^{t}(\xi^{t},\tilde{\lambda}^{t},q^{t},E^{t}\{\tilde{\gamma}\},E^{t}\{\tilde{\delta}\}) \qquad \text{(SH-46)}$$

Hence $\tilde{q}(\xi)$ maximises $h(\xi,\tilde{\lambda}(\xi),q,\tilde{\delta}(\xi),\tilde{\gamma}(\xi),\rho^{*}(\xi))$
if and only if $\tilde{q}^{t}(\xi)$ maximises: $\ell^{t}(\xi^{t},\tilde{\lambda}^{t},q^{t},E^{t}\{\tilde{\gamma}\},E^{t}\{\tilde{\delta}\})$
for all $t = 1,\ldots T$.

Since $\tilde{q}^{t}(\xi)$ maximises the latter expression irrespective of ρ, our ρ' satisfies the conditions required of $\tilde{\rho}$ and hence is the (a.s.) unique multiplier required.

Thus $\tilde{\rho}^{t}(\xi)(=\rho'^{\,t}(\xi))$ represents the difference between the true marginal value of water, $\tilde{\psi}^{t}(\xi)$, for period t of sequence ξ and the nonanticipative marginal value, $\tilde{\tilde{\psi}}^{t}(\hat{\xi}^{t})$, estimated at time t on the basis of information available at that time. Thus the "manager" of our reservoir, having made his decision at time t on the basis of $\tilde{\tilde{\psi}}(\hat{\xi}^{t})$ will, with hindsight, realise that he has suffered a marginal loss $\rho'^{\,t}$ due to the nonanticipative nature of his decision process. Thus ρ does indeed represent the "cost" of uncertainty and can be regarded as the multiplier on the nonanticipativity restriction (at least in the separable case).

8.4 CONCLUSIONS

In this chapter we have developed a stochastic model of a power system. Using some recent results on the optimal recourse problem, we have shown that this model can be decomposed using a pricing mechanism similar to that for the deterministic problem. The resultant thermal and exchange sub-problems are identical in form to their deterministic counterparts and so can be solved by the techniques already developed in Chapters 3 and 4.

The hydro sub-problem, (PSH), on the other hand, is rather complex. We have not developed a solution algorithm for it. We have shown, however, that if at each stage t we can estimate the marginal value of water in storage at the end of period t, then we can solve PSH by solving the series of sub-problems $\hat{PSH}^t$. We have also shown that, provided there is no head effect in the long-term reservoir, we can estimate this marginal value on the basis of the expected value of the multipliers on the storage constraints. In the next chapter we will consider some practicable approaches to the solution of the stochastic model. The difference between these various approaches lies in the approximate methods used to evaluate future marginal values of water. Finally we have examined the role of the multiplier ρ and shown that it does indeed represent the extra costs incurred by a nonanticipative nature of the decision process.

CHAPTER 9

PRACTICABLE STOCHASTIC MODELS

9.1 INTRODUCTION

In Chapter 8 we have developed a general stochastic model of a power system which, according to the theory, we can solve by finding, for "almost every" $\xi \varepsilon \Xi$, a vector of multipliers, $\lambda(\xi)$ on the constraints (SA-12). These multipliers act as energy "prices" just as in the deterministic case. That is, $\lambda_n^t(\xi)$ is the optimal price ensuring that the demand is met, in region n, for period t, given that ξ occurs and that the system is managed (nonanticipatively) optimally so as to maximise "profit" at those prices. In order to determine the optimal response of the system to a set of prices, λ, we need to solve a set of thermal, exchange and hydro sub-problems. The thermal and exchange sub-problems are identical to the equivalent deterministic problems and can be solved separately for each ξ. The hydro sub-problem (PSH), on the other hand, is much more complex and we have only developed some general principles relating to its solution.

If we imagine that each river system has a manager then he must solve the problem (PSH) of operating the river system so as to maximise his expected profit, given that, now and in any future period (t), he can only know the past inflow sequences $(\hat{\xi}^t)$ and hence the inflows into his own reservoir $(\hat{\xi}_h^t)$ and the prices relevant to that sequence $(\hat{\lambda}^{\xi t} = \hat{\lambda}^t(\xi))$. Just as in the deterministic case, these prices summarise, for each local manager, all that he needs to know about the state of the remainder of the system. Thus, if up to period t the system has experienced inflow sequence $\hat{\xi}^t$, then the reservoir h will have experienced flows $\hat{\xi}_h^t$ and prices $\hat{\lambda}_n^{\xi t}$. So the manager will make a release $q_h^{\xi t}$, taking into account the conditional probabilities of the various possible inflow sequences, $\check{\eta}$ (with $\hat{\eta}^t = \hat{\xi}^t$), and hence of the local inflows $\check{\eta}_h$ and prices $\check{\lambda}_n(\eta)$, from then until period T (where $\check{x}^t = (x^{t+1}, x^{t+2}, \ldots x^T)$). We should note that there may be several national inflow sequences giving identical local inflows but, in general, different prices. We should also note that, while we are really only interested in the decisions of a hypothetical local manager in the first period, these will be profoundly affected by his own expected future management. Therefore we need to attempt to solve the future management problem as accurately as possible to enable us to determine an optimal decision for the present period. This interaction is formalised by the value function in Theorem (S-1) or, alternatively, by the multipliers in Theorem (S-2).

Turning away from the theoretical structure of the model to consider practical solutions we are faced with two serious difficulties. We have noted above that we require a set of prices $\lambda(\xi)$ for each possible inflow sequence $\xi \varepsilon \Xi$, and also the probability of each such sequence $(\sigma(\xi))$. However, we have no accurate way of determining the likelihood of each possible sequence and, even if we did, we could never solve the ensuing problem using present-day computers. All the data we have is a set of (possibly fragmentary and probably inaccurate) historical inflows ($\xi^{\ell}, \ell=1,\ldots.L$). In view of our ignorance of the inflow patterns, and especially of the various serial and cross correlations involved, it would seem most appropriate to use a simulation approach, letting $\Xi = \{\xi^{\ell}\}^{\ell=1,\ldots L}$ and in general, $\sigma(\xi^{\ell}) = {}^{1}/_{L}$. Thus we intend to find, for each historical inflow sequence (ξ^{ℓ}), the optimal prices ($\lambda^{\ell} = \lambda(\xi^{\ell})$), thermal generation pattern ($g_T^{\ell} = g_T(\xi^{\ell})$), hydro releases ($q^{\ell} = q(\xi^{\ell})$) and exchange pattern $e^{\ell} = e(\xi^{\ell})$), under the assumption of nonanticipative management.

This means that our national (restricted) dual problem, DSA, adjusts a set of LxNxT prices to ensure that, for each recorded inflow sequence, the responses of the system are such that demand is met in each region, for each period. We have noted that the associated Lagrangian problem, PSA$'$, can be decomposed into a set of LxNxT thermal sub-problems and LxNx(N-1)xT exchange sub-problems, which are all straightforward, and local hydro problems which are not. A similar approach to the national dual problem has been suggested by the EDF. In the next section we consider two of their models along with one of our own, for the local hydro problem. In the following section we deal with the solution to the dual problem.

9.2 THE LOCAL HYDRO PROBLEM

9.2.1 Introduction

Within our general theoretical framework we could use any scheme capable of solving the local hydro problem

(PSH) provided that scheme does not require information which is not available. In particular, our national dual problem will only set λ prices for each of L historical inflow sequences, and hence strictly relevant only to the storage patterns actually attained in the course of our (simulated) management for those sequences. Thus any scheme which would require prices relevant to other storage patterns is not appropriate. We consider first a scheme which does not satisfy this criterion and then three which do.

9.2.2 Stochastic Dynamic Programming

An obvious candidate for the solution of PSH is stochastic dynamic programming. Consider, however the decision to be made in period t for a given previous inflow sequence, $\hat{\xi}_h^t$, and decision sequence, $\hat{q}_h^t$. If we suppose that there is no serial correlation of inflows then we can summarise $\hat{\xi}_h^t$ and $\hat{q}_h^t$ for our purposes by s_h^t. Then, if we have already determined the value function for period t as $\check{V}_h^t(s_h^t)$, we wish to choose q_h^t for each storage level s_h^{t-1}, so as to maximise the present (i.e., at period t) and expected future benefits. So we solve:

$$(\mathrm{PSH}^t) \quad \text{Find: } \max_{q_h^t \in Q_h^t} \left[\lambda_n^t\, g_h^t(q_h^t, s_h^{t-1}) + \check{V}_h^t(s_h^{t-1}(q_h^t, s_h^{t-1}))\right] \qquad \text{(SH-47)}$$

where: $\hat{q}_h^{t-1}$ is fixed.

(ignoring the effect of tributary inflows).

But how do we determine λ_n^t? DSA has provided us with many λ_n^t's, but each is relevant only to a particular storage pattern (i.e. vector of storage volumes in the various hydro reservoirs in the national system). Thus there may be no λ_n^t relevant to s_h^t at all, or there may be several, each corresponding to a different set of storage levels in the other reservoirs (resulting from different inflows to those reservoirs). Worse, we may have inflow series with $\eta_h < \xi_h$, but $\eta_i > \xi_i$ for $i \neq h$. The likely result of this is that $s_h(\eta) < s_h(\xi)$, but $\lambda_n(\eta) < \lambda_n(\xi)$. This makes any attempt at interpretation chaotic. We could allow λ as an additional random variable or state variable, but we really have very little information about its relationship to either ξ_h or s_h and so we would be little better off. Also λ, related as it is to aggregate storage, can be expected to exhibit very high serial correlation, introducing further difficulties. In fact the only way we could reasonably use stochastic dynamic programming would be under very restrictive assumptions as to the cross-correlations between reservoir inflows. We would need to assume, either that they were perfectly correlated (and so we could aggregate all the reservoirs into one), or that they were independent (and so we could use prices averaged over all inflow sequences). Since neither assumption is particularly realistic we are forced to look further afield for a solution method for PSH. (Although, as a compromise, we could assume that flows were perfectly correlated within each region, but not correlated with those in other regions).

9.2.3 Trajectory Methods

The next three models which we consider have many basic similarities. Recall that, in order to determine an optimal balance between releasing water in the present period and keeping it for release in the future, the hypothetical manager of a local hydro system must be able to determine its expected value if stored for the future. In order to determine this he must have some model of his own future management. Thus, roughly speaking, he must know the answers to the questions: "If I experienced the inflows ξ_h and prices $\lambda_n(\xi)$ what decisions would I make?", and: "What is the probability of each such occurrence (i.e., $(\xi_h(\xi),\lambda_n(\xi)))$?" Each of the models considered below answers this latter question by simulating future management over historical inflow sequences. (If we use all historical sequences they will be assumed to have equal probability, however we may wish to select just a few representative sequences). We have already seen that the first question can be reduced to a series of questions of the form: "If, up to period t, I had experienced inflows $\hat{\xi}_h^t$ and prices $\hat{\lambda}_n^{\xi t}$, what release would I make in the next period?". As we have also seen, the strictly consistent answer to this question would require the determination of the entire future (conditional) inflow distribution and the solution of our entire model (for the appropriate initial conditions) (i.e., PSH^t) to determine each such future decision. Worse still, it would involve the solution of an incredibly large dual problem in order to provide the prices required. This is out of the question and so we turn to more approximate methods for the solution of this hypothetical future management problem. Each of the following models uses a different approximation and we will examine the implications of each and their likely effect on the decision to be made in the current period.

The first model was proposed by the EDF as part of their GRAF model summarised in Section A.2 of Appendix A. According to this approach we should apply our national deterministic model to each of the L recorded inflow sequences separately, deriving for each one an optimal set of prices and trajectories and also a current water value, $\psi_h^{\ell o}$, for each reservoir $h \varepsilon H$. The optimal release for the current period is then found by treating $\bar{\psi}_h^o$, the mean water value, as the value of water in the current period (presumably adjusting λ^o until supply matches demand in the current period). In our general model this approach is equivalent to making the assumption that, although the manager of each reservoir does not know which of the L sequences he will experience, he does know that he will experience one of these sequences (or equivalently that the serial correlation is perfect). So, having observed the first period's inflows he will be able to predict future inflows and prices exactly. Thus his optimal future nonanticipative management is effectively deterministic. So the multipliers developed by the dual, DSA, for each sequence, will be identical to those for the deterministic optimisation on the same sequence.

Hence we can find the optimum by applying the model PA to each of the L sequences independently.

This approach is attractive in that it requires only the deterministic algorithm, multiplying the computational burden by the number of sequences, L. However the assumptions made about future management are highly unrealistic and may be expected to introduce consistent bias into the current decision. This is so because these assumptions allow us to suppose that, in the future, the hydro sector, knowing flows in advance, will be able to absorb an unrealistically high proportion of the variation in inflows. This allows us to suppose that the thermal can be kept at a more constant level than would realistically be possible in a nonanticipative environment. This in turn leads to lower expected average (future) thermal costs as in Figure (9-1).

Here we have not only extended the thermal cost curve by a very high cost segment to account for shortage, but also by a "no cost" negative generation segment to account for spill in the hydro sector. That is, we suppose that there is a fictitious thermal station absorbing the energy from the spilled water.

The effect of the deterministic simulated management policy on marginal values depends, however, on the nature of the marginal cost curve. The marginal cost curve for the thermal sector of the NZED system is shown in Figure (9-2). The concave increasing function shown would seem to be a reasonable approximation to this curve over the range of likely thermal generation levels. This type

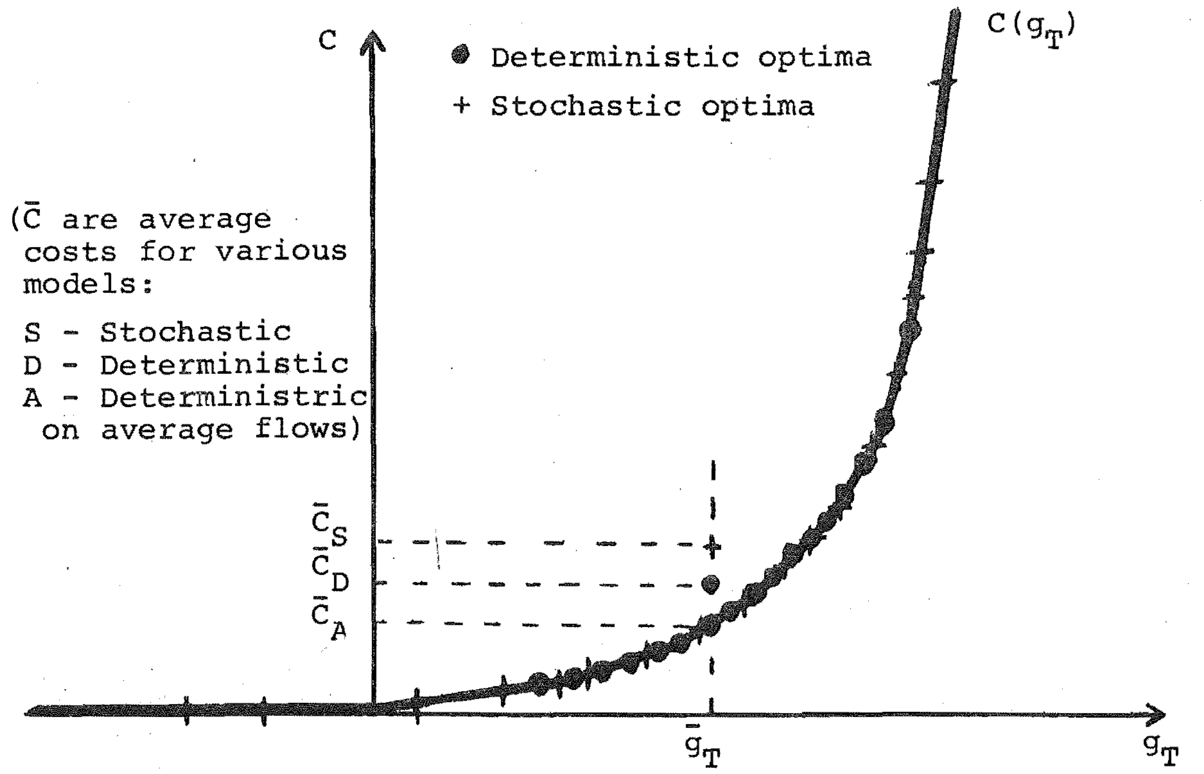


FIGURE (9-1): Comparison of average costs

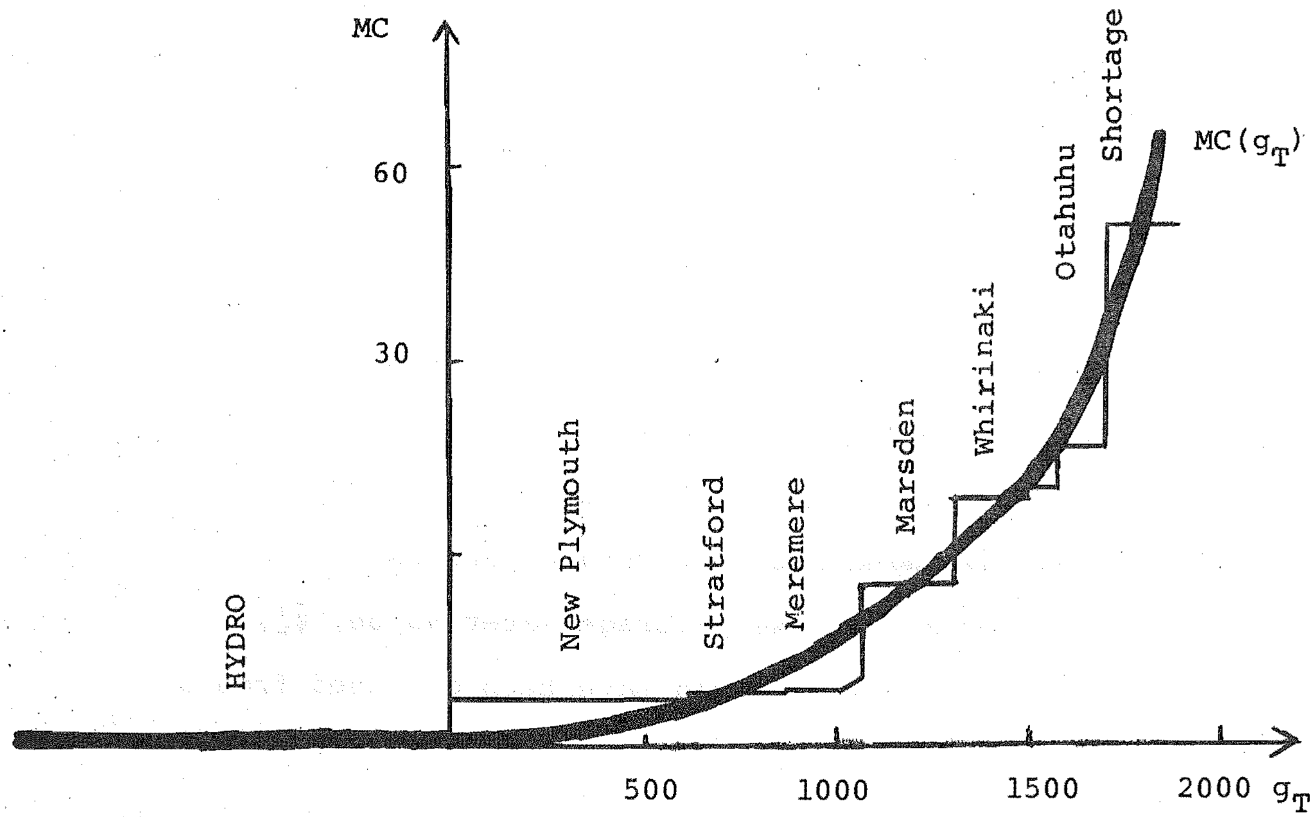


FIGURE (9-2): Marginal costs for NZED system of marginal cost curve would seem typical of mixed hydro-thermal systems involving large, cheap, base-load plants and smaller, dearer, peaking plants, and in which there are significant probabilities of spill or shortage. In this case the artificially decreased spread of the deterministic water values corresponding to the various sequences will decrease the current (expected) water value. (See Figure (9-4)). This effect may be enhanced by the fact that a truly nonanticipative model would have to adopt fairly similar policies for the first few periods of any sequence. This will lead to much more rapid divergence of the simulated storage trajectories than for the equivalent deterministic trajectories. Consequently the nonanticipative model will be forced to adopt more extreme policies as time progresses so as to compensate for the more "average" decisions in earlier periods. Thus the distribution of eventual values of water left in storage in the first period and managed nonanticipatively thereafter will tend to be more widely spread than for the average values discussed above (see Figure (9-3). Thus, if we average these to determine the current water value, the bias introduced by the proposed deterministic simulation (in the case of convex marginal costs) will be more serious than we might be led to suppose from a consideration of marginal values averaged over the future periods.

So we will obtain a lower average water value, $\bar{\psi}_h^1$, for the current period than would have been obtained from a truly nonanticipative model. Since total production and the thermal cost structure for the current period will be the same for any model, this will lead us, on average, to consistently over-estimate the release for the current period. Thus we

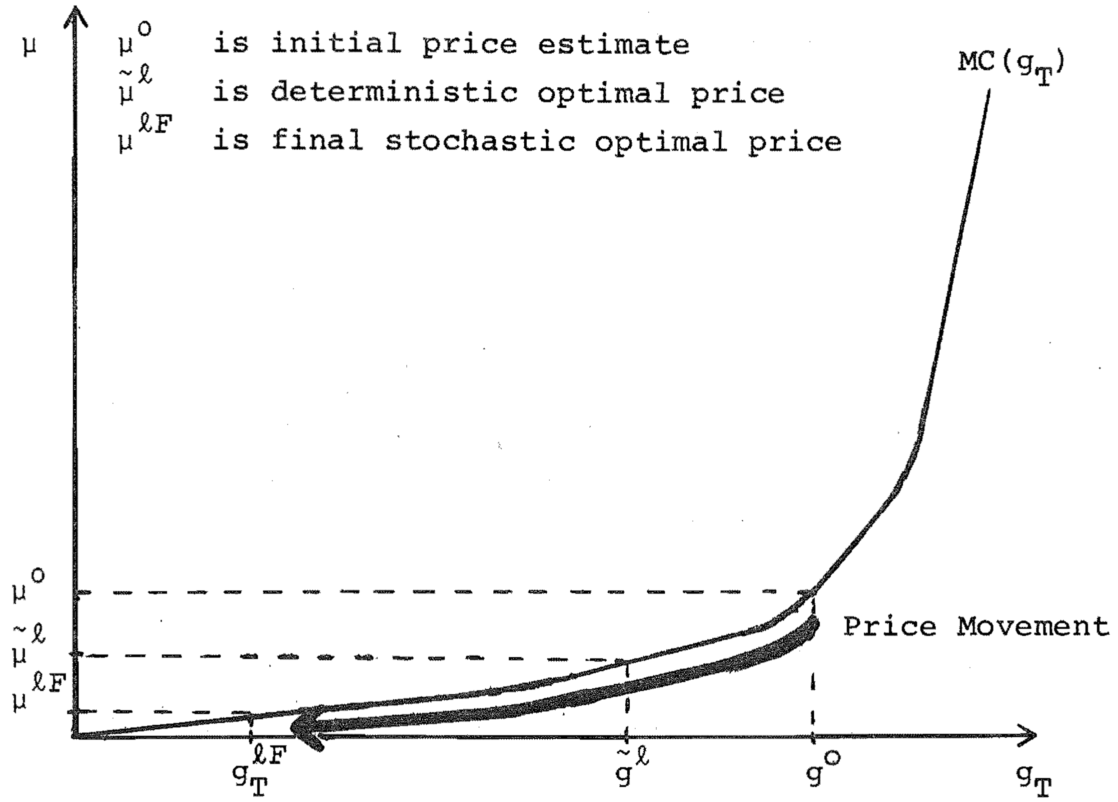


FIGURE (9-3): Movement in marginal costs for one sequence

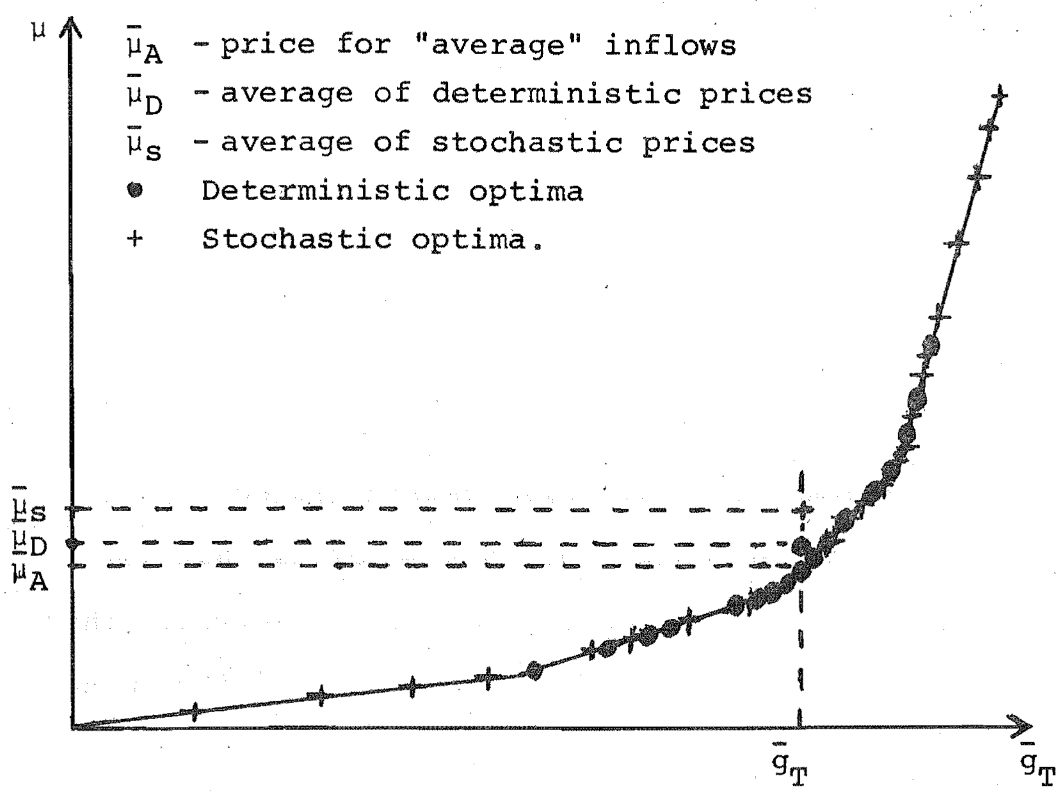


FIGURE (9-4): Comparison of stochastic and deterministic averages.

come to the, scarcely surprising, conclusion that, if we assume that in the future we will be able to predict flows more accurately than is in fact the case, this will lead us to be less cautious than we should be in determining the current release. This conclusion applies to any such "over-anticipative" model.

The second approach has also been proposed by the EDF, as part of their SGEP model and is summarised in Section A.3 of Appendix A. This approach considers all the sequences together in the dual problem, DSA, which develops prices relevant to each. However the local problem, PSH_h, is solved by a method which finds only one "desired trajectory", $\tilde{x}_h$, for each reservoir. The releases, $\tilde{q}_h^{\ell}$ (and hence the trajectory $\tilde{s}_h^{\ell}$), relevant to each inflow sequence are then found under the assumption that, in the future, the manager will attempt to keep a close as possible to this desired trajectory. So, if $s_h^{\ell t} > x_h^t$, he will release enough water to lower $s_h^{\ell t+1}$ to x_h^{t+1}, and likewise refrain from releasing if $s_h^{\ell t} < x_h^t$. In our general scheme this approach involves the implicit assumption that, in the future, the manager will learn nothing whatsoever from his observations of inflows and prices. Rather, he will blindly attempt to follow the trajectory set at the beginning of the planning horizon (although the possible values of ξ and λ were all taken into account in the determination of this desired trajectory).

The most attractive feature of this model is that it involves only one desired trajectory for each reservoir, the proposed trajectories for each inflow sequence ξ being found by simulation around this trajectory. This means that, in the solution of PSH, we can deal directly with this much smaller set of desired values.

So a mathematical programming approach to the solution of PSH is feasible. This approach can more readily be generalised to deal with several interconnected reservoirs. However the assumptions made about future management are as unrealistic as those of the earlier model, if at the other extreme. These assumptions will also lead to a consistent bias in the decision made in the current period. Here the thermal sector is supposed to have to absorb, in the future, an unrealistic proportion of the variability in the inflows. In fact it is supposed that, if it were not for the flow restrictions, any variability in the inflows would be immediately reflected in the hydro output, making compensating adjustments necessary in the thermal system. This undue fluctuation in thermal output leads to an expectation of unduly high average thermal costs in the future (cf. Figure (9-3)). With convex marginal cost it also leads to an unduly high current water value, ψ_h^1, for each reservoir. Analogously to the previous model, this results in current releases which are on average too low. This is again in accord with our intuition, in that a model which assumes that, in the future, we will not be able to make proper use of the information available to us, leads to over-conservative policies for determining the optimal current release. Again this conclusion applies to all such "under-anticipative" models.

From this heuristic discussion it can be seen that we can expect the solutions from these two EDF models to bracket the optimum. Hence we could use these two approaches to determine, not only the range in which the solution lies, but the maximum extent of the gains which could be derived from a more accurate model. Tests of this nature are currently being undertaken using a simpler single reservoir model due, primarily, to Boshier ([6]). Before discussing a more realistic stochastic model we wish to reiterate that in the above discussion we are not supposing that future management will actually follow the patterns described (of deterministic optimisation on the one hand or blind trajectory following on the other). It is merely that, in making the current decision, it is assumed that these patterns will be followed and thus a bias is introduced. Of course the problem is in fact re-optimised at each stage, starting from the current storage which is, on average, too high or too low depending on the model used. So, as storage moves away from what would have been the optimal nonanticipative trajectory, other factors come into play which tend to counteract the effects of the bias introduced by the solution algorithm. This leads to a kind of false equilibrium, at either a higher or lower average storage volume than the optimum. Thus the long-term effects of a biased solution algorithm may not be as serious as they would at first appear.

9.2.4 A New Method

Consideration of the deficiencies of the approaches so far put forward leads us to suggest the following approach. We retain the same general framework, approximating the actual (unknown) probability space by the recorded inflows and using the dual, DSA, to develop a set of prices relevant to each sequence. We require that the algorithm for the local hydro problem, PSH, use only information available from this scheme. Further, we insist that (as opposed to the first EDF scheme) the simulated future management of the reservoir be non-anticipative, but that (as opposed to the second) each future simulated decision be as nearly optimal as is practicable, given the circumstances under which it is assumed to be made.

We have shown, in Section 8.3.6, that we can solve PSH by solving the series of problems PSH^t. Further, we have seen that, since $\check{V}$ is concave and differentiable, we can solve PSH^t by finding, for each $(\hat{\xi}^t, \hat{q}^{\xi t}), \tilde{q}^{\xi t}$ such that:

$$\lambda^{\xi t} \frac{\partial g^t(\hat{q}^{\xi t}, \hat{\xi}^t)}{\partial q^{\xi t}} \Bigg|_{\tilde{q}} = -\frac{\partial \check{V}^t(\hat{\xi}^t, \hat{q}^{\xi t})}{\partial q^{\xi t}} \Bigg|_{\tilde{q}} \qquad \text{(SH-23)}$$

(letting $\tilde{q}^{\xi t} = \bar{Q}^{\xi t}$ or $\underline{Q}^{\xi t}$ as appropriate if there is no feasible solution to (SH-23)).

We have also shown, in Section 8.3.7, that the marginal water value can be found from the expected values of the multipliers on the storage constraints. That is:

$$\frac{\partial \check{V}^t(\hat{\xi}^t, \hat{q}^{\xi t})}{\partial q^{\xi t}} = -\bar{\psi}^{\xi t} \qquad \text{(SH-40)}$$

Where: $\bar{\psi}^{\xi t} = E\{\psi^{\eta t} \,|\, \hat{\eta}^t = \hat{\xi}^t\}$

$$= \sum_{r=t}^{T} \left[E\left\{ (\gamma^{\eta r} - \delta^{\eta r}) \,\middle|\, \hat{\eta}^t = \hat{\xi}^t \right\} \right] \qquad \text{(SH-37)}'$$

So then, if we have some method for estimating $\bar{\psi}^{\ell t}$, for each $\ell=1,\ldots L$, $t=1,\ldots T$, we can solve PSH (via the series of problems PSH^t). This approach amounts to simulating optimal future nonanticipative management over the L sequences. We can state our general algorithm as:

(1) Let: $\ell=1$

(2) Let: $t=1$

(3) Estimate $\bar{\psi}^{\ell t}$

(4) Find $q^{\ell t}$ via (SH-40) and (SH-23).

(5) Let: $$s^{\ell t} = s^{\ell t-1} + \xi^{\ell t} - q^{\ell t} \qquad \text{(SA-16)}^{\ell t}$$

(6) Let: $t=t+1$

IF $t \leqslant T$ THEN GO TO (3)

ELSE GO TO (7)

(7) Let: $\ell=\ell+1$

IF $\ell \leqslant L$ THEN GO TO (2)

ELSE STOP.

When this process terminates in Step (7) we will have a complete collection of simulated trajectories, one for each inflow sequence. Each of these will start with the same initial release, q^1, this release being based on the estimated marginal water value for that period, $\bar{\psi}^1$. (This value being identical for all sequences because they share the same initial storage, s^o, and the same initial forecasts, based on the current real data). However the true value of water for each sequence, ℓ, assuming that in the future the reservoir would be managed in accord with our simulation. We assume (as do the EDF ([20]) and

SSPB ([55])) that this is given by ψ_F^{ℓ}, the marginal value of water at the end of the first arc of the simulated trajectory. (The maximum ψ attained during that arc would provide an alternative estimate). So the best estimate we have of the initial marginal water value becomes:

$$\tilde{\psi}^1 = \sum_{\ell=1}^{L} \sigma^{\ell} \psi_F^{\ell} \qquad \text{(SH-48)}$$

We can then use this value to determine the best current release. Note that we should, properly, go through the complete algorithm (Steps 1-7) again, using $\tilde{\psi}^1$ as our initial estimate of $\bar{\psi}^{\ell 1}$ for all $\ell=1,\ldots L$.

This, in turn, gives us a new estimate for ψ^1, and we could continue in this fashion until $\tilde{\psi}^1$ remains constant in successive iterations. However, except perhaps in the final iteration of the dual algorithm (DSA), the extra effort involved is not likely to be justified. We may however use the initial water value determined from the previous iteration of DSA as our estimate of $\bar{\psi}^1$ in Step 3.

The accuracy of this whole approach depends on the accuracy with which we can evaluate $\bar{\psi}^{\ell t}$ in Step 3. Recall that $\bar{\psi}^{\ell t}$ is defined as:

$$\bar{\psi}^{\ell t} = E\left\{\psi^{\eta t} \middle| \hat{\eta}^t = \hat{\xi}^{\ell t}\right\} \qquad \text{(SH-37)}$$

In order to evaluate this exactly we would need to know the conditional distribution of η (at least).

Since this is not known (nor can be known) in detail, we will deal with some approximation to it. We propose that, at stage t, the manager be supposed to:

(a) Forecast one or more future inflow sequences,

$\overset{\vee}{\eta}^{t}(\overset{\vee}{\eta}^{t}=(\eta^{t+1},\eta^{t+2},\ldots\eta^{T}))$, and assign probabilities to these (all on the basis of whatever information, particularly $\hat{\xi}^{\ell t}$, would be available to him a time t).

(b) Forecast corresponding future price sequences, $\overset{\vee}{\lambda}^{t}(\eta)$, (on the same basis, especially considering $\hat{\lambda}^{t}(\xi^{\ell})$).

(c) Use the deterministic trajectory method of Section 5.4.3 to optimize $\overset{\vee}{q}^{t-1}(\eta)$ for each such sequence. Determine $\bar{\psi}^{\ell t}$ on the basis of these optimisations.

For obvious reasons we suppose that, in fact, only one expected inflow/price sequence is forecast at each stage. So we simulate the manager undertaking, in each future period, a deterministic optimisation on (conditional) expected inflows and assume that the water value derived is sufficiently close to the true expected value to enable a reasonable decision at that stage. This is in accord with Theorem (S-4) which assures us that, for a separable problem such as ours, we should make decisions at each stage which are optimal in the light of (conditional) expected future "prices", in this case not only the λ prices, but the multipliers on the storage constraints (and hence the water value ψ_h^t). Here the extent of our departure from the optimal decision for stage t is the extent of the error involved in approximating the expected value of water, $\bar{\psi}^{t}$, by the value, $\psi^{t}(\overset{\vee}{\eta}^{t}(\hat{\xi}^{t}))$, for the expected inflow sequence. We could reduce this inaccuracy by forecasting more than one future sequence (assigning appropriate probabilities) then averaging the water values derived from deterministic optimisation on each of these. Of particular interest would be sequences which necessitate spill or force a shortage. However the scheme which we have proposed seems likely to achieve reasonable accuracy while incurring an acceptable computational burden.

In practice we will require that the forecasts of inflows and prices to be made at each stage do not require too much computation. For the inflows which generally, in New Zealand at any rate, have a relatively small serial correlation, we will assume that the manager applies a simple Markov lag k model to modify his expectation of the inflows. Thus we let our forecast for the expected inflow sequence, $\check{\eta}^t(\hat{\xi}^t)$, be defined by

$$\eta^r(\hat{\xi}^t) = \bar{\eta}^r + \sum_{j=r-k}^{t} \omega_j \, \xi^j \quad \text{for all } r \geqslant t \qquad \text{(SH-49)}$$

where the ω_j co-efficients are determined from some hydrological model. We similarly allow the ξ sequences themselves to be modified on the basis of information available at the time of the optimisation (i.e. period 1). Thus, if it is expected that inflows for the first month in the planning horizon will be (say) 10% above the average for that month, we will modify the L historically observed sequences accordingly.

The forecasting of energy "prices" of the kind with which we are dealing is a rather different matter since these "prices" are entirely internal to our algorithm, having no direct connection with any externally observable phenomena. However we assume that our manager is able to study the whole set of prices proposed by the dual DSA (which he obviously is). So he is able to form estimates of their distribution, especially their mean, seasonality and serial correlation. Initial estimates of these can be established from a study of the distribution of optimal deterministic prices for these inflow sequences. We could suppose that the manager uses a lagged Markov model similar to that for the inflows. However the price sequence will tend to look like that in Figure (7-10). Here the price jumps very markedly as each thermal station is brought onto (or off) base load. Thus the most important characteristic of the future price sequence which will have to be forecast will be the period in which each such jump is likely to occur. This problem is not inherently difficult. However further experience with these price sequences will be required before we can definitely recommend any particular method for producing such forecasts.

In summary we do not intend to use the most sophisticated possible forcasting techniques for these future forecasts, but rather simple rules which can be carried out efficiently during the process of the deterministic optimisation for each stage. So these will add little to the computational burden of the algorithm.

Now we can summarise our algorithm for the (stochastic) local hydro problem as:

(0) Modify the inflow sequences on the basis of current inflow predictions.

(1) Let: $\ell=1$

(2) Let: $t=1$

(3) If $t=1$, then let $\psi^{\ell t} = \tilde{\psi}^{1}$ from previous iteration of DSA.

ELSE (a) Forecast an expected inflow sequence, $\check{\eta}^t$, and price sequence, $\check{\lambda}^t$, on the basis of $(\hat{\xi}^t, \hat{\lambda}^t)$.

(b) Use the deterministic trajectory algorithm to optimise the first arc of the future trajectory starting from $s^{\ell t-1}$ and assuming $\check{\lambda}^t, \check{\eta}^t$. This yields $\bar{\psi}^{\ell t}$.

(4) Find: $q^{\ell t}$, via (SH-40) and (SH-23).

(5) Let:
$$s^{\ell t} = s^{\ell t-1} + \xi^{\ell t} - q^{\ell t}. \qquad \text{(SA-16)}^{\ell t}$$

(6) Let: $t = t + 1$

If $t \leqslant T$ THEN GO TO (3).

ELSE GO TO (7).

(7) Let: $\ell = \ell + 1$

If $\ell \leqslant L$ THEN GO TO (2)

ELSE STOP, $$\bar{\psi}^1 = \sum_{\ell=1}^{L} \sigma^{\ell} \psi_F^{\ell} . \qquad \text{(SH-48)}$$

This process is demonstrated for a very simple example in Figure (9-5).

For this example we have assumed that the inflows exhibit no serial correlation and that there is no significant variation in (λ) prices from those expected. For each period we assume that the "manager" makes a release decision, q^t, then experiences an inflow, ξ^t, resulting in a storage level: $s^t = s^{t-1} + \xi^t - q^t$. He then determines the optimal deterministic trajectory, starting from s^t, using the expected inflows and prices for the remainder of the planning horizon (noting that, under our assumptions for this example, these expectations are not modified by experience). We show this management simulated for the first five periods of three inflow sequences. The actual inflows are shown in Table (9-1).

We note that, since we are only interested in the water value, $\bar{\psi}^t$, in Step 3(c), we need only find the first arc of the optimal deterministic trajectory. This represents a considerable computational advantage. Also, as each simulated trajectory approaches the final period, the storage constraints will begin to dominate the problem. We propose to imitate realistic system operation by abandoning these constraints after some period. From then on we assume that the manager will ignore the final "target" and aim for a "target" at the end of the following year. Thus the requirement of essentially complete recourse causes no computational difficulty.

9.2.5 Evaluation.

It is obvious that, in the new model, we have not completely eliminated the bias inherent in the EDF algorithms, but merely removed it a step further from the current decision. We expect that the simulated future decisions will, on average, be under-cautious. However, since a new decision is made at each stage in the light of the storage level actually attained at that stage, this bias will be compensated. So the simulated trajectories represent a false equilibrium with a little less storage than the true optimum. In fact, since our simulated future decisions are assumed

sequence	Period 2	3	4	5	6
1	0	0	8	0	2
2	4	7	10	4	3
3	9	9	1	12	2
(expected)	4	6	5	7	7

(expected inflows for periods 7,8,9 are 5,5,4)

Note: Figure (9-5) demonstrates our stochastic trajectory algorithm as explained in text.

TABLE (9-1): Inflows for example in Figure (9-5).

Lake	$\bar{L}$	Lake	$\bar{L}$
Taupo	25	Waitaki	6.5
Cobb	25	Hawea	12
Coleridge	8.2	Manapouri	6

Here $\bar{L}$ has been calculated as the average over all t of $\tilde{L}^t$, the number of periods from t to the next active storage constraint at the optimal solution as calculated in Section 7.4.

TABLE (9-2): Average initial arc lengths.

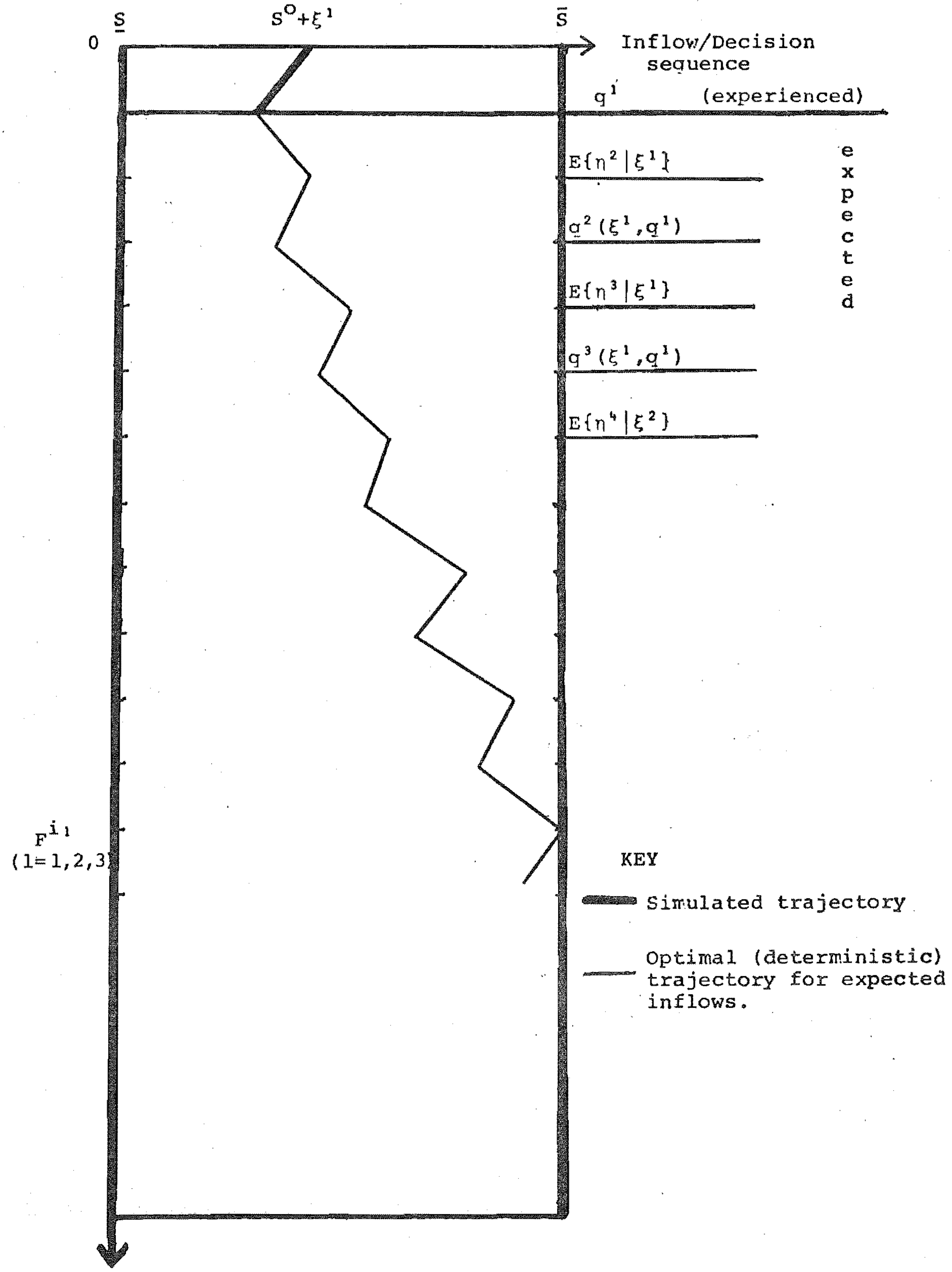


FIGURE (9-5a): Trajectories at end of first period

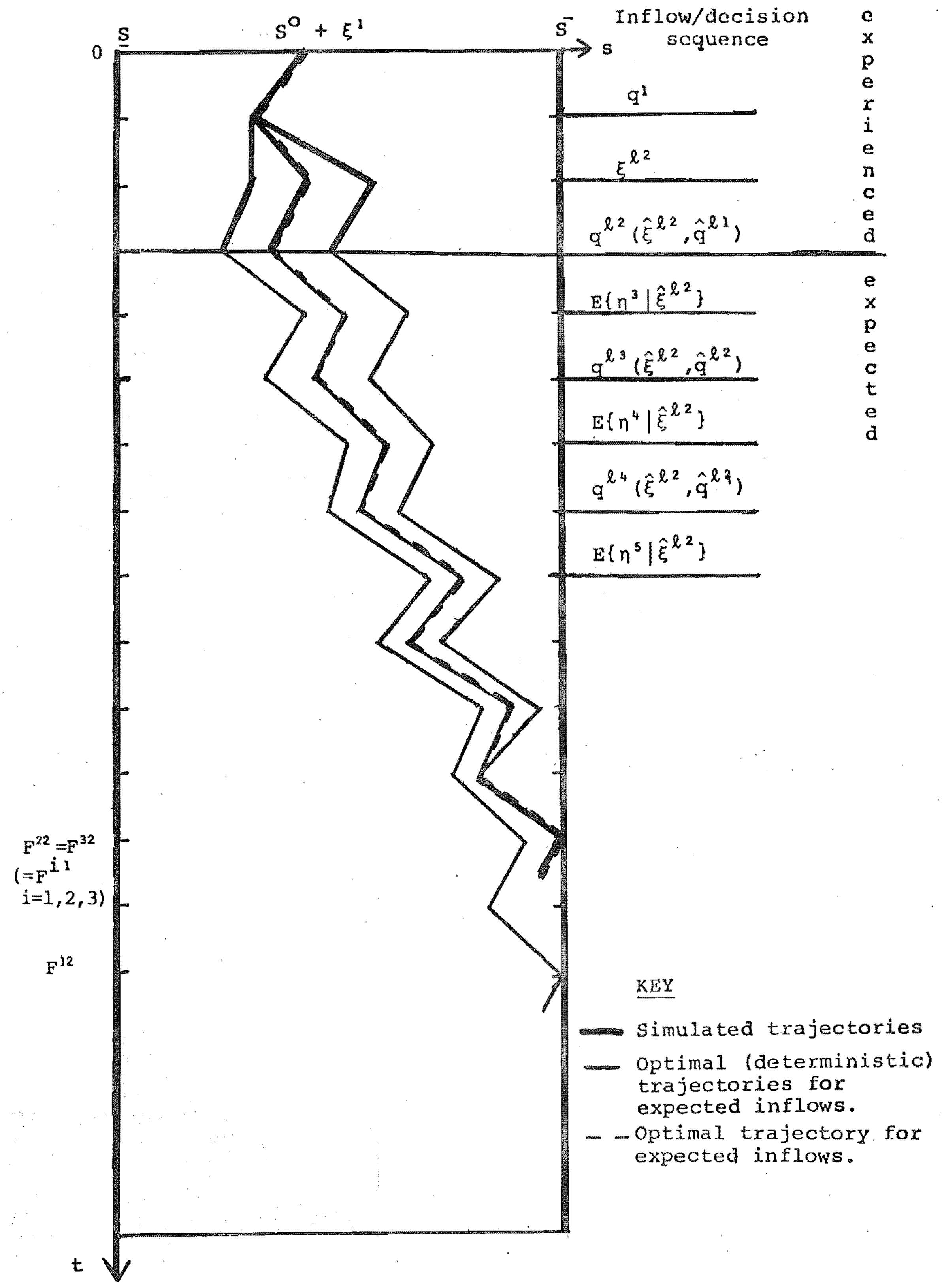


FIGURE (9-5b): Trajectories at end of second period

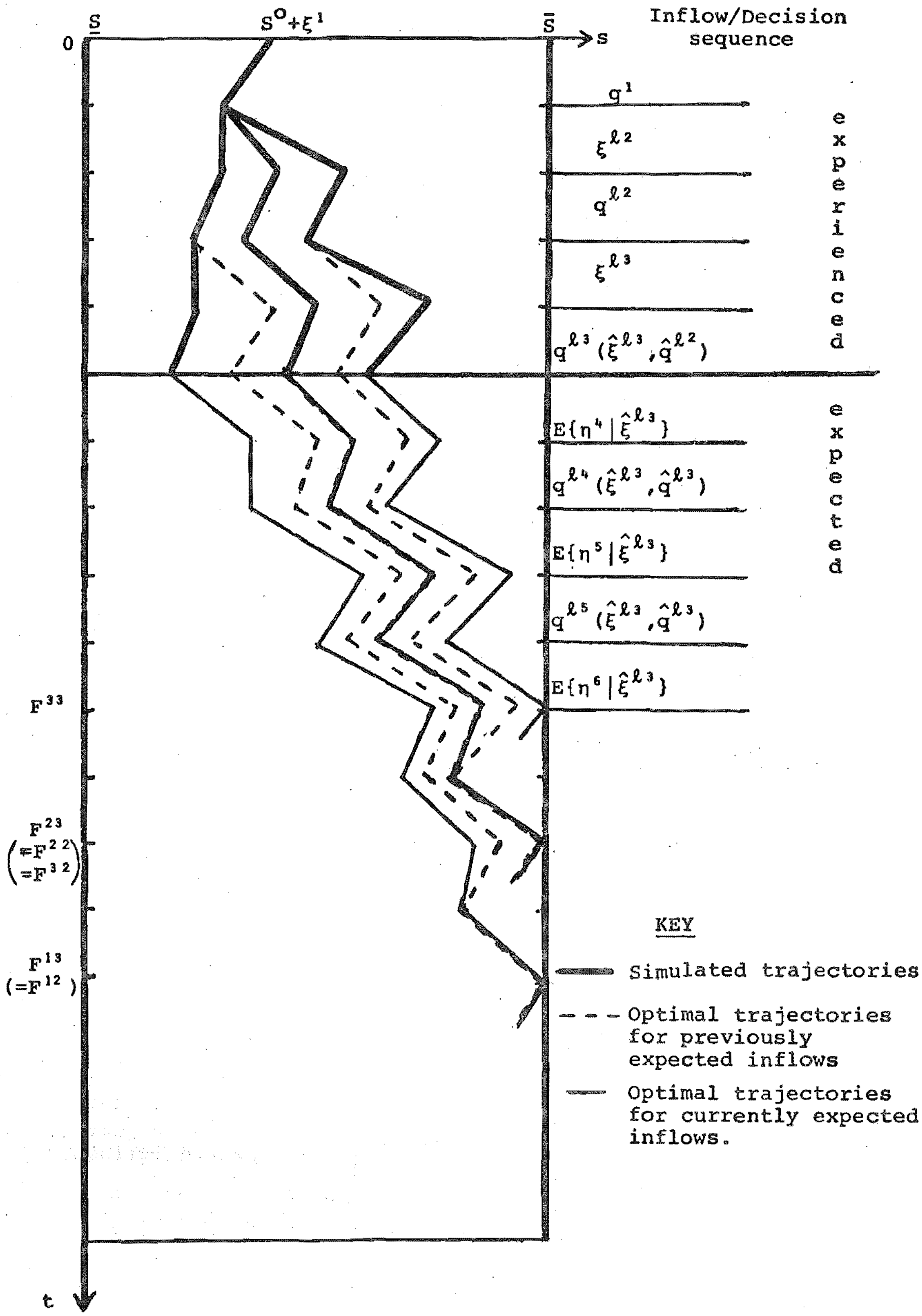


FIGURE (9-5c): Trajectories at end of third period.

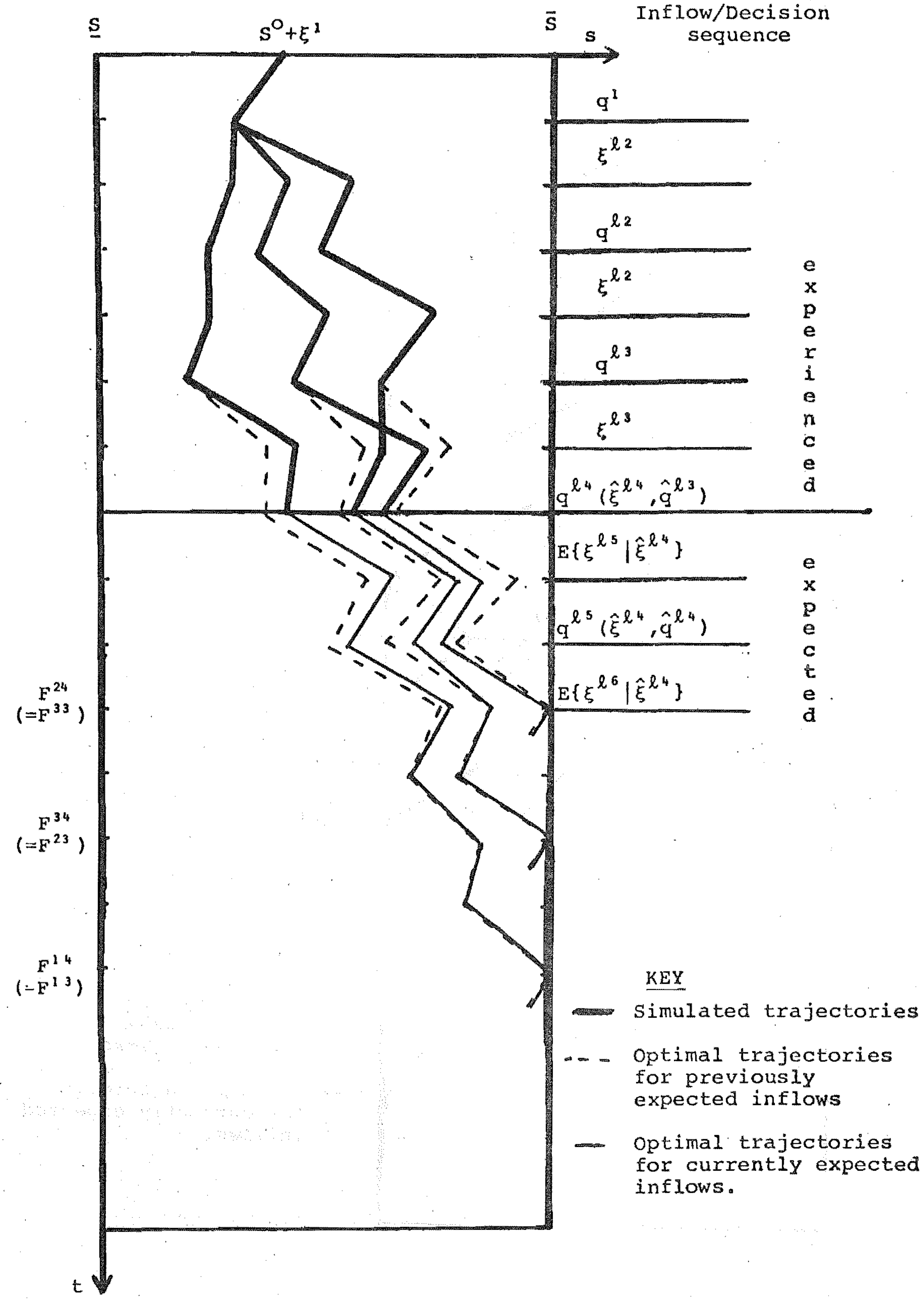


FIGURE (9-5d): Trajectories at end of fourth period

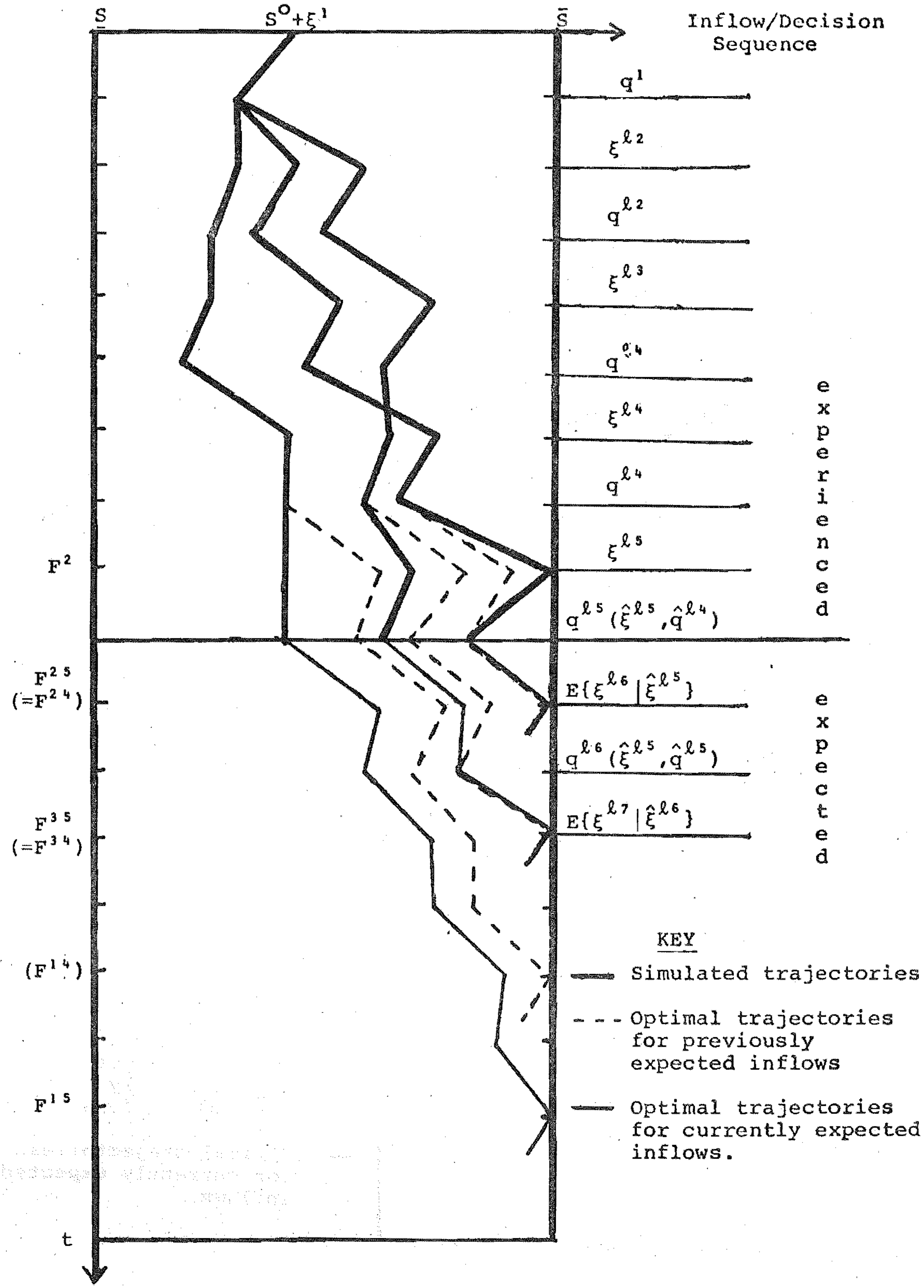


FIGURE (9-5e): Trajectories at end of fifth period to be made on the basis of a rather less than perfect utilisation of observed information, we expect that the derived decisions for the current period will be a little over-cautious. However this is a reasonably realistic model of future management and should be expected to produce current release decisions near to the optimum. Further, the slight conservative bias means that the results of this model can be applied to the system without any fear of prejudicing long-term security considerations. This observation applies to any stochastic model which assumes that future decisions will be made nonanticipatively but with less than perfect accuracy. Such conservatism is quite an attractive feature from a management point of view.

The improved realism has been achieved at the expense of an obviously increased computational burden. In this model we not only have to solve a deterministic optimal trajectory problem for each sequence, but also for each stage of each sequence. Thus, if the computation time, τ_D, for the determination of an optimal trajectory for a K period problem is given by $\tau_D(K) = \alpha K$, and we were to continue our simulated management right up to time T, the computation time, τ_S, for each sequence would be given

$$\tau_S(T) = \sum_{k=1}^{T} \tau_D(k) = \sum_{k=1}^{T} \alpha k = \alpha\ T\left(\frac{T+1}{2}\right)$$

$$= \tau_D(T)\left(\frac{T+1}{2}\right) \qquad \text{(SH-50)}$$

Thus we have that, apart from any time taken by the forecasting at each stage, the computation time for this algorithm could be $(\frac{T+1}{2})$ times greater than that for the EDF scheme 1. However, this estimate is misleading because:-

(i) At each stage we only ever have to optimize the first arc of the deterministic trajectory based on future inflows. The expected length of this arc depends on the lake. Consideration of the optimal solution to the problem shown in Section 7.5 gives the average initial arc lengths shown in Table (9-2). Experience with the deterministic algorithm indicates that we can expect computation times to be approximately proportional to the length of each such arc.

(ii) The optimal water value for the first arc will vary little, and in a predictable way, from period to period. So we can use the water value from period t to determine the water value for period t + 1 fairly closely. Specifically, from the calculations for period t, we will have $\frac{ds^F}{d\bar{\psi}}$ (where F is the end of the first arc from t) and an expected storage level $s^{t+1^*}(E\{\xi\})$ for period t + 1. The new storage level will be:

$$s^{t+1^*}(E\{\xi\}) + \xi^t - E\{\xi^t\} \qquad \text{(SH-51)}$$

so that, if $\xi^t = E\{\xi^t\}$ (i.e., the expected inflows occur), our previously calculated trajectory (and hence water value) will still be optimal for t + 1 (ignoring any change in our expectations about $\check{\xi}$ and $\check{\lambda}^t$). However, if (say) $\xi^t > E\{\xi^t\}$, then we will need to lower $\bar{\psi}$ so as to release that extra water over the next trajectory segment. Thus our initial estimate for $\bar{\psi}^{t+1}$ will be:

$$\bar{\psi}^{t+1} = \bar{\psi}^{t} - \frac{(\xi^{t} - E\{\xi^{t}\})}{\frac{ds^{F}}{d\bar{\psi}}} \tag{SH-52}$$

If the generation functions are, in fact, quadratic, $s^{F}(\psi)$ will be linear within limits and so this expression will often be exact.

(iii) In view of the above it may often be reasonable to suppose that future management blindly follows the expected trajectory for more than one period before re-optimising. In particular, there are often extensive portions of the trajectory in which the release is always at either a maximum or a minimum level. In this case there is no point in re-optimising. This could be used to particular advantage on long trajectory arcs (remembering that we will eventually take only the final water value for the arc into account to determine the initial water value).

These considerations taken together can be expected to considerably reduce the, otherwise formidable, computational burden to an acceptable level. The total burden will of course depend very heavily on the number (L) of inflow sequences under consideration. Only experience can finally determine whether it is better to solve a sophisticated model for a few sequences or a simple model for a large number of them. However, if we choose our sequences so as to form a good representation of the observed flow distribution (and assign appropriate probabilities to the sequences so chosen), we would expect the former approach to yield better results (since, if a method is biased, any number of sequences will not remove the bias).

Finally we note the similarity between this method and the single-reservoir method employed by the Swedish State Power Board ([31]). They were able to utilise a stochastic dynamic programming method because they aggregated all the reservoirs into a single aggregate reservoir. They then estimated a set of trial water value curves, which assigned a water value, $\psi^t(s)$, to each storage level, s^t, at time t. (In our framework, this involves implicitly assuming serial independence and perfect spatial correlation of inflows so that the whole prior sequence of inflows ($\hat{\xi}^t$) and decisions ($\hat{q}^t$) can be summarised by the present state ($s^t(\hat{\xi}^t,\hat{q}^t)$ of the system). This estimated water value curve is then recursively modified by, for each s^t, simulating "optimal" future management for the next twenty periods (or until a storage constraint is reached), assuming various (historically observed) inflow sequences. The new water value $\psi^t(s^t)$, is then taken to be the mean of the water values associated with the storage levels attained at the end points of these simulated trajectories. The "optimal" future decisions are (presumably) made under the assumption that the load must be met and that the hydro sector can be regarded as a thermal plant with fuel cost $\psi^r(s^r)$ for each $r > t$.

This process is continued until the water value curves cease to change. An attempt has been made to generalise this system to allow two reservoirs using successive approximations. This has apparently met with little success however ([17]).

9.3 THE STOCHASTIC DUAL

We are concerned here with the problem of determining optimal energy prices in our "practicable" stochastic model. We know that these can be characterised as the λ components of the (a.s. unique) saddle point of the restricted Lagrangian (SA-32). We can define a "dual objective function", $P_s(\lambda)$, by:

$$P_s(\lambda) = \inf_{z \in Z \cap N_\infty} R(z,\lambda)$$

$$= \inf_{z \in Z \cap N_\infty} \sum_{\ell=1}^{L} \sigma^\ell \left\{ \sum_{t=1}^{T} \sum_{n=1}^{N} \left[C_n^t(g_{nT}^{\ell t}) - \lambda_n^{\ell t}\, g_{nT}^{\ell t} \right.\right.$$

$$- \lambda_n^{\ell t}\Big(\sum_{h \in n} g_h^t(\hat{q}_h^{\ell t}, \hat{\xi}_h^{\ell t}) \Big)$$

$$- \lambda_n^{\ell t}\Big(\sum_{m=1}^{N} (f_{mn}^{\ell t} - e_{nm}^{\ell t}) \Big)$$

$$\left.\left. + \lambda_n^{\ell t} D_n^{\ell t} \right] \right\} \qquad \text{(SA-34}'\text{)}$$

$P_s(\lambda)$ can be estimated by solving the Lagrangian problem, PSA$'$. Our restricted dual problem, DSA, can be stated as:

Find: $\sup\limits_{\lambda} P_s(\lambda)$ (SA-37)

such that: $\lambda_n^{\ell t} \geqslant 0$ for all $\ell=1,\ldots L$
$n=1,\ldots N$
$t=1,\ldots T.$ (SA-38)

Note that we cannot use either Theorem (S-3) or Theorem (S-4) directly to derive conclusions about the dual problem because we are dealing only with the restricted Lagrangian and our problem is not separable, since we are, in this national model, using the storage constraints to define the feasible region, $Q(\xi)$, rather than incorporating them in the Lagrangian. Instead we use the known properties of the solutions to our sub-models to determine the nature of $P_s(\lambda)$.

From our consideration (in Section 8.3.4) of the structure of the Lagrangian problem used to evaluate $P_s(\lambda)$ we can restate $P_s(\lambda)$ as:

$$P_s(\lambda) = \sum_{\ell=1}^{L} \sigma^{\ell} \Bigg[\sum_{t=1}^{T} \sum_{n=1}^{N}$$

$$\Bigg[C_n^t\left(g_{nT}^{\ell t*}(\lambda_n^{\ell t})\right) - \lambda_n^{\ell t}\, g_{nT}^{\ell t*}(\lambda_n^{\ell t})$$

$$- \lambda_n^{\ell t}\left(\sum_{h\varepsilon n} g_h^{\ell t*}(\lambda)\right)$$

$$- \lambda_n^{\ell t}\left(\sum_{m=1}^{N} f_{mn}^{\ell t*}(\lambda_m^{\ell t},\lambda_n^{\ell t}) - e_{nm}^{\ell t*}(\lambda_n^{\ell t},\lambda_m^{\ell t})\right)$$

$$+ \lambda_n^{\ell t}\, D_n^{\ell t} \Bigg] \Bigg] \qquad \text{(SA-34'')}$$

Thus:

$$\frac{\partial P_s}{\partial \lambda_n^{\ell t}} = \sigma^\ell \left\{ \left(\frac{dg_{nT}^{\ell t*}}{d\lambda_n^{\ell t}} \times \left(\left. \frac{dC_n^t(g_n^{\ell t})}{dg_{nT}^{\ell t}} \right|_{g_{nT}^{\ell t*}} \right) - g_{nT}^{\ell t*} - \lambda_n^{\ell t} \frac{dg_{nT}^{\ell t*}}{d\lambda_n^{\ell t}} \right) \right.$$

$$- \sum_{m=1}^{N} \left[\lambda_n^{\ell t} \left(\frac{\partial f_{mn}^{\ell t*}}{\partial \lambda_n^{\ell t}} - \frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} \right) + \lambda_m^{\ell t} \left(\frac{\partial f_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} - \frac{\partial e_{mn}^{\ell t*}}{\partial \lambda_n^{\ell t}} \right) \right]$$

$$- \sum_{m=1}^{N} (f_{mn}^{\ell t*} - e_{nm}^{\ell t*})$$

$$\left. - \lambda_n^{\ell t} \left[\left(\sum_{j=1}^{L} \sum_{r=1}^{T} \sum_{h \varepsilon n} \frac{\partial g_h^{jr*}}{\partial \lambda_n^{\ell t}} \right) \right] - \sum_{h \varepsilon n} g_h^{\ell t*} \right\} \quad \text{(SA-39)}$$

$$= - \sigma^\ell \left[g_{nT}^{\ell t*} + \sum_{h \varepsilon n} g_h^{\ell t*} + \sum_{m=1}^{N} (f_{mn}^{\ell t*} - e_{nm}^{\ell t*}) - D_n^{\ell t} \right.$$

$$\left. + \lambda_n^{\ell t} \left(\sum_{h \varepsilon n} \sum_{j=1}^{L} \sum_{r=1}^{T} \frac{\partial g_h^{jr*}}{\partial \lambda_n^{\ell t}} \right) \right] \quad \text{(SA-40)}$$

for all $\ell=1,\ldots L$
$n=1,\ldots N$
$t=1,\ldots T$

(SA-40) follows since we have, from the solution to the thermal and exchange sub-problems, that, firstly:

$$\text{Either:} \quad g_{nT}^{\ell t*} \varepsilon \left\{ \underline{G}_{nT}^t, \bar{G}_{nT}^t \right\} \Rightarrow \frac{dg_{nT}^{\ell t*}}{d\lambda_n^{\ell t}} = 0 \quad \text{(SA-41)}$$

$$\text{Or:} \quad \left. \frac{dC_n^t(g_{nT}^{\ell t})}{dg_{nT}^{\ell t}} \right|_{g_{nT}^{\ell t*}} = \lambda_n^{\ell t*} \quad \text{(T-2)}$$

$$\text{So:}\quad \frac{dg_{nT}^{\ell t*}}{d\lambda_n^{\ell t}}\left(\frac{dC_n^t(g_{nT}^{\ell t*})}{dg_{nT}^{\ell t}} - \lambda_n^{\ell t}\right) = 0 \qquad \text{(SA-42)}$$

for all $n=1,\ldots N$
$\ell=1,\ldots L$

Secondly:

$$\text{Either:}\quad e_{nm}^{\ell t*}\ \varepsilon\left\{\underline{E}_{nm}^t,\ \bar{E}_{nm}^t\right\} \Rightarrow \frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} = \frac{\partial f_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} = 0 \qquad \text{(SA-43)}$$

$$\text{Or:}\quad \lambda_m^{\ell t}\frac{\partial f_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} = \lambda_m^{\ell t}\left(\frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} - \frac{dL_{nm}^t}{de_{nm}^t}\frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{t}}\right) = \lambda_n^{\ell t}\frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{t}}$$

(SA-44) cf.(F-2)

$$\text{So:}\quad \sum_{m=1}^{N}\left[\lambda_n^{\ell t}\left(\frac{\partial f_{mn}^{\ell t*}}{\partial \lambda_n^{\ell t}} - \frac{\partial e_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}}\right) + \lambda_m^{\ell t}\left(\frac{\partial f_{nm}^{\ell t*}}{\partial \lambda_n^{\ell t}} - \frac{\partial e_{mn}^{\ell t*}}{\partial \lambda_n^{\ell t}}\right)\right] = 0$$

(SA-45)

for all $n=1,\ldots N$
$\ell=1,\ldots L$
$t=1,\ldots T$.

Thus the contribution of the thermal, demand and exchange systems to the Hessian, $H(\lambda)$, will be just as in the deterministic case (for each sequence ℓ).

Ignoring the hydro sector, the Hessian breaks up as before into (LxT) blocks from the leading diagonal. Each of these blocks is of size NxN and can be dealt with separately. However the hydro sector introduces a complex interaction between periods and sequences, while the nonanticipative nature of the decision process destroys the symmetry of the interaction. This is because, for sequence ℓ say, the future prices, $\lambda_n^{\ell r}$ $(r > t)$, only affect the optimal decision, $q_h^{\ell t*}$ (at period t), indirectly, through their effect on the expected future prices, $E^{\ell t}\{\lambda_n^{\ell r}\}$ (exactly as do the future prices λ_n^{jr} of the other sequences). On the other hand the past prices, $\hat{\lambda}_n^{\ell t}$, affect the decision through the effect they will have had on the storage level, $s_n^{\ell t}$, (at time t), and the expectations at time t about future prices ($E^{\ell t}\{\lambda_h^{jr}\}$). Also, the prices, $\hat{\lambda}_h^{jt}$, corresponding to other sequences will have a lesser effect on the storage level (through their effect on earlier decisions via the water values expected at those earlier periods). These interactions are expressed in the gradient vector by the terms:

$$- \sigma^{\ell}\lambda_n^{\ell t}\left(\sum_{h\varepsilon n} \sum_{j=1}^{L} \sum_{r=1}^{T} \frac{\partial g_h^{jr*}}{\partial \lambda_n^{\ell t}} \right) \qquad (SA\text{-}40)_H$$

for each $n=1,\ldots N$
$\ell=1,\ldots L$
$t=1,\ldots T$

When this term is differentiated again it will result in terms dispersed throughout the Hessian matrix. In Section 6.3.6 we have described a method to account for the equivalent terms in the deterministic Hessian. There we found that, in certain cases at least, we could ignore the inter-period interaction and deal with each NxN block separately. We noted that, in practice, this method converged rather slowly and outlined a modification which has proved to converge very quickly. Here the terms involved will, generally, be smaller and more diffuse. We intend to adopt a similar strategy for the stochastic price adjustment process to that adopted for the deterministic process. Only experience will enable us to decide whether or not some modification is desirable in order to speed convergence.

9.3 CONCLUSIONS

We have discussed some ways in which the theoretical model discussed in the previous chapter could be approximated so as to be solvable in practice. We have seen the biases inherent in the two, otherwise attractive, EDF schemes. A more accurate model of the local hydro problem has been developed which should produce realistic solutions while still being feasible computationally. Finally we have discussed the dual problem of adjusting the regional energy prices.

An important inference from this theory is that, given convex marginal costs, we may use a pair of models, one 'under-anticipative", the other "over-anticipative" to determine the range in which the true stochastic optimum must lie. Such a range is in many ways more useful for the purposes of long-term scheduling than a single estimate, no matter how accurate. Further, the extent of the range allows us to evaluate the maximum gains which could result from more sophisticated stochastic models. In this way we may hope to reach a reasonable balance between accuracy and costs of development and computation. Such studies have been initiated at NZED, using a simpler single-reservoir model due, primarily, to Boshier. (This approach is discussed in [48], where the two types of policy are referred to as "stupid" and "cheating").

CHAPTER 10

ECONOMIC IMPLICATIONS OF THE MODEL

10.1 INTRODUCTION

The model we have developed has, as its primary objective, the determination of optimal policies for the internal operation of an electrical supply system. Our concern in this chapter is with the interaction between the electrical system and the remainder of the economy. Here our decomposition approach yields an additional bonus. In a market economy the various units interact by means of prices. In our model we have treated the electrical system as a competitive economic system using prices to co-ordinate the decisions of (hypothetical) profit maximising managers of the various components of the system. Our dual problem can in fact be heuristically summarised by the statement:

> "We find energy prices such that the optimal response of the supply system to these prices satisfies the demand".

At various points we have suggested economic interpretations for these prices and policies. Here we discuss some ways in which they can be utilised for purposes other than scheduling. The contributions of this approach fall in two major areas. The first contribution is to development studies where our prices can be used to estimate the worth of any proposed development. The second, and perhaps more important, contribution is to the setting of those prices over which the utility has direct control. Of these, the most important in our context are the prices paid by consumers. In Section 10.3 we develop a more general model to determine such optimal tariffs.

10.2 DEVELOPMENT STUDIES

Every electric utility is continually faced with the question:

> "What new plant should be installed? when? where? and of what type?"

This question is particularly complex when the structures under consideration have uses other than electricity generation or when there is a conflict of interest between electrical and other interests. For instance, in New Zealand, there are schemes currently under consideration or construction involving combined irrigation and generation (Rakaia), the diversion of water from electricity generation for irrigation (Upper Waitaki Basin), and the flooding of highly productive agricultural land to enable electricity generation (Upper Clutha). We wish to be able to determine the overall net benefit from such schemes, taking into account the costs and benefits to all potential and actual users of the resources.

Our model can be used to provide information relevant to these decisions in two ways. Firstly, it is possible to use this, or any other, scheduling model in a simulation role. Thus, if we optimise system operation, given the demand for some target year, with and without the proposed development, or with various alternatives, we can easily determine the contribution of the development to the system. While this may be appropriate for major projects it is likely that a much simpler model will give similar information at a lower computation cost.

Secondly, we can use the prices developed by this model directly, to identify areas of need or to evaluate in detail the contribution of any new scheme. Since the aim of optimal system operation is equivalent, in our model, to bringing the prices throughout the system and the planning horizon as near as possible to equality (thus minimising transmission losses and thermal costs), we can use any differences in these 'prices' to recognise situations where the installation of further plant may be appropriate. Thus overall high prices in a region may indicate the need for more base load plant, while a large difference between peak and off-peak prices would suggest the need for more peaking plant or, if appropriate, pumped storage plant. Similarly, a large price difference between adjacent regions in the same period indicates that the installation of further transmission lines could be justified on economic grounds to lower losses or alleviate a bottleneck. Again, a large difference between prices (or equivalently water values) in different times of the year in the same region would indicate the desirability of further long-term hydro storage capacity in that region.

Further, suppose that we have a specific proposal.

In evaluating the proposal it would be most useful to be able to put a cash value on its potential contribution to the total system. For the NZED system, there is at present no direct connection between the tariff revenue from energy sales and either the cost of producing the energy or the benefits derived from its use. So this revenue is quite irrelevant as a measure of the contribution of any scheme. Provided that the scheme under consideration is not so large as to significantly alter the energy prices, we can use the prices developed by our model (and hence, strictly speaking, only relevant to the system as modelled) to evaluate the contribution of the scheme. For instance, we could very quickly evaluate the contribution from a small hydro scheme, running a local hydro model if necessary to determine an optimal generation pattern.

We can similarly evaluate the economic benefit from a proposed new transmission link by considering the gains given by:

$$\int_0^{e_{nm}^{t*}} \left[\lambda_n^t \left(1 - \frac{dL_{nm}}{de_{nm}}\bigg|_{e_{nm}^t}\right) - \lambda_m^t\right] de_{nm}^t \qquad \text{(P-1)}$$

Where e_{nm}^{t*} is determined in the usual way.

For a pumped storage scheme we can make a similar calculation, considering the price difference between periods at the one site (see [51]).

Thus the prices developed in the solution of our scheduling problem can be used to give useful insights into the development needs of the system and also to provide detailed information on the worth of specific proposed developments.

10.3 THE OPTIMAL TARIFF PROBLEM

10.3.1 Introduction

There are two sets of prices within the control of the utility: the price it pays to independent suppliers of energy and the price which it charges to consumers.

In New Zealand there are a number of small generating plants under the control of independent concerns (generally county councils or major industrial plants) which supply energy to the national system at rates determined by the NZED. In order to determine appropriate rates we must consider the real value of the contribution made to the national system. This can be derived, exactly as for a plant owned (or proposed) by the utility, from the energy prices supplied by our scheduling model. Of course it is not feasible to set a rate which varies hour by hour, as these prices may, but this information gives us the basis on which to set practicable rates which reflect the real value of the energy supplied.

In order to accurately resolve the question of appropriate tariffs for consumers we need to generalise our model. The remainder of this chapter is devoted to this problem, the "optimal tariff problem".

10.3.2 The Situation

In recent years there has been a sharp rise in the price of oil coupled with the realisation that reserves of all fossil fuels are limited while new energy producing technologies may not be as attractive as might have been supposed. These considerations have focussed attention, not only on energy conservation in general, but on the question of the optimal balance between the use of various resources and technologies. This is particularly so in New Zealand,where virtually all oil must be imported while coal,and now natural gas,are available locally and electricity has traditionally been generated largely from hydro-electric plants. One important mechanism by which this balance may be changed is that of price manipulation.

The NZED was previously,[44],empowered to:

> "Secure or promote...A continuous programme of works providing adequate supplies of electricity" ([44],S.7).

Here the adequacy of supply was, apparently, to be measured by the (exogenously determined) "need for electricity" ([44],S.7). However the terms "secure or promote" have since,[45], been modified to "arrange or execute" while the department has been charged with the additional task, to:

> "Undertake or promote measures to achieve greater economy and efficiency in the use of electricity as a means of reducing the future

rate of growth of electricity requirements"
([45],S.3).

A general or selective increase in tariffs would be one obvious measure. Once the use of tariffs to modify demand has been accepted in principle we have a potentially very complex problem. On the one hand we have the question of balancing the use of various energy technologies. Should the electricity price be increased so as to cut demand and reduce the amount of fuel oil consumed, or lowered so as to encourage the installation of electric rather than oil fired equipment? Many of these questions are really beyond the scope of the electric utility's decision-making, requiring a more general, though necessarily less detailed, model of the energy sector.

A New Zealand energy model has been developed ([57]). We do not concern ourselves with this problem.

On the other hand we have the possibility of modifying the form of the load curve by setting tariffs which vary according to the time of day, or season, or for different regions or types of load. This is already done to some extent in New Zealand. Some large industrial concerns have special contracts for agreed loads at reduced prices, while some domestic loads, such as water heating, are supplied at a reduced rate at the option (within limits) of the local supply authorities (via the "ripple control" mechanism). Also, the local supply authorities are charged according to a formula which takes some account of the height of the peaks in each month.

We concern ourselves here with this latter problem. That is, we wish to find tariffs which will result in efficient production and utilisation patterns for electricity (assuming fixed prices for other fuels). Much has been written on this subject. We will not attempt to catalogue or analyse the approaches taken by others. Our intention is to give a broad indication as to the way in which our long-term scheduling model may be modified so as to give much insight into the structure of optimal tariffs.

10.3.3 A Price-Elastic Scheduling Model

How could we use the model of Chapter 2 (or the stochastic version of Chapter 8) to determine optimal tariffs for electricity? Firstly, we have noted that these models do, in fact, directly manipulate energy prices (λ), arriving at 'optimal prices' for each load segment appropriate to each region and period (and to each historical streamflow sequence in Chapter 8). At first glance we might conclude that these are the prices we require. However, we recall that these prices were derived under the assumption of fixed demand and our task here is to modify demand. Thus we need some model which includes the customer's reaction to the prices set. In this model we must again search for an equilibrium where the 'prices' are set so that supply and demand for electrical energy are equated and energy is produced, transmitted and utilised optimally.

In economic terms, our long-term scheduling model corresponds to the situation shown in Figure (10-1). Here we adjust prices so as to match supply, $g^*(\lambda)$, to a fixed (inelastic) demand. Now the optimal tariff problem corresponds to the situation in Figure (10-2). Here we have a price-elastic demand curve $(d^*(\lambda))$ and we wish to find a price, $\tilde{\lambda}$, which matches supply and demand at $\tilde{D}$. (Note that, so as to conform to the conventions adopted for the scheduling problem, the orientation of the "price" and "quantity" axes in these figures is the reverse of normal economic practice).

We deal with a model identical to the complete model (PC) of Section 2.1 except for the following modification.

We require the existence of a concave increasing social benefit function B(d) (where $d = (d^r_{ia})^{r=1,\ldots R}_{i\varepsilon I,\ a=1,\ldots A}$ i and r having their usual interpretation and $a=1,\ldots A$ indexing the alternative types of energy use (e.g., domestic, industrial)). So d^r_{ia} is the demand for energy for use a, in instant r, at node i. This energy has decreasing marginal benefit as in Figure (10-3).

We ignore the practicalities involved in the empirical derivation of such a function because we shall never in fact need an explicit formula for it. Its existence (and form) is sufficient to guarantee solutions, while all of its characteristics required by the solution procedure are in fact observable (see Section 10.3.4). Note that we have not yet made any assumption about the separability of B - we leave this until it is required.

We can also place limits on the acceptable variation in d:

$$\underline{D}^r_{ia} \leqslant d^r_{ia} \leqslant \bar{D}^r_{ia} \qquad \text{for all } a=1,\ldots A,\ i\varepsilon I,\ r=1,\ldots R \qquad \text{(P-2)}$$

And define:

$$d^{r\prime}_i = \sum_{a=1}^{A} d^r_{ia} \qquad \text{for all } i\varepsilon I,\ r=1,\ldots R \qquad \text{(P-3)}$$

Discussion of appropriate limits and benefit functions is left to Section 10.3.4. We continue with our exposition of the model.

Our objective is now to maximise the net benefit from the electricity sector, so we can state our optimal tariff problem, PP, as:

$$\text{Find} \ \underset{g,e,d}{\text{MAX}} \left[B(d) - \sum_{r=1}^{R} \sum_{i\varepsilon I} C^r_i \, (g^r_{iT}) \right] \qquad \text{(P-4)}$$

Such that:

$$g^r_{iH} + g^r_{iT} + \sum_{j\varepsilon I} (f^r_{ji} - e^r_{ij}) - \sum_{a=1}^{A} d^r_{ia} \geqslant 0 \qquad \text{for all } i\varepsilon I,\ r=1,\ldots R \qquad \text{(P-5)}$$

$$\underline{D}^r_{ia} \leqslant d^r_{ia} \leqslant \bar{D}^r_{ia} \qquad \text{for all } i\varepsilon I,\ r=1,\ldots R,\ a=1,\ldots A \qquad \text{(P-2)}$$

$$\underline{E}^r_{ij} \leqslant e^r_{ij} \leqslant \bar{E}^r_{ij} \qquad \text{for all } i,j\varepsilon I,\ r=1,\ldots R \qquad \text{(P-6)}$$

$$\underline{G}^r_{iT} \leqslant g^r_{iT} \leqslant \bar{G}^r_{iT} \qquad \text{for all } i\varepsilon I,\ r=1,\ldots R \qquad \text{(P-7)}$$

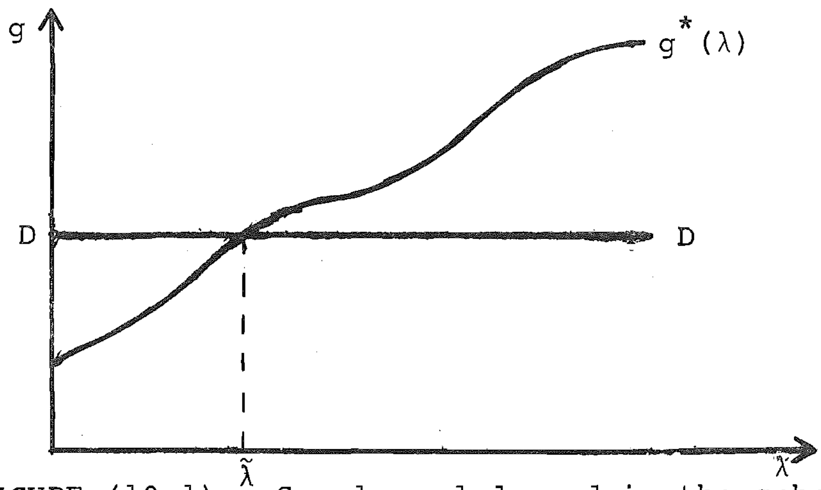


FIGURE (10-1): Supply and demand in the scheduling program.

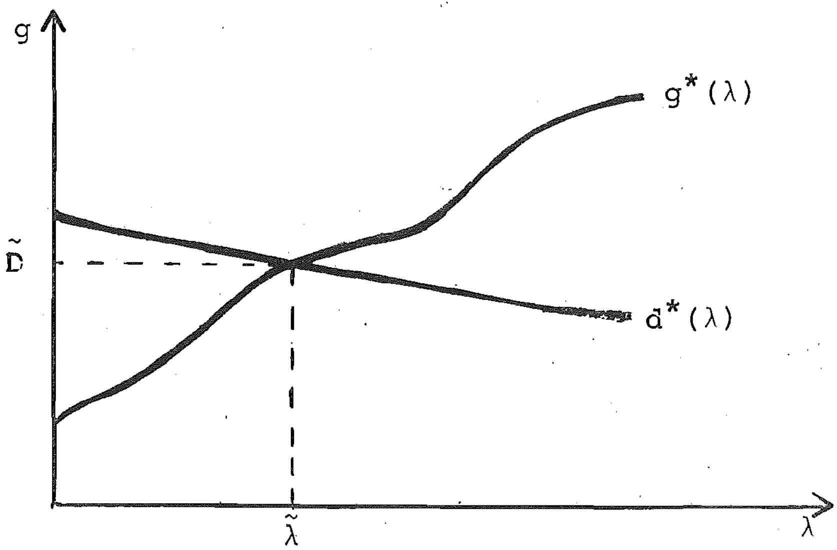


FIGURE (10-2): Supply and demand in the tariff program.

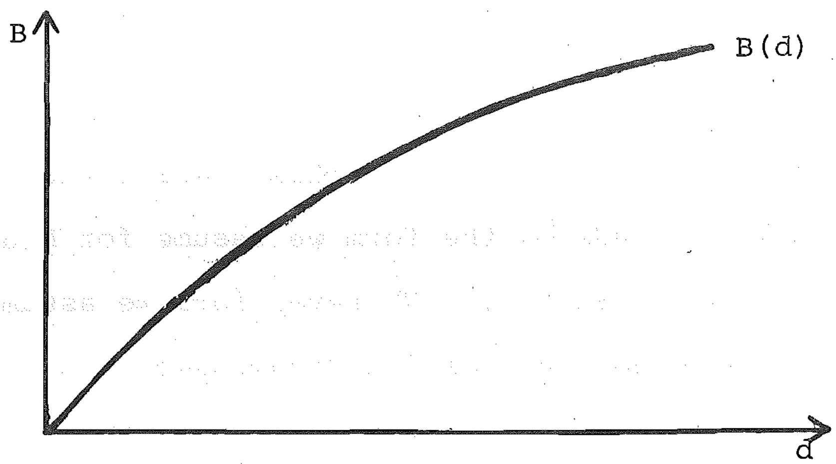


FIGURE (10-3): The benefit function.

$$q_h \in \mathcal{Q}_h \qquad \text{for all } h\varepsilon H \qquad \text{(P-8)}$$

Let Z_P = {(g,e,d) satisfying (P-2),(P-6) - (P-8)}.

It is clear that this problem satisfies the same conditions as did PC (maximising a concave objective rather than minimising a convex one).

Thus we can form the Langrangian:

$$\mathcal{L}_P(g,e,d) = B(d) - \sum_{r=1}^{R} \sum_{i\varepsilon I} \left[C_i^r(g_{iT}^r) - \mu_i^r\left(g_{iT}^r + g_{iH}^r + \sum_{j\varepsilon I} (f_{ji}^r - e_{ij}^r) - d_i^r \right) \right]$$

(P-9)

Define:

$$P_P(\mu) = \underset{(g,e,d)\varepsilon Z_P}{\text{MAX}} \quad \mathcal{L}_P(g,e,d,\mu) \qquad \text{(P-10)}$$

Now we have the dual problem, DP:

Find $\underset{\mu}{\text{MIN}} \; P_P(\mu)$

such that: $\mu_i^r \geqslant 0$ for all $i\varepsilon I$, r=1,...R (P-11)

This can be solved by the same basic algorithm as was PC. That is:

1. Set μ
2. Determine $P_p(\mu)$ by solving PP′ (the Lagrangian problem)
3. If converged THEN STOP. ELSE GO TO (4).
4. Adjust μ, GO TO (2).

How much more difficult to solve is this problem than PC? The extra burden depends on the form we assume for B(d).

We look first at Step 2. Whatever form we assume, the Lagrangian problem, PP′, may be decomposed into the following sub-problems:

PT - Nodal thermal problems (as for PC′)

PH - Local hydro problems (as for PC′)

PE - Inter-nodal exchange problems (as for PC′)

PD - Demand problem.

The problems PT, PH and PE can be handled by the techniques of Chapters 3 - 5 so we turn our attention to PD, the demand problem. This may be stated as:

$$\text{Find} \quad \underset{d}{\text{MAX}} \left[\left(B(d) - \sum_{r=1}^{R} \sum_{i \varepsilon I} \mu_i^r \left(\sum_{a=1}^{A} d_{ia}^r \right) \right] \right] \qquad \text{(P-12)}$$

$$\text{Such that:} \quad \underline{D}_{ia}^r \leqslant d_{ia} \leqslant \bar{D}_{ia}^r \qquad \text{for all } i \varepsilon I,\ r=1,\ldots R,\ a=1,\ldots A \qquad \text{(P-2)}$$

One can imagine a great variety of possible forms for B(d). For instance, we could take into account such factors as equipment purchase decisions based on high consumption in some periods leading to greater benefit from consumption in others. The empirical estimation of such functions would be very difficult. We assume here a very simple form for B(d), for which the solution of problem PD is feasible, both from a theoretical viewpoint and because the necessary data is readily available. Later in this section we consider the impact of more general forms. In the meantime we will assume that B(d) is completely separable and differentiable.

That is:

$$B(d) = \sum_{r=1}^{R} \sum_{i \varepsilon I} \sum_{a=1}^{A} B_{ia}^r (d_{ia}^r) \qquad \text{(P-13)}$$

Then problem PD is obviously decomposable into IxAxR, nodal instantaneous demand problems, PD_{ia}^r, for various types of energy use.

Problem PD^r_{ia} may be stated as:

$$\text{Find} \quad \underset{d^r_{ia}}{\text{MAX}} \quad B^r_{ia}(d^r_{ia}) - \mu^r_i \, d^r_{ia} \qquad (P\text{-}12)^r_{ia}$$

Such that: $\underline{D}^r_{ia} \leqslant d^r_{ia} \leqslant \bar{D}^r_{ia}$ $\qquad (P\text{-}7)^r_{ia}$

This is easily solved by finding $\tilde{d}^r_{ia}$ such that:

$$\left.\frac{\partial B^r_{ia}}{\partial d^r_{ia}}\right|_{\tilde{d}^r_{ia}} = \mu^r_i \qquad (P\text{-}14)$$

Then setting:

$$d^{r*}_{ia} = \text{MIN}\{\,\text{MAX}\{\,\tilde{d}^r_{ia},\ \underline{D}^r_{ia}\},\ \bar{D}^r_{ia}\} \qquad (P\text{-}15)$$

In economic terms this corresponds to the consumers maximising their benefit from use of electricity by equating their marginal benefit with the marginal cost, μ.

Note that this solution procedure requires no knowledge of B^r_{ia}, but only of $\frac{\partial B^r_{ia}}{\partial d_{ia}}$ - the estimation of which is considered in Section 10.2.3.

So we can determine d^{r*}_{ia} for each μ^r_i and hence obtain a "demand response" function, $d^{r*}_{ia}(\mu^r_i)$. These instantaneous nodal solutions can, if desired, be aggregated and tabulated exactly as for the hydro, thermal and exchange problems.

We now turn our attention to the dual problem, DP.

Just as for P_C we have that:

(a) $P_P(\mu)$ is differentiable for all $\mu \geqslant 0$

(b) $$\frac{\partial P_P(\mu)}{\partial \mu_i^r} = g_{iT}^{r*}(\mu_i^r) + g_{iH}^{r*}(\mu_i) + \sum_{j\varepsilon I}\left[f_{ji}^{r*}(\mu_j^r,\mu_i^r) - e_{ij}^{r*}(\mu_i^r,\mu_j^r)\right] - d_i^{r\prime *}(\mu_i^r) \qquad \text{(P-16)}$$

Hence:

$$\frac{\partial^2 P_P(\mu)}{\partial \mu_i^r \partial \mu_j^s} = \begin{cases} \dfrac{\partial g_{iT}^{r*}}{\partial \mu_i^r} + \dfrac{\partial g_{iH}^{r*}}{\partial \mu_i^r} + \sum_{j\varepsilon I}\left[\dfrac{\partial f_{ji}^{r*}}{\partial \mu_i^r} - \dfrac{\partial e_{ij}^{r*}}{\partial \mu_i^r}\right] - \dfrac{\partial d_i^{r\prime *}}{\partial \mu_i^r} & \text{if } i=j,\ r=s \quad \text{(P-17)} \\ \dfrac{\partial g_{iH}^{r*}}{\partial \mu_i^s} & \text{if } i=j,\ r\neq s \quad \text{(P-18)} \\ \dfrac{\partial f_{ji}^{r*}}{\partial \mu_j^r} - \dfrac{\partial e_{ij}^{r*}}{\partial \mu_j^r} & \text{if } i\neq j,\ r=s \quad \text{(P-19)} \\ 0 & \text{if } i\neq j,\ r\neq s \quad \text{(P-20)} \end{cases}$$

Thus the Hessian matrix, $H_P(\mu)$, has exactly the same structure as $-[H_C(\mu)]$, the only difference being the addition of the $\left(\dfrac{\partial d_i^{r\prime *}}{\partial \mu_i^r}\right)$ terms on the leading diagonal.

These terms, being always positive, will have the effect of enhancing the convexity of the Hessian and thus improving the convergence properties of the technique. In fact, while $\underline{D}_i^r < d_i^{r\prime} < \bar{D}_i^r$, there will always be at least a gentle curvature on the dual objective function, $P_P(\mu)$, and hence the modifications suggested earlier to the thermal response curve to achieve this would be unnecessary. Note also that, again, we do not require any knowledge of B(d), but only of $\dfrac{\partial d_i^{r\prime *}}{\partial \mu_i^r}$, the evaluation of which is discussed in Section 10.3.4.

It is in fact difficult to obtain detailed information on the benefit function, so that the model outlined above should be sufficiently accurate to capture what we know of the reaction of consumers to prices. However a more accurate representation can be incorporated into the aggregated version of this model without much difficulty. In line with Section 2.4 we may divide our planning horizon into periods and our nodes into regions. Then we may form an aggregate model in which we ensure that aggregate constraints are met by adjusting "aggregate prices". Detailed prices, $(\mu_i^r)_{i\varepsilon n}^{r\varepsilon t}$, may be derived from these aggregate prices. Suppose that the benefit function and constraints are separable between regions and concave, that is:

$$B(d) = \sum_{n=1}^{N} \sum_{t=1}^{T} B_n^t(d_n^t) \qquad \text{(P-21)}$$

and:

$$\frac{\partial^2 B_n^t}{\partial d_{ia}^r \partial d_{jb}^s} \leqslant 0 \qquad \text{for all } r,s\varepsilon t, \quad i,j\varepsilon n, \quad a,b=1,\ldots A \qquad \text{(P-22)}$$

Then problem PD_n^t will be reasonably easy to solve and the results may be aggregated and used to summarise the demand response. Also, the dual algorithm, dealing as it does with aggregate prices and quantities for regions and periods, will be no less amenable to solution than in the case of completely separable benefit function. This approach could be used in a limited fashion, say, to model the interaction between the benefit arising from using peak and off-peak energy for domestic water heating while leaving the rest of the benefit function separable. Obviously one could increase the extent of the inseparability so as to model inter-regional or inter-period interaction, with a corresponding increase in difficulty for both the Lagrangian problem, PP′, and the dual, DP.

Of course all of this can be extended into a stochastic framework (as in Chapter 8), with different prices appropriate to different streamflow sequences (and/or demand sequences). Such weather dependent pricing is unlikely to be acceptable in practice but see Section 10.4 on this.

10.3.4 The Benefit Function

As has been pointed out, we do not require the evaluation of B(d) at any stage in our solution procedure. We only require:

(i) $\dfrac{\partial B_{ia}^{r}}{\partial d_{ia}^{r}}$ - in order to solve the demand sub-problem PD_{ia}^{r}

(ii) $\dfrac{\partial d_{i}^{r,*}}{\partial \mu_{i}^{r}}$ - in order to solve the dual problem DP.

If we knew $\frac{\partial B^r_{ia}}{\partial d^r_{ia}}$, we could easily derive $\frac{\partial d^{r'*}_i}{\partial \mu^r_i}$ from the solutions of problem PD^r_{ia}. A suitable estimate for $\frac{\partial B^r_{ia}}{\partial d^r_{ia}}$ could be found from a more general model of the energy sector such as that of [57]. We prefer, however, to derive $\frac{\partial B^r_{ia}}{\partial d^r_{ia}}$ from $\frac{\partial d^{r*}_{ia}}{\partial \mu^r_i}$.

We take as our measure of social welfare the "consumer's surplus", defined by:

$$B(d) = \sum_{r=1}^{R} \sum_{i\varepsilon I} \sum_{a=1}^{A} \int_{0}^{d^r_{ia}} [D^{r*}_{ia}]^{-1}(x)\,dx \qquad \text{(P-23)}$$

Where demand for use a at node i in instant r is given by the function $D^{r*}_{ia}(\mu^r_i)$. Thus $[D^{r*}_{ia}]^{-1}(x)$, the inverse demand function, gives the price which category a users at node i would pay for the x^{th} unit of electricity supplied in instant r.

This approach is standard in partial equilibrium analysis and has been used in a number of previous studies on the electricity sector (e.g., [40], [59] and [65]). It involves the assumption that "income effects" are not significant. It has been shown, however, that the errors involved in this approach are quite small ([66]). We must also assume that there are no "externalities" involved and that the benefit to society may be measured by the sum of the benefits to individuals. This assumption is reconsidered in Section 10.4

The advantage of this approach is that the properties of B which we require can easily be derived from available data.

Firstly, it is clear that:

$$(\text{P-22}) \Rightarrow \left.\frac{\partial B^r_{ia}}{\partial d^r_{ia}}\right|_{\tilde{d}^r_{ia}} = [D^{r*}_{ia}]^{-1}(\tilde{d}^r_{ia}) \qquad (\text{P-24})$$

In other words, the marginal benefit derived from consumption at level $\tilde{d}^r_{ia}$ can be measured by the price which consumers are prepared to pay at that consumption level (cf. problem PD^r_{ia}). The demand function, D^{r*}_{ia} can be determined from historical data. Suppose, for instance, that ρ is the estimated price elasticity of demand and μ^{ro}_i, the current price level, leads to demand d^{ro}_{ia}. Then, assuming constant elasticities, we would have:

$$D^{r*}_{ia}(\mu^r_i) = d^{ro}_{ia}\left(\frac{\mu^r_i}{\mu^{ro}_i}\right)^{\rho} \qquad (\text{P-25})$$

(Here the elasticity, ρ, is defined by

$$\rho = \frac{\mu}{d}\;\frac{dD^*(\mu)}{d\mu} \qquad (\text{P-26})$$

Further, if we assume consumers to be rational and consistent, we can deduce their future reactions to tariff changes from their past reactions. Thus we can derive $\frac{\partial d^{r*}_{ia}}{\partial \mu^r_i}$ from historical records either directly or indirectly via the elasticity, i.e.,

$$(P\text{-}26) \Rightarrow \frac{\partial d_{ia}^{r*}}{\partial \mu_i^r} = \rho \left(\frac{d_{ia}^r}{\mu_i^r}\right) \qquad (P\text{-}27)$$

In order to implement this approach we only require an estimate of the elasticity, ρ. The data to enable such an estimate is generally on record and, in fact, such studies have been done. [60] gives elasticities for residential demand in each region of New Zealand, while [15] deals with national elasticies (for residential demand). This latter study estimates a short-term elasticity of the order of -0.1 and a long-term elasticity of about -0.45. This difference arises via the following mechanism. In the short term, consumers are unable to change their equipment and can only change their consumption to a limited extent. For example, an all-electric household must use electricity for heating, or freeze, no matter what the price of electricity. However, in the longer term the consumers are free to change their consumption to a far greater degree because they can change their equipment (e.g., by installing oil-fired central heating). Thus long-term elasticities are much higher than those for the short term. We should use the elasticities appropriate to our planning horizon.

The magnitude of the short-term elasticities has been confirmed by recent experience at NZED. Tariffs have successively been raised by 40% and

40% again. The resultant short-term fall in demand was about 4% in each instance. Records have been kept from which more detailed elasticities or estimates could be computed if desired. Thus it may be seen that all relevant properties of B(d) may readily be determined on the basis of available data.

We turn our attention to the limits $\underline{D}$ and $\bar{D}$. These are most naturally seen as limits to the acceptable growth or decline in demand for electrical energy. Such limits could be imposed, for instance, by limits to expansion, government policy on fuel imports, concern for conservation or the political acceptability of rationing. In general they will reflect decisions beyond the scope of our study. For example, in the short run we may wish to hold demand to at most its present level, but consider that any more than 7% "rationing" is unacceptable. So we would have:

$$\underline{D}^r_{ia} = 0.93\ d^{or}_{ia} \leqslant d^r_{ia} \leqslant d^{or}_{ia} = \bar{D}^r_{ia} \qquad \text{(P-28)}$$

for all $i \varepsilon I$

$a=1,\ldots A,$

$r=1,\ldots R.$

Where d^{or}_{ia} is the demand at current prices.

We must also exercise care as to the interaction between this provision for demand modification and the provision made, in our extended thermal response curve, for shortages. We must ensure that thermal "production" never moves into the "shortage" region before the lower demand limit, $\underline{D}_{ia}^{r}$, is reached. This can be achieved by making the shortage cost greater than the marginal benefit from consumption.

10.4 CONCLUSIONS

In order to find "optimal" tariffs we have introduced a separable concave increasing social benefit function, B(d), representing the benefits from consumption of electrical energy for various end uses, at various times, in various places. This has allowed us to form a problem (PP) which we can decompose in a manner similar to that of the earlier (fixed demand) model (PC). The resultant problem structure is shown in Figure (10-4). Neither the dual problem (DP) nor the Lagrangian problem (PP′) are significantly more difficult to solve than their fixed-demand analogues (DC and PC′). This model could be approximated by aggregating instants and nodes to get a model involving periods and regions. This could be solved in the same way as PA, utilising more complex "response surfaces" derived from the hydro, thermal, exchange and now demand sub-models. These sub-models can be generalised, within limits, to allow for non-separable cost or benefit functions, without disturbing the dual

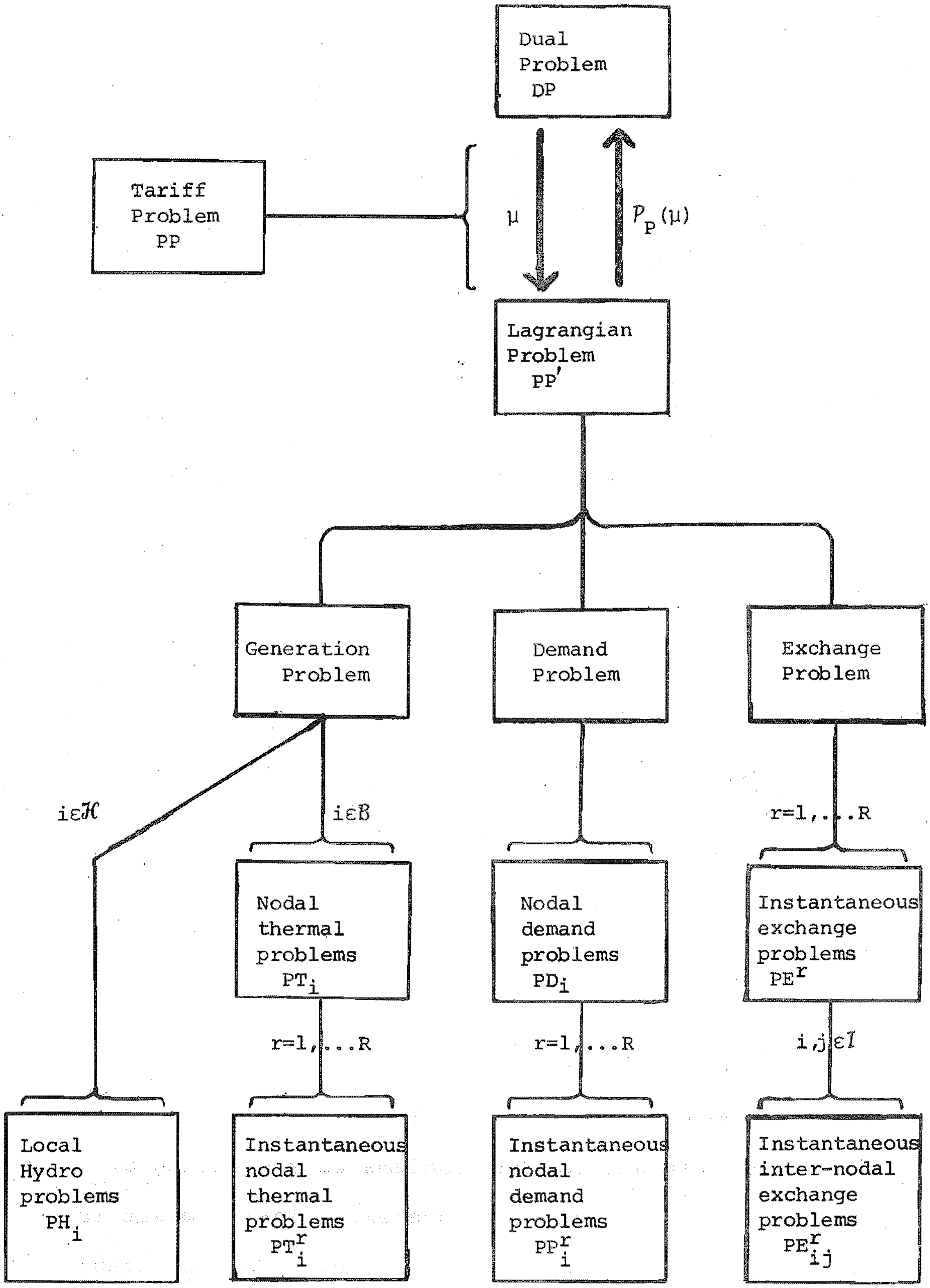


FIGURE (10-4): Decomposition of the tariff problem.

optimisation. The model does not require the explicit evaluation of the benefit function, but only of certain of its characteristics which can be deduced from available data.

Thus we can arrive at a solution to our model, but what interpretation should we place on these "tariffs" and their associated schedules? They are, to the extent that the cost function is accurate and social welfare is in fact represented by our benefit function, "socially optimal". In other words the schedules so derived are the best balance between production cost (as measured by $C(g_T)$) and benefit to society (as measured by $B(d)$) from consumption of electrical energy. The tariffs derived are the best tariffs in the sense that rational consumers faced with these tariffs will consume energy according to the best schedule, and a profit maximising utility, faced with these tariffs, would produce energy according to this same schedule. However, we may legitimately question the accuracy of both $C(g_T)$ and $B(d)$.

Firstly, $C(g_T)$ reflects only the fuel costs not the capital or running costs. However, provided that these other costs do not depend on the level of production, they are not relevant to the problems of scheduling or setting tariffs in an existing system. Their impact is in the area of development and staffing. To the extent that costs (e.g. maintenance) are proportional to production, they can, in fact, be incorporated in cost functions. The inclusion of such costs need not disturb the structure of any of the sub-models.

Secondly, in our derivation of B we assumed that there were no "externalities" involved. Thus each consumer received all the benefit from his own consumption, no other consumer receiving any benefit or suffering any penalty therefrom. Thus his "selfish" decision (cf. problem PD) is "socially optimal".

The tariffs determined by our model will favour "considerate" users who do not strain system capacity by using power at peak times or in regions where production is expensive. They do not, however, make any concessions to social welfare, energy conservation or regional development considerations (except inasmuch as these are reflected in fuel costs or consumer preferences). For instance, since it costs just as much to supply power to old age pensioners or to key local industries as to any other consumer, these groups would not receive any concessions from our tariffs. These tariffs would maximise both the profit to the utility and the direct benefits to users. However, if the pricing of energy is to be used as an instrument of government policy, we must modify our model so as to reflect the relevant externalities. Such price manipulation seems, in practice, to be determined on the basis of rather subjective criteria. As an example, suppose that the Government has decided to charge a special tariff, μ_{ia}^{r} 10% lower than μ_{i}^{r}, on electricity used for purpose a, at node i, in instant r. Then this corresponds to assuming that the total social benefit from this consumption, $B_{ia}^{\prime r}$

is, on average, 11% higher than the direct benefit to the consumer. That is:

$$B'^{r}_{ia}(d^{r}_{ia}) = (1.11)x(B^{r}_{ia}(d^{r}_{ia})) \qquad \text{(P-29)}$$

Now, if we were to use $B'^{r}_{ia}(d^{r}_{ia})$ in our model in place of $B^{r}_{ia}(d^{r}_{ia})$, problem PD^{r}_{ia} is solved by finding $\tilde{d}^{r}_{ia}$ such that:

$$\left.\frac{\partial B'^{r}_{ia}(d^{r}_{ia})}{\partial d^{r}_{ia}}\right|_{\tilde{d}^{r}_{ia}} = \mu^{r}_{i} \qquad \text{(P-30)}$$

Or, equivalently:

$$\left.\frac{\partial B^{r}_{ia}(d^{r}_{ia})}{\partial d^{r}_{ia}}\right|_{\tilde{d}^{r}_{ia}} = \frac{\mu^{r}_{i}}{1.11} = 0.9\ \mu^{r}_{i} = \mu^{r}_{ia} \qquad \text{(P-31)}$$

Here the consumer, faced with a lower tariff μ^{r}_{ia}, increases consumption to its "socially optimal" level, the increased production cost to the system being balanced by the increased benefits to persons other than the immediate consumer.

Any tariff concession or surcharge involves, explicitly or implicitly, a modification to the social welfare function as derived from observed consumer behaviour. Decisions as to appropriate modifications for this kind of purpose are beyond the scope of our study. However, where such modifications have been exogenously determined, they can be translated directly into the demand model as in the example just discussed. Thus the benefit function in our model may be modified so as to be an accurate reflection of social welfare as perceived by the legislators. Then the (modified) "optimal" tariffs and schedules found by our model will be the best possible (as measured by the criteria laid down). So they are, in theory, the tariffs we require.

However, this optimum is in fact unattainable because of the practical impossibility of setting tariffs which vary widely according to the time of day, season and location of the consumption (and even according to the weather) and are, moreover, revised at regular intervals.

Even if such a flexible tariff structure were imposed, the consumers at large would not be capable of optimising their consumption to it and so would not consume according to the "best" schedule. In fact some simple pricing formula must be worked out on the basis of these prices so as to produce an approximation to the optimal consumption pattern. However, in order to satisfy this new consumption pattern, a new production schedule, and hence a new set of optimal tariffs, will be needed. This gives rise to a new approximation to the optimal tarriff and yet another consumption pattern and so on. Thus, while our model now answers the question "What is the theoretical socially optimal price pattern for electrical energy?", and so gives a good basis from which to determine the best form and level for a realisable tariff structure, it does not quite answer the question - "If, on the basis of production costs, we set tariffs of a particular form, what will the resultant optimal demand/transmission/generation schedule be (assuming rational consumers and production)?"

If we wished to answer this question exactly we could modify our model so that, having set our λ prices, we derived from them, not only a detailed price curve, $\mu(\lambda)$, but a realisable tariff, $\sigma(\lambda)$ say. We would then solve the demand problem, PD, for this price curve rather than for μ or λ. This requires little more computational effort in the solution of the modified primal. However it destroys the separability of the demand response function in the dual. Thus the terms associated with this, rather than being concentrated on the leading diagonal, are scattered diffusely throughout the Hessian. They are, however, correspondingly small and could, perhaps, be safely ignored or dealt with as were terms resulting from inter-period interaction in the hydro system. If the function $\sigma(\lambda)$ is convex and differentiable then $d^*(\lambda)(=d^*(\sigma(\lambda)))$ will be convex and differentiable and the dual solvable.

APPENDICES

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APPENDIX A - THE EDF MODEL

A.1 INTRODUCTION

The intention of this appendix is to summarise the approach which Electricité de France (EDF) have adopted to the long-term scheduling problem. It is essentially a condensation of Section 5 and Appendix C of our earlier survey paper [17]. This, in turn, was based primarily on a collection of internal reports supplied by the EDF. We have adapted the notation of [17] so as to conform as far as possible with that of this thesis. A summary of the additional notation required may be found in the appropriate section of Appendix B.

Over the years EDF have tried a number of approaches to the long-term scheduling problem. A number of variations to each basic approach have also been suggested. Here (and in [17]) we describe two integrated systems which contain the ideas on which we have based our approach. We restrict our attention here to the mathematical models - the interested reader is referred to [17] for a discussion of their implementation and some evaluation of their merits and deficiencies.

A.2 SUCCESSIVE APPROXIMATIONS: GRAF.

This approach, reported in [1], is based on the trajectory method described in Section 5.4.3. We will not repeat that discussion here. We will, however, describe the way in which that method may be extended so as to model a multi-reservoir system with stochastic inflows.

In the GRAF system successive approximations are used to generalise the trajectory method to handle several independent reservoirs. That is, assuming trajectories for all but one reservoir, the conditional optimal trajectory for that reservoir is determined. This conditional optimal trajectory now becomes one of the assumed trajectories and a new conditional optimal trajectory is determined for another reservoir, etc., until all reservoirs have been evaluated. This constitutes one iteration of the optimization procedure. Additional iterations are made until the trajectories of successive iterations differ by less than some specified tolerance. The trajectories found at the last iteration are taken as the optimal trajectories. If the "profit" functions of all river systems are concave, this procedure will converge to the optimal solution. The number of iterations required is usually small, i.e., rarely exceeds three or four, if the number of independent river systems is small. The better the initial assumed trajectories, the fewer the iterations needed to find the optimal solution.

A similar approach could also be applied to multi-reservoir river systems. In this case however convergence is not rapid.

Because of the non-linear nature of the thermal cost functions, deterministic optimisation on average inflows does not yield an appropriate policy for system operation in a stochastic environment. In [20] the following approach is used in order to represent the stochastic nature of the problem. Each of the L historically observed sequences of yearly streamflows is taken as a dependent sequence. Correlations are thus maintained. This set of L sequences is considered as representative of future streamflows. The optimal (deterministic) trajectory is determined separately for each annual sequence, yielding L different ψ values and L different releases in each period of the planning horizon. The average of these ψ values is used to determine the optimal decision in the face of probabilistic streamflows. The optimal decision is thus based on the average of optimal deterministic water values - or the average of optimal values with hindsight.

In this approach L optimal trajectories have to be determined. The computational effort is thus about L times higher than for the deterministic model. This approach is discussed and evaluated in Section 9.2.3 where it is referred to as EDF scheme 1.

Further to the above, one may wish to modify the definition of the water value so as to account for head effects if these are significant for any long-term controllable reservoir. Such a modification is formalised in the next section.

This system could be used to find optimal first period release decisions for a multi-reservoir hydro-thermal system in which the system load (and generation) are treated as if it occurred at one point. It may also be used to determine a set of water value curves or contours for each reservoir (assuming average values for the remainder of the reservoirs). These curves, giving marginal water values as a function of the storage level and the time of year, could then be utilised to determine future release decisions. Such a scheme is described in [20].

For a number of reasons this system was never actually applied on a regular basis at EDF. Instead, after considerable experimentation, the system described in the next section was approved for implementation.

A.3 DECENTRALISATION VIA PRICES: SGEP.

A.3.1 Introduction

The basic idea of the SGEP system is appealingly simple. Each week, the "centre" provides the local decision makers - the "subsystems" consisting of a single river system or of all thermal plants - with a list of energy prices covering each week in the one-year planning horizon. On the basis of these prices, each subsystem determines an optimal generation schedule for each week. The centre collates the resulting generation decisions and determines whether the total generation of all subsystems is equal to the system's load in each period. If this is so, the generation schedule is optimal for the system as a whole.

If the total generation of all subsystems does not match the system's load in each period the centre adjusts the energy prices, increasing them if total generation exceeded system load, and reducing them in the reverse case. The adjusted prices are again communicated to the subsystems for another trial. This procedure is continued until total generation matches the system load in each week.

This process is repeated each week and only the first week's results implemented. In practice all computations would be done by the centre, including the optimization for the subsystems, and only the final results communicated to the subsystems for implementation.

The proposed system consists of 15 individual river systems plus one subsystem covering all other hydro generation on an aggregate basis.

In order to account for the stochastic nature of the problem the model actually develops twenty sets of prices (and corresponding decisions), one set for each of twenty observed historical inflow sequences. These prices are used in the solution of the local long-term hydro problem discussed in Section A.3.3. Further, in order to account for short-term requirements, separate prices are required for "peak" and "off-peak" load segments. From these a "price duration curve" is derived and used in the solution of the short-term hydro scheduling problem discussed in Section A.3.4.

The system discussed here consists of three computer programs P1, P2 and P3. (A fourth program, P4, is intended to perform a more detailed optimisation of the thermal sector than that currently incorporated in P3). P3 manipulates energy prices (and hence thermal generation levels) so as to ensure that national demand is met in each load segment of each week for each streamflow sequence. These prices are used by P2 to solve a long-term hydro sub-problem. P1 is a heuristic short-term hydro scheduling program which is used to provide input data for P2. Each of these programs is described in the following sections.

A.3.2 The Global Optimisation - P3

P3 has the dual role of optimizing the thermal production and adjusting the weekly energy prices ([8]).

The first phase of P3 determines the thermal generation pattern that minimizes the total annual fuel and shortage cost, given the same set of 20 years of weekly energy prices for each load segment as was used in the previous iteration for the evaluation of the hydro energy (via P2). The optimal thermal generation in each week is found by scheduling an amount of thermal generation that equates the marginal thermal cost with the corresponding weekly energy price for each load segment. The output of this phase consists of the (thermal) energy produced for each load segment and each week of the 20 data years of weekly energy prices.

The second phase of P3 compares the predicted load for each load segment in each week of the planning horizon with the corresponding total hydro and thermal energy produced in each of the 20 data years. Note that the same sequence of load predictions is always used, while the total generation may vary from data year to data year. If total supply and demand match in each load segment in each week, the overall optimal solution has been found. If not, the energy prices are decreased for all those load segments in all those weeks in which total supply exceeds the total energy demand and increased if total supply is insufficient to cover the demand. The adjusted energy prices are then referred back to the P2 models for a new round of optimizations, i.e., a new iteration of P2/P3.

This iterative P2/P3 scheme is an application of Uzawa's algorithm ([64]). Provided we are sufficiently cautious in our price adjustments, this scheme can be guaranteed to converge to the optimum if the fuel cost function is convex and hydro generation is a concave function for each river system. This holds provided there is no head variation within each week for the main hydro station in the river system.

Limited tests indicate that 4 or 5 iterations of P3 are sufficient for covergence within 1%. It is hoped that as more experience is gained in setting the initial set of energy prices, the number of iterations can be reduced to about 2.

When P3 has converged the trajectories produced in the final round of P2 optimizations are 'optimal' and the average energy to be generated in the first period by each valley, and the corresponding thermal generation, are communicated to the operators as the basis for the next week's scheduling pattern. One full iteration of P2/P3 requires about 8 minutes of computer time on an IBM 370/165. It is planned to run the iterative process of P2/P3 only every second week, as the energy prices tend to change only slowly, while the P2 programs would be run every week.

The remainder of this section gives the mathematical basis for the P3 program.

In the EDF system we have 52 weekly periods. Each week is divided into K load segments (indexed by superscript $k=1,\ldots K$). These segments are not (necessarily) connected, e.g., the 'peak' segment may consist of four hours on each working day. It is assumed that the transmission system is such that all loads can be aggregated into one single load and D^{tk} indicates the demand for energy in segment k of week t.

The aggregate thermal production in segment k of week t is g_T^{tk} and it has a cost function $C^t(g_T^{tk})$. (A fictitious very high cost extension to this curve represents shortage and ensures that feasible solutions exist to the mathematical problem). The hydro system is divided into H river systems or 'valleys' (indexed by subscript $h=1,\ldots H$). It is clear that for each valley the volume of controllable inflows (F_h^t) and uncontrollable inflows (A_h^t) for a particular year will affect the optimal release pattern and hence the generation and water value. Since we do not know the future inflows we must make some kind of probabilistic forecast. The distribution actually used for this purpose is the data from past years and from now on this will be assumed in the formulation. (The flows for the first week or two in each year may actually be adjusted in the light of the current flow forecasts). To facilitate this we will index the data years by superscripts ℓ, (ℓ=1,...L) and denote the inflows in week t of year ℓ by $F_h^{\ell t}$ (controllable) and $A_h^{\ell t}$ (tributary). The resultant storage levels become $s_h^{\ell t}$, and the release $q_h^{\ell t}$, producing energy $g_h^{\ell k}$ in each segment of the week.

Each of the L sequences is considered to be equiprobable. So our objective will be to find a desired trajectory, x_h for each valley h, so as to minimize the expected value, over the L years, of the thermal costs which would be incurred to make up the deficit in the supply for year ℓ if each valley were managed so as to conform as closely as possible to x_h, under the streamflow conditions of year ℓ.

So we wish to find: g_T^{ℓ} for ℓ=1,...L and x_h for h=1,...H, so as to minimize the expected cost of thermal generation while satisfying all constraints. We take as objective function:

$$\frac{1}{L} \sum_{\ell=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} C^t(g_T^{\ell tk}) \tag{F-0}$$

And, as constraints, that:

Demand is satisfied:

$$g_T^{\ell tk} + \sum_{h=1}^{H} g_h^{\ell tk}(x_h^1, \ldots x_h^t) \geq D^{tk} \quad \text{for all } \ell=1,\ldots L,\ t=1,\ldots T,\ k=1,\ldots K. \tag{F-1}$$

Initial storage is fixed:

$$x_h^0 = S_h^0 \qquad \text{for all } h=1,\ldots H \tag{F-2}$$

Desired final storage is fixed:

$$x_h^T = S_h^T \qquad \text{for all } h=1,\ldots H \tag{F-3}$$

Storage in all other periods is feasible:

$$\underline{S}_h^t \leqslant x_h^t \leqslant \bar{S}_h^t \qquad \text{for all } h=1,\ldots H \quad t=1,\ldots T-1 \qquad \text{(F-4)}$$

Thermal production is feasible:

$$g_T^{\ell tk} \geqslant 0 \qquad \text{for all } \ell=1,\ldots L \quad T=1,\ldots T \quad k=1,\ldots K. \qquad \text{(F-5)}$$

Our global problem then is to minimise (F-0) subject to (F-1) to (F-5).

Let us assume the following (apparently reasonable) hypotheses:

(i) The thermal cost function is convex and the hydro production functions are concave.

(ii) The sets of permissible trajectories, x_h, and thermal generating levels, g_T, are convex and there is some feasible solution.

(iii) The solution set is bounded.

Then minimizing the objective (F-0) subject to the constraint (F-1) under the conditions (F-2) to (F-5) can be shown ([8]) (via Karlin's Theorem) to be equivalent to finding the saddle point of the Lagrangian $\mathcal{L}_F^3$ under conditions (F-2) to (F-5). Where $\mathcal{L}_F^3$ is defined as:

$$\mathcal{L}_F^3(\lambda, g_T, x_1, \ldots x_H) = \frac{1}{L} \sum_{\ell=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} [C^t(g_T^{\ell tk}) - \lambda^{\ell tk}(g_T^{\ell tk} + \sum_{h=1}^{H} g_h^{\ell tk}(x_h) - D^{tk})] \qquad \text{(F-6)}$$

Where $\lambda^{\ell tk}$ is the Kuhn-Tucker multiplier on the constraint (F-1) corresponding to the k^{th} segment of the t^{th} week of data year ℓ, and hence represents the price of energy delivered in that segment. Uzawa's algorithm ([64]) may be used to solve such saddle point problems, that is, to find $\tilde{\lambda}$, $\tilde{g}_T$ and $\tilde{x}$ ($\tilde{x} = \tilde{x}_1, \ldots \tilde{x}_H$) such that:

$$\mathcal{L}_F^3(\lambda, \tilde{g}_T, \tilde{x}) \leq \mathcal{L}_F^3(\tilde{\lambda}, \tilde{g}_T, \tilde{x}) \leq \mathcal{L}_F^3(\tilde{\lambda}, g_T, x)$$

for all λ, g_T, x satisfying (F-2) to (F-5). Applied to this problem the procedure is:

1. Set: n=0.

   Initialize: $\lambda(0) = \lambda$ (arbitrary)

2. Solve the Lagrangian problem:

$$\text{Find } \underset{g_T, x}{\text{MIN}}\ \mathcal{L}_F^3(\lambda(n), g_T, x) \tag{F-6$'$}$$

subject to (F-2) to (F-5)

To get $\tilde{g}_T, \tilde{x}, \tilde{g}_h(\tilde{x}_h)$ for all h=1,...H.

Let:

$$g_T(n) = \tilde{g}_T \tag{F-7}$$

$$x(n) = \tilde{x} \tag{F-8}$$

$$g_h^\ell(n) = \tilde{g}_h^\ell(\tilde{x}_h(n)) \qquad \text{for all } h=1,\ldots H,\ \ell=1,\ldots L \tag{F-9}$$

3. Let

$$\lambda^{\ell tk}(n+1) = \text{MAX}\{0, \lambda^{\ell tk}(n) - \theta(g_T^{\ell tk}(n) + \sum_{h=1}^{H} g_h^{\ell tk}(n) - D^{tk})\} \tag{F-10}$$

(where the maximization is designed to avoid negative 'prices' and θ is chosen by experience).

4. If $\lambda(n+1) \neq \lambda(n)$

   then let n=n+1 and GO TO (2),

   otherwise STOP.

   Convergence is assured, under the conditions (i) - (iii) above, given a good choice of θ in Step (3). The smaller θ is the better the prospect of a solution but more iterations are required.

The major difficulty in the above is in Step (2):

$$\text{Find } \underset{g_T,x}{\text{MIN}}\ \mathcal{L}_F^3(\lambda(n), g_T, x) \tag{F-6'}$$

Subject to (F-2) to (F-5)

But ([8]) under the conditions (i) - (iii) above this problem is separable into a thermal problem (PL_0) and local hydro problems (PL_h) of the following forms:

$$(PL_o)\ \text{Find } \underset{g_T}{\text{MIN}}\ \frac{1}{L} \sum_{\ell=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} C^t(g_T^{\ell tk}) - \lambda^{\ell tk} g_T^{\ell tk} \tag{F-11}$$

$$\text{Such that: } g_T^{\ell tk} > 0 \text{ for all } \ell = 1, \ldots L \quad t=1,\ldots T \quad k=1,\ldots K \tag{F-5}$$

$$\underset{h=1,\ldots H}{(PL_h)} \quad \text{Find } \underset{x_h}{\text{MAX}}\ \frac{1}{L} \sum_{\ell=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} \lambda^{\ell tk} g_h^{\ell tk}(x_h) \tag{F-12}$$

Such that:

$$x_h^o = S_h^o \tag{F-2}$$

$$x_h^T = S_h^T \tag{F-3}$$

$$\underline{S}_h^t \leqslant x_h^t \leqslant \bar{S}_h^t \quad \text{for all } t=1,\ldots T-1 \tag{F-4}$$

The solution of PL_0 is an easy task - we merely set

$$\frac{\partial C^t}{\partial g_T^{\ell tk}}(g_T^{\ell tk}) = \lambda^{\ell tk} \tag{F-13}$$

(This step has been included in P3 although P4 is intended to solve this problem in a more sophisticated way - taking more account of short-term factors).

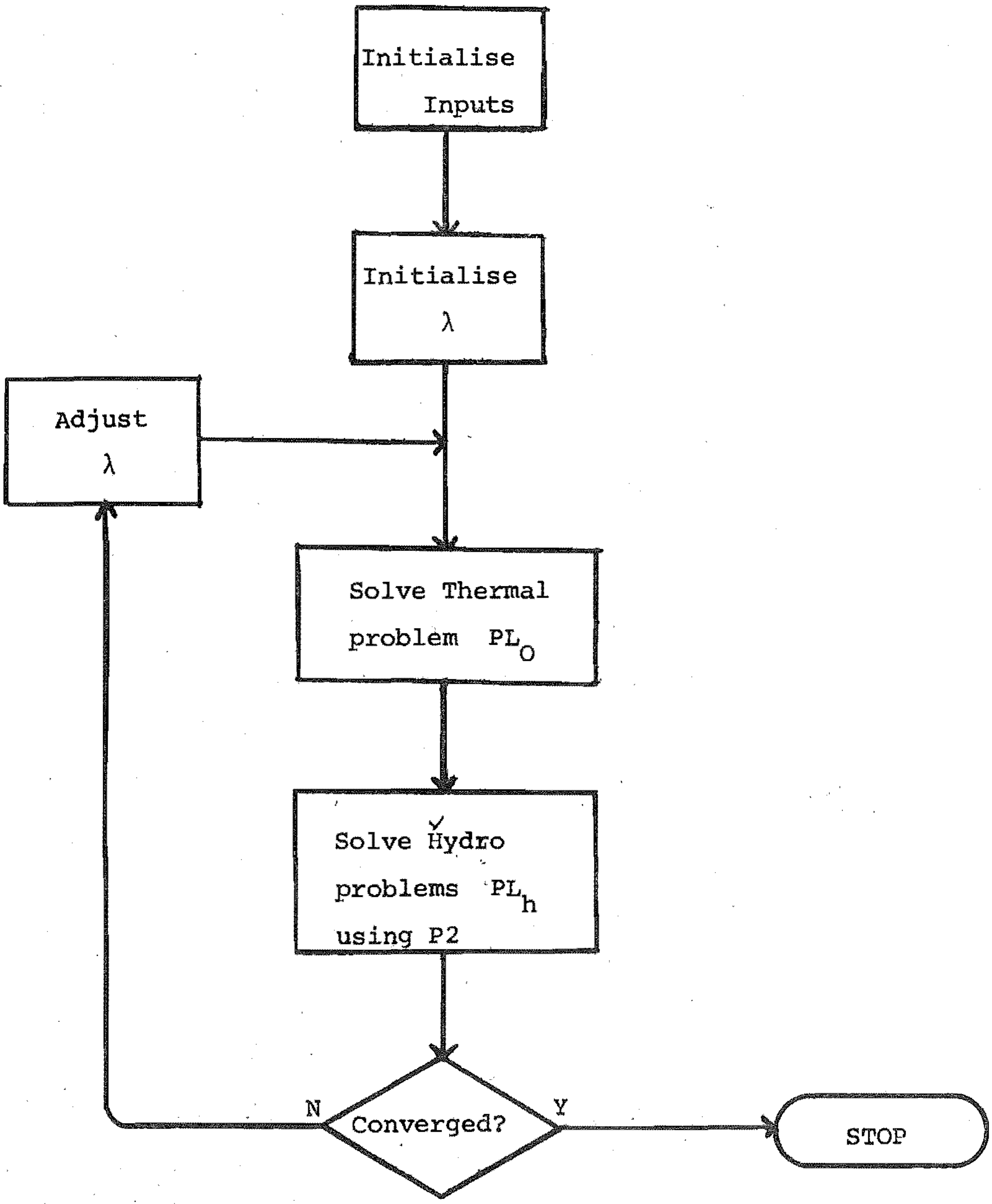


FIGURE (A-1): Flow chart for P3

The PL_h (h=1,...H) problems are solved by P2 which finds the optimal "desired" trajectory given the set of prices λ^{tk}.

The global algorithm works by adjusting the 'prices' ($\lambda^{\ell tk}$) in Step (3) so as to adjust the supply to match the demand, achieving equality at the optimum. Thus we have a model which decentralizes the production decisions by means of a price mechanism.

In the next section we discuss the solution of the local problems using P2. A flow chart for P3 is shown in Figure (A-1) (cf. Figure (2-6)).

A.3.3 The Local (Long-term) Hydro Program - P2

P2 determines the optimal generation schedule for a river system ([7], [20]). Since the non-controllable tributary streamflows have to be processed or else will go to waste the main task of P2 is to determine the optimal release schedule from the top reservoir. It thus performs a similar role to that of the trajectory method discussed in Section 5.4.3. In fact one experimental version of P2 is based on the trajectory method ([20]). However the version proposed to be part of the SGEP production system is based on optimal control theory, although the solution method is of a more conventional form. This approach facilitates dealing with more than one reservoir, either in parallel or in series ([7],[56]). The current version of P2 can handle at most two reservoirs.

P2 is supposed to be run once a week, with only the first period optimal decisions to be implemented. Its planning horizon is 52 weekly periods. The objective function is to maximise the expected value of the energy generated by the reservoir releases. The decision variables are not, however, the releases, but the change in the reservoir levels. This implies that the amount of generation becomes a random variable in response to the random nature of the reservoir inflows.

P2 uses as input:

- the curves showing the energy produced during each load segment by a weekly release as a function of the tributary streamflow index and the head of the top hydro station. (These are the output of P1.)
- a list of 20 years of weekly reservoir inflows and the value of the corresponding tributary streamflow index.
- a list of weekly average energy prices for each load segment which, in theory, should be a function of the tributary streamflow index. In practice, since the streamflows are not represented by probability distributions but by a sample of actual observed streamflows over 20 years, these weekly energy prices are specified in terms of a one-to-one correspondence with the streamflow data for a 20-year period.

Note that it is possible to incorporate streamflow predictions into these 20 years of data. For instance, if current and predicted weather conditions indicate that the streamflows during the coming two weeks are likely to be larger by a certain percentage than the average reservoir inflows and tributary streamflows for the corresponding two weeks in the data, the entries in the data are adjusted accordingly while maintaining the shape of their distribution.

On the basis of these data, P2 determines one optimal desired trajectory defined by the desired reservoir levels at the beginning of each week in the 52-week planning horizon. This trajectory originates from the current reservoir level at the beginning of the planning horizon and is constrained to meet the target reservoir level at the end of the planning horizon. Physical or policy imposed constraints on the reservoir level or the throughput at the top hydro station may prevent following the desired trajectory for any one of the 20 years of reservoir inflows taken as representative of the random nature of these inflows. Minimum levels are imposed so as to provide some protection against not reaching the target reservoir level at the end of the planning horizon, while constraints on the maximum levels are imposed so as to reduce the chance of spilling. The actual trajectory is simulated for each one of the 20 data years. If the reservoir inflows for any given year are such that the optimal trajectory cannot be followed, the program attempts to follow it as closely as possible within the operating constraints imposed. The energy produced for each load segment in each week is evaluated at the corresponding energy price. The average of these 20 simulations is used as the value of the objective function.

The method of solution used is an iterative primal-dual algorithm by Uzawa ([64]). Starting with an initial trajectory specified as input into the program, the trajectory is displaced at each iteration in response to the values of the dual variables. At each iteration the primal problem is solved by a steepest ascent method, then the dual variables are adjusted so as to move the trajectory towards a saddle point where the expected marginal value of water stored is equal to that of the water released in each period. Computer run times for this optimization by P2 on an IBM 370/165 system amount to between 5 and 10 seconds per river system.

The program gives as part of its final output for use in P3 the hydro generation of the river system for each load segment and each week of the 20 data years of streamflows and energy prices.

One version of P2 ([20]), mainly used for separate detailed analysis of the operation of a river system to evaluate the effect of changes in equipment, changes in the volume of controllable streamflows, maintenance scheduling, or the optimal strategies in the face of exceptional weather conditions (e.g. extremely dry winters), produces isograms for the water values over time, which can be used as scheduling aids in simulations of the river system.

The remainder of this section gives mathematical details of the approach taken in P2.

For each valley, $h(h=1,\ldots H)$ we wish to solve problem PL_h, that is to find a desired trajectory x_h, so as to maximise the value of production from the valley if it were managed so as to follow that trajectory.

Thus we have as objective function:

$$\text{MAX} \sum_{\ell=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} g_h^{\ell tk}(x_h)\lambda^{\ell tk} \tag{F-12}$$

And as constraints:

$$x_h^0 = s_h^0 \tag{F-2}$$

$$x_h^T = s_h^T \tag{F-3}$$

$$\underline{s}_h^t \leqslant x_h^t \leqslant \bar{s}_h^t \quad \text{for all } t=1,\ldots T-1 \tag{F-4}$$

Where the vector of energy prices, λ, is specified by P3.

From now on we shall be dealing with only one valley so the subscript h will be dropped. There may, however, be more than one controllable seasonal reservoir in the valley. Let these reservoirs be indexed by $i=1,\ldots I$, where $i=1$ denotes the top reservoir.

Hence $x^t = x_h^t = (x_{h1}^t,\ldots x_{hI}^t) \qquad t=1,\ldots T$

and so forth for s^t, q^t, and u^t.

Now, if we let $\bar{Q}^t$ and $\underline{Q}^t$ be the maximum and minimum releases allowed in period t, we must modify the desired trajectory, x, so as to conform to these. We try to keep as close as possible to x, and so, for a particular inflow $F^{\ell t}$ and storage level $s^{\ell t}$, we can obtain the actual release, $q^{\ell t}$, for year ℓ as a function of the desired storage level, x^t, and the desired decrease in storage level u^t.

$$q^{\ell t}(x^{t-1},u^t) = \text{MIN}\ \{\bar{Q}^t, \text{MAX}\{s^{\ell t-1}+F^{\ell t}-(x^{t-1}-u^t),\underline{Q}^t\}\}$$

$$\text{for all } \ell=1,\ldots L \qquad t=1,\ldots T \tag{F-14}$$

Where:

$$u^t = x^{t-1} - x^t \qquad \text{for all } t=1,\ldots T \qquad \text{(F-15)}$$

(These u variables are used as control variables).

From this release, $q^{\ell t}$, the energy, $g^{\ell tk}$, produced in each time segment k of this data week, is given as a function of the release and the storage level:

$$g^{\ell tk} = g^{\ell tk}\,(s^{\ell t},q^{\ell t}) \qquad \text{(F-16)}$$

Here the function $g^{\ell tk}$ takes into account the downstream inflows for year ℓ, $(A^{\ell t})$, and the various inefficiencies, time delays and so on. This function is determined by P1. (See Section A.3.4). (There are in fact many possible production patterns for a particular release $q^{\ell t}$, but P1 determines (as best it can) the optimal one, thus $g^{\ell tk}$ is a function.) Now, since:

$$s^{\ell t} = s^{\ell t-1} + F^{\ell t} - q^{\ell t}, \qquad \text{for all } t=1,..T, \qquad \text{(F-17)}$$

and, in general: $q^{\ell t} \neq F^{\ell t} - u^t$,
then, in general: $s^{\ell t} \neq x^t$,
and in particular: $s^{\ell T} \neq x^T = S^T$.

So we need to consider the value of the water in storage in the final period in order to be able to evaluate the particular trajectory followed in year ℓ. We give this water the value σ^T per unit and discuss the choice of this value later.

Also we note that, for a given inflow sequence ℓ, the storage trajectory s becomes, by (F-17), a function of the release trajectory q which, by (F-14), is a function of the desired vectors x and u.

Thus we have:

$$q^{\ell} = q^{\ell}(x,u) \qquad \text{for all } \ell=1,\ldots L \qquad \text{(F-18)}$$

and: $$s^{\ell} = s^{\ell}(q^{\ell}) = s^{\ell}(x,u) \qquad \text{for all } \ell=1,\ldots L \qquad \text{(F-19)}$$

So we can write:

$$V(x,u) = \frac{1}{L}\sum_{\ell=1}^{L}\left[\left(\sum_{t=1}^{T}\sum_{k=1}^{K}[g^{\ell tk}(s^{\ell t},q^{\ell t})\lambda^{\ell tk}]\right) + (s^{\ell T} - S^{T})\sigma^{T}\right] \qquad \text{(F-20)}$$

The problem becomes:

$$\text{Find } \underset{x,u}{\text{MAX}}\ V(x,u) \qquad \text{(F-20)}'$$

Such that:

$$x^t = x^{t-1} - u^t \qquad \text{(F-21)}$$

$$x^o = S^o \qquad \text{(F-22)}$$

$$x^T = S^T \qquad \text{(F-23)}$$

$$x^t \geqslant \underline{S}^t \qquad \text{for all } t=1,\ldots T-1 \qquad \text{(F-24)}$$

$$x^t \leqslant \bar{S}^t \qquad \text{for all } t=1,\ldots T-1 \qquad \text{(F-25)}$$

(Renumbering constraints as appropriate).

If we let the multipliers on constraints (F-24) and (F-25) be γ^t and δ^t respectively, we get the Lagrangian:

$$\mathcal{L}_F^2(\gamma,\delta,x,u) = V(x,u) + \sum_{t=1}^{T-1}[\gamma^t(x^t-\underline{S}^t) - \delta^t(x^t-\bar{S}^t)] \qquad \text{(F-26)}$$

The problem above can be treated as one of finding a permissible $(\tilde{x},\tilde{u})$ and the appropriate $(\tilde{\gamma},\tilde{\delta})$ such that $(\tilde{\gamma},\tilde{\delta},\tilde{x},\tilde{u})$ is a saddlepoint of $\mathcal{L}_F^2$, i.e.,:

$$\mathcal{L}_F^2(\tilde{\gamma},\tilde{\delta},x,u) \leqslant \mathcal{L}_F^2(\tilde{\gamma},\tilde{\delta},\tilde{x},\tilde{u}) \leqslant \mathcal{L}_F^2(\gamma,\delta,\tilde{x},\tilde{u}) \qquad \text{(F-27)}$$

for all permissible γ,δ,x,u.

This can again be done by Uzawa's method ([64]) which may be stated in terms of successive primal dual iterations (cf. our approach of evaluating a "dual objective" using a "Lagrangian problem" in Section 2.3.2).

At the n^{th} iteration we have first to solve a "primal problem" (cf. our "Langrangian problem"):

$$\max_{x,u} \mathcal{L}_F^2(\gamma(n),\ \delta(n),\ x,u) \qquad \text{(F-26)}'$$

Where $\gamma(n)$ and $\delta(n)$ have been determined by the previous dual iteration and x and u satisfy (F-21) to (F-23).

From this we get: $\quad x(n) = x(\gamma(n),\ \delta(n)) \qquad$ (F-28)

and: $\quad u(n) = u(\gamma(n),\ \delta(n)) \qquad$ (F-29)

as the optimal solutions.

We then wish to solve a "dual problem", that is to adjust γ and δ so as to minimise:

$$\mathcal{L}_F^2(\gamma,\delta,x(n),u(n)) \qquad \text{(F-26)}''$$

where x(n) and u(n) have just been determined and $\gamma \geqslant 0$, $\delta \geqslant 0$.

Hence, at the $n+1^{th}$ iteration, we put:

$$\gamma^t(n+1) = \text{MAX}\{0,\gamma^t(n) - \rho(x^t(n) - \underline{S}^t)\} \qquad \text{(F-30)}$$

$$\delta^t(n+1) = \text{MAX}\{0,\delta^t(n) + \rho(x^t(n) - \bar{S}^t)\} \qquad \text{(F-31)}$$

(the maximization being designed to avoid negative multipliers) Here ρ is chosen (by experience) to be small enough to ensure convergence.

We turn our attention to the solution of the primal problem so as to determine x(n) and u(n) for each iteration.

We first use the state equation to modify the constraints and get the problem:

$$\text{Find} \quad \max_{x,u} V(x,u) + \sum_{t=1}^{T-1} [\gamma^t(n)(x^t - \underline{S}^t) - \delta^t(n)(x^t - \bar{S}^t)] \qquad \text{(F-26)}$$

Such that:

$$x^t = x^{t-1} - u^t \quad \text{for all } t=1,\ldots T \tag{F-21}$$

$$x^o = S^o \tag{F-22}$$

$$\sum_{t=1}^{T} u^t = S^o - S^T \tag{F-23)'$$

which may be solved by a method of steepest ascent.

At each iteration on the primal problem we have $\tilde{u}$ and $\tilde{x}$ as the current control and trajectory vectors and let $\tilde{V} = V(\tilde{x},\tilde{u})$. We consider displacing the control $\tilde{u}_i^t$ at the rate $\frac{\partial u_i^t}{\partial r}$ (introducing r as a dummy variable) and normalising the displacement vector by the constraint

$$\sum_{t=1}^{T} \sum_{i=1}^{I} \omega_i \left[\frac{\partial u_i^t}{\partial r}\right]^2 = 1 \tag{F-28}$$

where the 'weights' ω_i are to be determined.

We wish to find $\frac{\partial u^t}{\partial r}$ for t=1,...T, so as to maximise the increase in the objective. So we wish to maximise $\frac{dL}{dr}$ under the constraints, and thus obtain the problem:

$$\text{Find: MAX} \sum_{t=1}^{T} \frac{\partial V}{\partial x^t}\frac{\partial x^t}{\partial r} + \frac{\partial V}{\partial u^t}\frac{\partial u^t}{\partial r} + (\gamma^t - \delta^t)\frac{\partial x^t}{\partial r} \tag{F-29}$$

$$\text{Such that:} \quad \frac{\partial x^t}{\partial r} = \frac{\partial x^{t-1}}{\partial r} - \frac{\partial u^t}{\partial r} \quad \text{for all } t=1,\ldots T \tag{F-30}$$

$$\sum_{t=1}^{T} \frac{\partial u^t}{\partial r} = 0 \tag{F-31}$$

$$\sum_{t=1}^{T} \sum_{i=1}^{I} \omega_i \left[\frac{\partial u_i^t}{\partial r}\right]^2 = 1 \tag{F-32}$$

$$\frac{\partial x^o}{\partial r} = 0 \tag{F-33}$$

The decision variables for this problem are $\frac{\partial x^t}{\partial r}$ and $\frac{\partial u^t}{\partial r}$.

If we let ϕ_i^t be the Kuhn-Tucker multiplier attached to the constraint (F-30) and σ_i^T be the multiplier on the constraint (F-31) we can define ψ by:

$$\psi_i^t = \phi_i^{t-1} + \sigma_i^T \tag{F-34}$$

It is shown in [7] that the new control u can be defined by:

$$u_i^t = u_i^t(\theta) = \tilde{u}_i^t + \theta\left[\frac{\partial V}{\partial u_i^t} - \psi_i^{t+1}\right] \tag{F-35}$$

for $t=1,\ldots T$

$i=1,\ldots I$

(i.e. $\omega_i = -\frac{\Delta r}{2\varepsilon\theta}$ for all $i=1,\ldots I$ where ε is the multiplier on (F-32)).

Here $\theta > 0$ must be chosen to be small enough so that the value of the objective is strictly increased. The trajectory x is then easily determined from the state equation.

But what are these multipliers ψ_i^t, and how do we determine them or $\frac{\partial V}{\partial u_i^t}$? It is shown in [7] that in fact:

$$\begin{aligned}\psi_i^{t+1} &= \phi_i^t + \sigma_i^T \\ &= \sum_{r=t}^{T}\left[\frac{\partial V}{\partial x_i^r} + \gamma_i^r - \delta_i^r\right] + \sigma_i^T \end{aligned}\tag{F-36}$$

for $i=1,\ldots I$.

Thus, apart from σ^T (to be discussed later), the quantities to be determined are:

$$\left.\frac{\partial V}{\partial x_i^t}\right|_{(\tilde{x},\tilde{u})} \quad \text{and} \quad \left.\frac{\partial V}{\partial u_i^t}\right|_{(\tilde{x},\tilde{u})}$$

Recall that:

$$V(x,u) = \frac{1}{L}\sum_{\ell=1}^{L}\left[\left(\sum_{t=1}^{T}\sum_{k=1}^{K} g^{\ell tk}(s^{\ell t},q^{\ell t})\lambda^{\ell tk}\right) + (s^{\ell T}-S^{T})\sigma^{T}\right] \tag{F-20}$$

So, for a given trajectory $(\tilde{x},\tilde{u})$, we may determine for each streamflow year ℓ, s^{ℓ} and q^{ℓ}. From P1 we may obtain curves relating production in the various segments and productivity of releases to the volume of water released, q, the head of the top station, s^{ℓ}, and the downstream inflows, A^{ℓ}, all of which are known. So we may, for each ℓ, read off these curves $\frac{\partial g^{\ell tk}}{\partial q^{\ell t}}$ and $\frac{\partial g^{\ell tk}}{\partial s^{\ell t}}$ and hence obtain $\left.\frac{\partial V}{\partial x}\right|_{(\tilde{x},\tilde{u})}$ and $\left.\frac{\partial V}{\partial u}\right|_{(\tilde{x},\tilde{u})}$.

Thus all the information required to solve the primal problem at each iteration is available.

These (primal) iterations are repeated until no further improvement is possible. We then return to the dual problem for another iteration of Uzawa's algorithm.

Note that, in (F-36), $\frac{\partial V}{\partial x_i^r}$ represents the marginal gain from having a greater head in reservoir i for period r while γ_i^r and δ_i^r are the multipliers (current for this iteration of Uzawa's algorithm) on the minimum and maximum storage constraints. Thus the expression, $[\frac{\partial V}{\partial x_i^r} + \gamma_i^r - \delta_i^r]$ represents the marginal gain in period r from an increase in storage. An increase in storage in period t would have an effect on all future periods if the original control trajectory, $\tilde{u}$, were to be followed from then on. The marginal gain from storing water in period t is the sum of the gains over all future periods plus the final value of water σ^T. So ψ_i^t is the marginal value of water in storage at time t.

When convergence is achieved we have:

$$\frac{\partial V}{\partial u_i^t} = \psi_i^t \qquad \text{for all } i=1,\ldots I \quad t=1,\ldots T \tag{F-37}$$

So the marginal value of the release in period t is equal to the marginal value of the water stored at the end of period t. This solution is similar to that derived by an equimarginal water value method (if no constraints are violated).

Alternatively, we may define R_i^t, the "marginal rent" for reservoir i for period t, by:

$$R_i^t = \frac{\partial V}{\partial u_i^t} - \psi_i^t \qquad \text{for all } i=1,\ldots I \quad t=1,\ldots T \tag{F-38}$$

Observe that the (primal) algorithm works to reduce all these to a zero level.

Convergence of this entire iterative primal-dual method (i.e.P2) is again assured by conditions (i) - (iii) on page 358. When it is achieved we have the optimal trajectory for this valley given the energy prices set by P3. When all valleys and the thermal sector have been optimised, control is returned to P3 to re-adjust the prices to better match supply to demand.

Finally we consider the estimation of $\sigma^T (=\psi^T)$. It seems reasonable to assume that $\psi^T = \psi^o$ so we adopt the following scheme:

1) Pick initial value for σ^T.
2) Do optimisation (i.e., whole of Uzawa's algorithm)
3) If $\sigma^T = \psi^T = \psi^o$ STOP,
   otherwise put $\sigma^T = \psi^o$ and GO TO (2).

In fact, if P2 were run once a week we would expect little change in σ^T so that last week's ψ^o would become this week's σ^T.

A flow chart of the whole of P2 is shown in Figure (A-2).

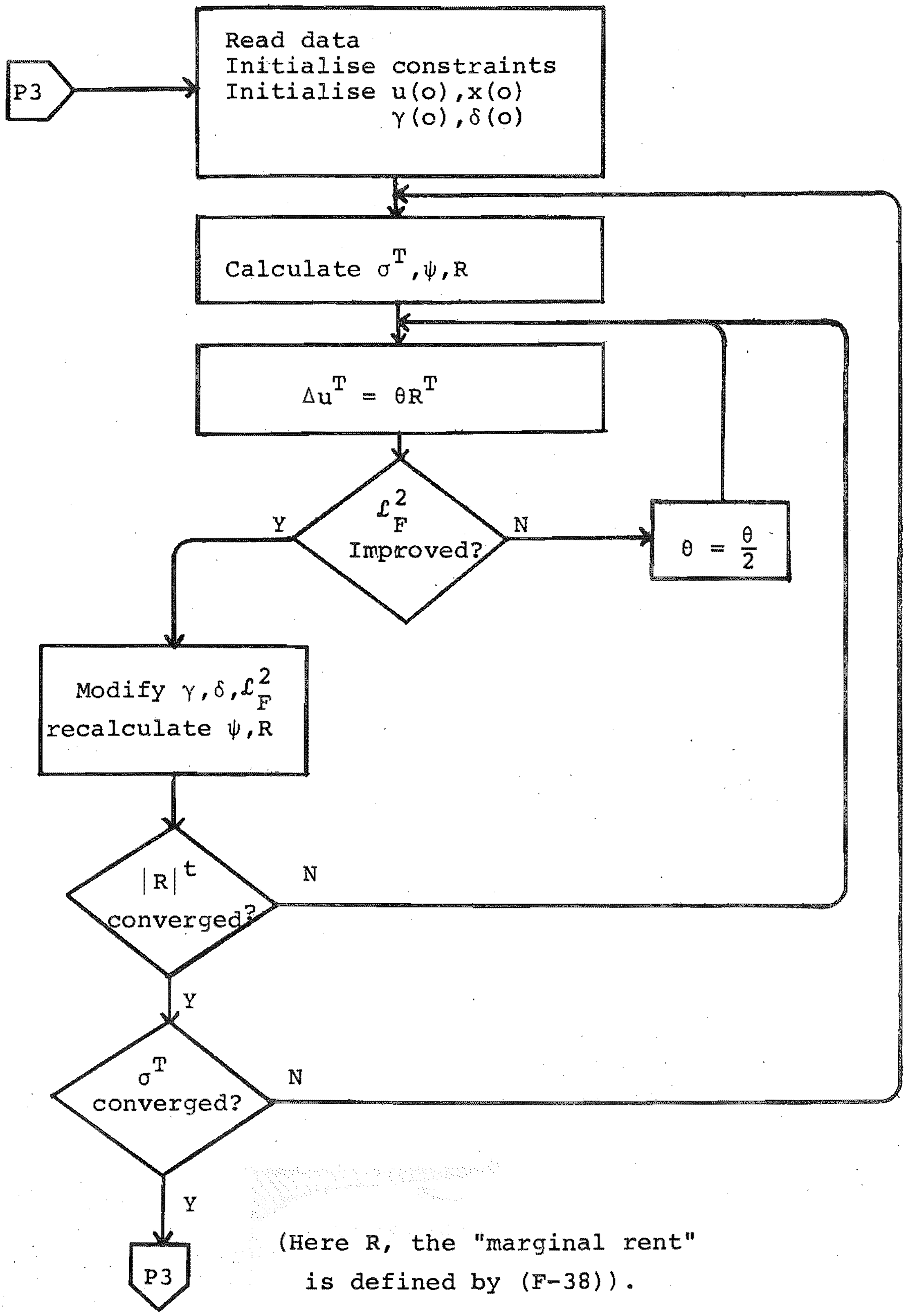
In Section 9.3.2 we consider the implications of the P2 approach (referred to as EDF Scheme II). There we concentrate on the assumption that, in the future, optimal management of each hydro reservoir will involve attempting to follow a fixed "desired trajectory". In the next section we turn our attention to the short-term hydro sub-program, P1, used to prepare input tables for P2.

A.3.4 The Short-Term Hydro Sub-Problem - P1

P1 determines the energy produced in a river system as a function of the non-storable tributary streamflows and the weekly release from the main hydro reservoir (assumed to be upstream of the first power station)([18],[19]). All storable streamflows are assumed to be optimally placed during the week so as to maximise the total value of the energy produced.

The program makes the following assumptions:

(a) The curve of hourly energy prices (or price duration curve) is as shown in Figure (A-3). In order to define the curve the intermediate and peak segments are amalgamated into one segment henceforth referred to as normal. The shape of the curve for each day of the week is assumed constant under all conditions and so all the prices may be considered as functions of one reference price for the day (λ^j for day j), in this case the average price in the normal segment. The price duration curve during the normal segment is assumed to slope so that energy delivered in the highest



(Here R, the "marginal rent" is defined by (F-38)).

FIGURE (A-2): Flow chart for P2.

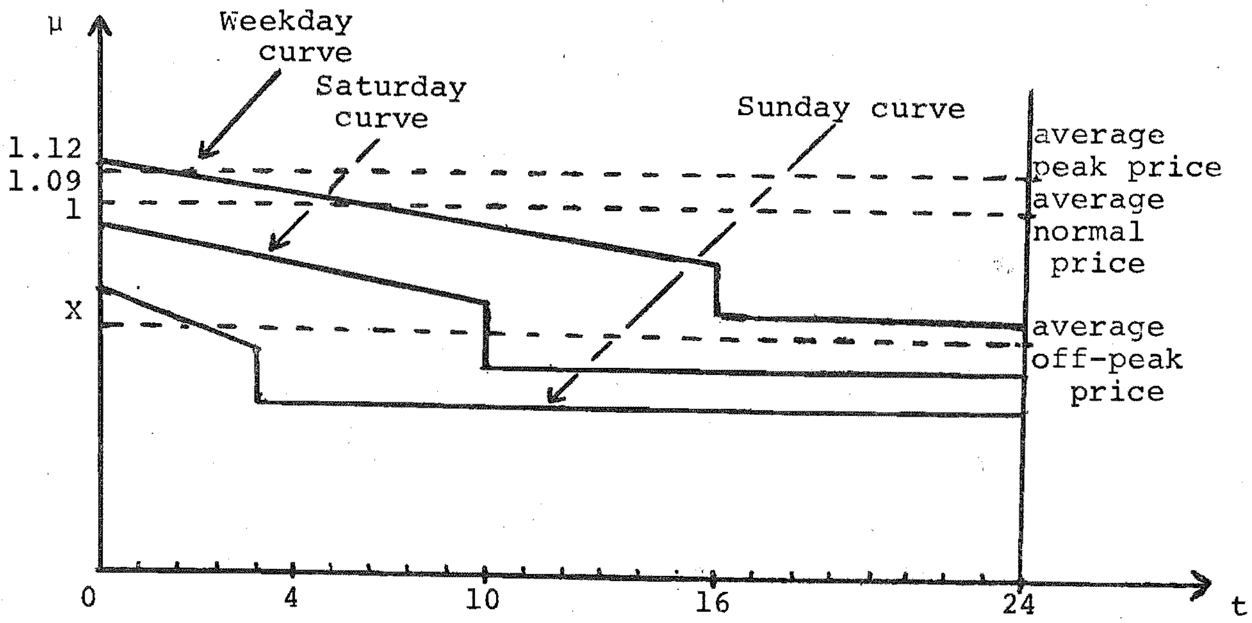


FIGURE (A-3): Price duration curve.

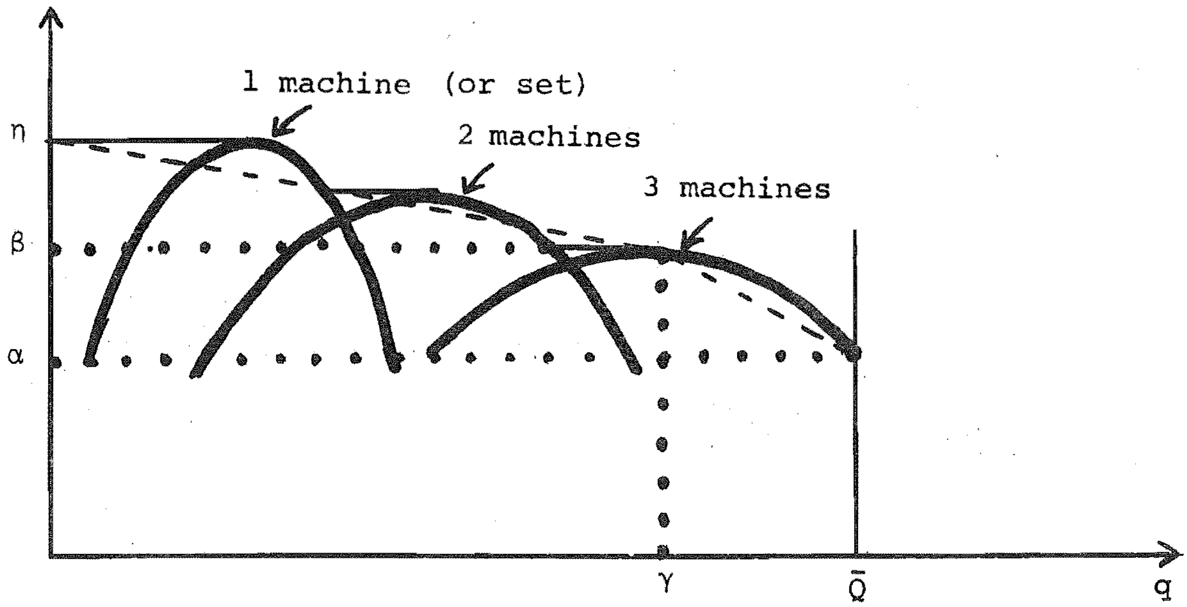


FIGURE (A-4): Generation efficiency curve

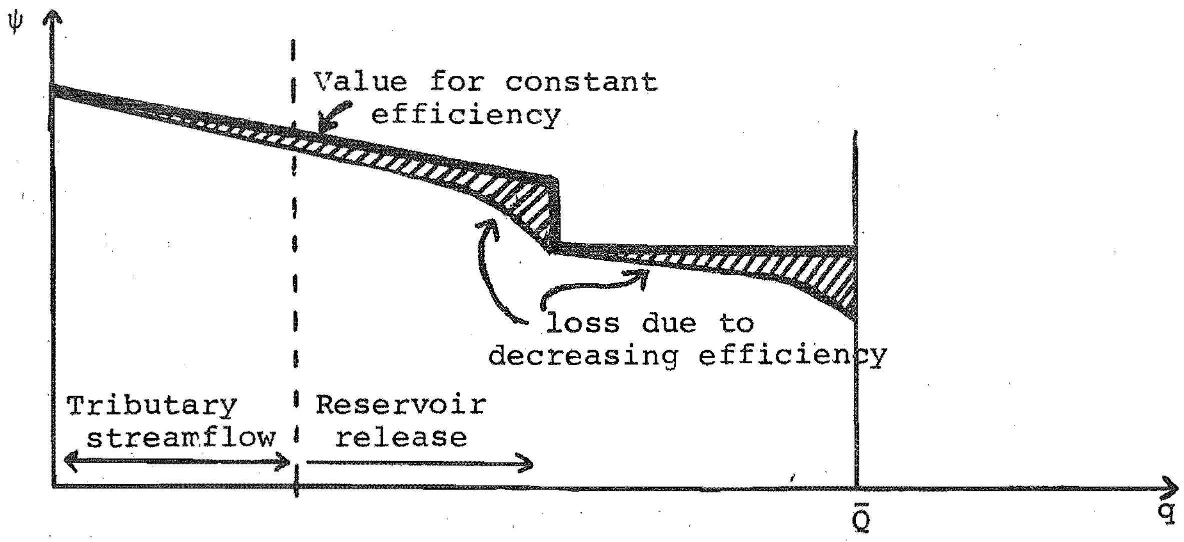


FIGURE (A-5): Marginal value of production from release peak hour of the week is valued at $1.12\lambda^j$. The average price of energy delivered in the off-peak segment is assumed to be $X\lambda^j$. (The values of X used so far are 0.7 and 0.85) and the curve slopes over the off-peak segment so that the highest price in this segment is $1.04\ (X\lambda^j)$. Actually the λ^j are not specified for each day, but we are given an average price λ for energy in the normal segment of a weekday. So $\lambda^j = \lambda$ for $j = 1,...5$. We then assume

$$\lambda^6 = \frac{(1+X)\lambda}{2} \tag{F-39}$$

$$\lambda^7 = X\lambda. \tag{F-40}$$

We also specify the number of hours in each day which are considered to be part of each segment.

	Peak	Intermediate	Off-peak
Weekdays	4	12	8
Saturday	-	10	14
Sunday	-	6	18

(b) The tributary streamflows for each week over the entire river system downstream of the main reservoir are expressed as linear functions of one streamflow index. If the tributary streamflows are not homogeneous, different parts of the river system may have a different index. No more than two indices have been used so far. The factors of proportionality have been determined on the basis of 20 years of historical streamflows.

(c) The tributary streamflows are assumed to occur at a constant rate during the entire week.

The marginal value of a given daily release from the main hydro reservoir is determined separately for each hydro station in the river system, by taking into account all physical constraints of the plants as well as minimum streamflow requirements.

Consider first the top hydro station immediately below the main reservoir. If it has several generating sets the marginal energy output for a given head depends on the number of sets in operation, as indicated by the solid parabolas in Figure (A-4). However the actual marginal output for each number of sets is given by a horizontal line at the level of the highest marginal output for that number of sets, since it is always possible to run the sets for only part of the day at optimal level. There are thus as many horizontal segments as there are sets. P1 approximates the actual marginal output for each head by at most two linear segments that can be defined by 4 parameters, as shown in Figure (A-4), where η is the maximum efficiency for a given head.

The first cubic meter released is considered to be processed at maximum marginal output and also valued at the maximum price for peak time. Each additional cubic meter released is processed at a progressively lower marginal output and also valued at a progressively lower price. The marginal value of a release is thus obtained as a product of the marginal output and the associated energy price. Since both the marginal output and the marginal energy price are decreasing, this product is decreasing as shown in Figure (A-5). This curve is shown in terms of the average price for normal load standardized to unity.

The releases are also processed by all the downstream hydro stations. These can be grouped as follows:

(a) River stations with sufficiently large regulation ponds to allow the generation to be scheduled when it is most valuable, i.e., as much as possible during peak times. These stations are analysed in the same way as the top reservoir station except that no head effect is present. However all tributary streamflows are assumed to be processed first. The marginal value of the release is thus obtained by a displacement on the q axis equal to the sum of all upstream tributary streamflows.

(b) Run-of-river stations that are directly influenced by the volume and pattern of release of the preceding station with a minimum of elapsed time have their output valued in the same time pattern as the preceding station, except that any tributary streamflows are again assumed to be processed first.

(c) Run-of-river stations that are sufficiently far away from the preceding station that a release from the preceding station is unlikely to affect the timing but only the volume of generation are assumed to process the release uniformly over the hours of the day, except that all tributary streamflows are again assumed to be processed first.

It is thus possible to find for each station a marginal value curve for the throughput water which is then displaced to the left by an amount equal to all upstream tributary streamflows, as shown in Figure (A-6). The residual curve gives the marginal value of the release from the main reservoir. The marginal value of the release for the entire river system is found by summing the residual curves of all stations. This is done separately for each day of the week, resulting in a set of curves as shown in Figure (A-7).

According to economic principles an optimal weekly schedule of releases requires the marginal value, ψ, of the daily release to be equal during the entire week. We can thus associate with each marginal value ψ a total weekly release q, given by the sum of the corresponding daily releases. By allowing ψ to run over all values we can derive a relationship between ψ and q as a function of the streamflow index and the head at the top station. Figure (A-8) shows the general shape of these curves for various values of the streamflow index for a fixed head.

The energy produced over the three daily load segments as a function of the release, the streamflow index, and the head of the top station can be computed accurately at the same time as the marginal value curves are constructed. P1 uses a functional approximation method that slightly overextimates the power output during periods of high streamflows, but gives good estimates during normal and low flows. This forms the input into model P2.

The reliability of these curves in terms of supplying sufficiently accurate estimates of the amount of energy produced by a river system is highly dependent on how well the various intermediate streamflows and the reservoir inflows can be expressed as a function of one or two streamflow

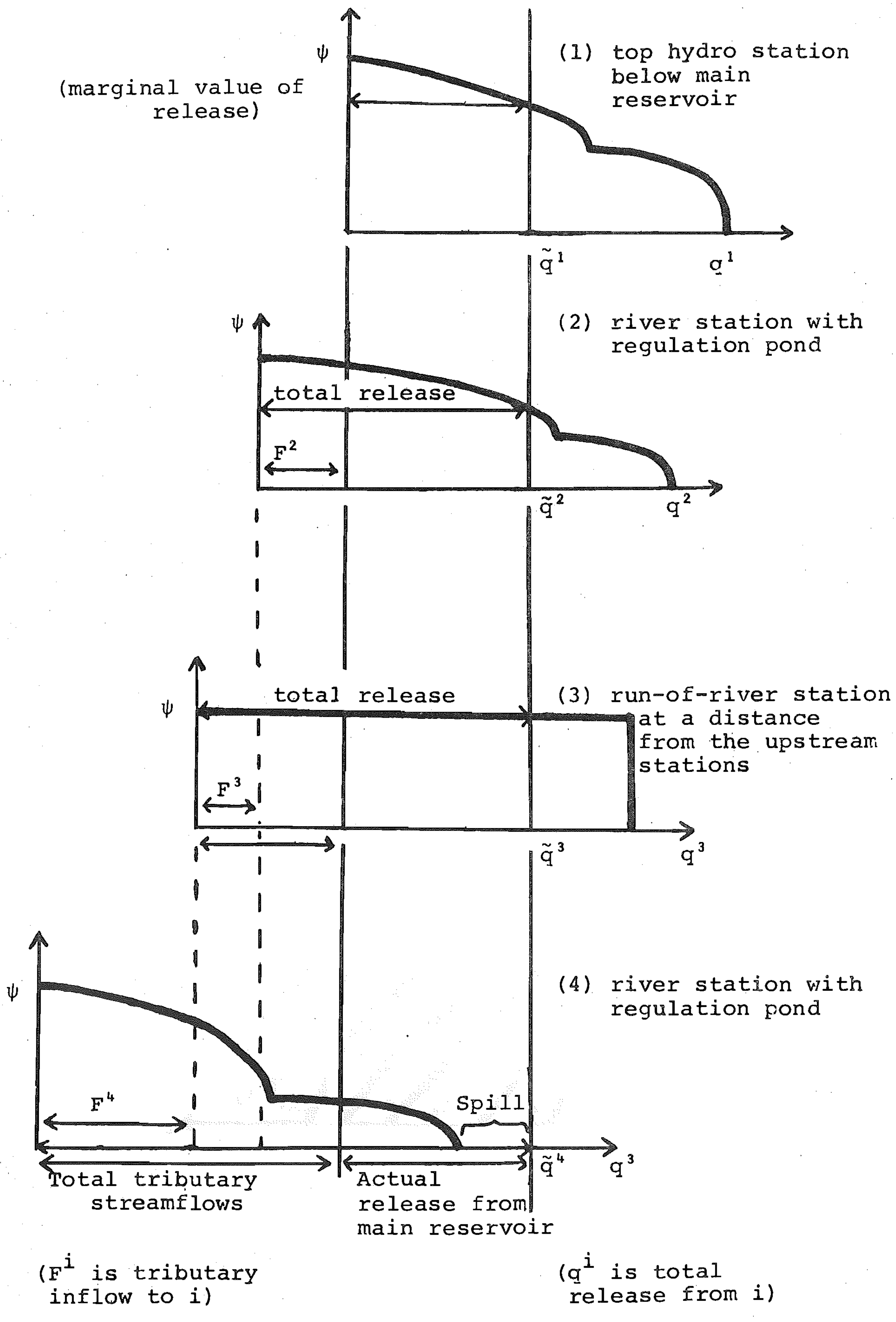
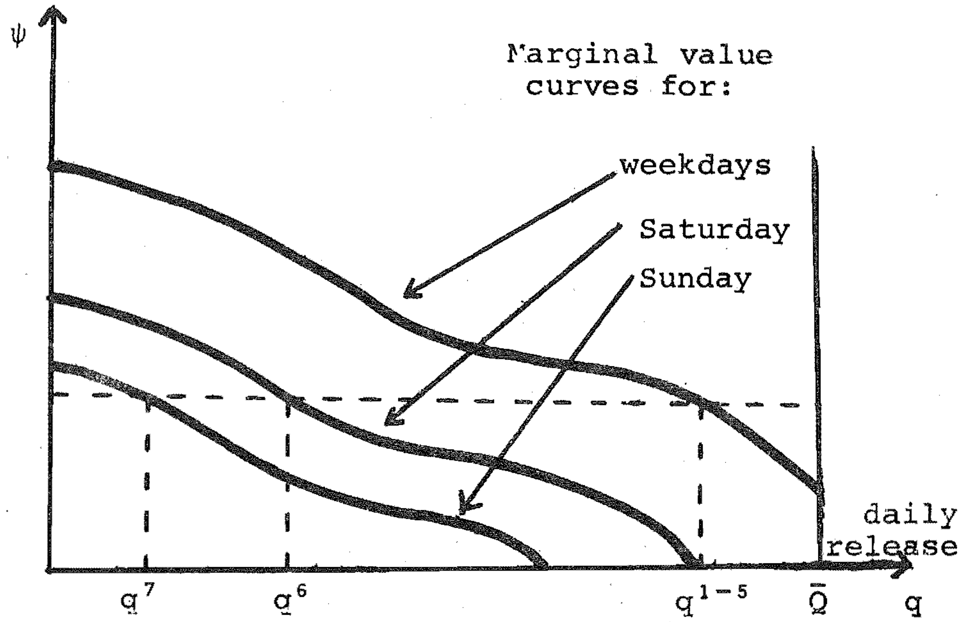


FIGURE (A-6): Treatment of downstream stations



(ψ is marginal value of release (from whole valley) for a given inflow index)

FIGURE (A-7): Determination of weekly release

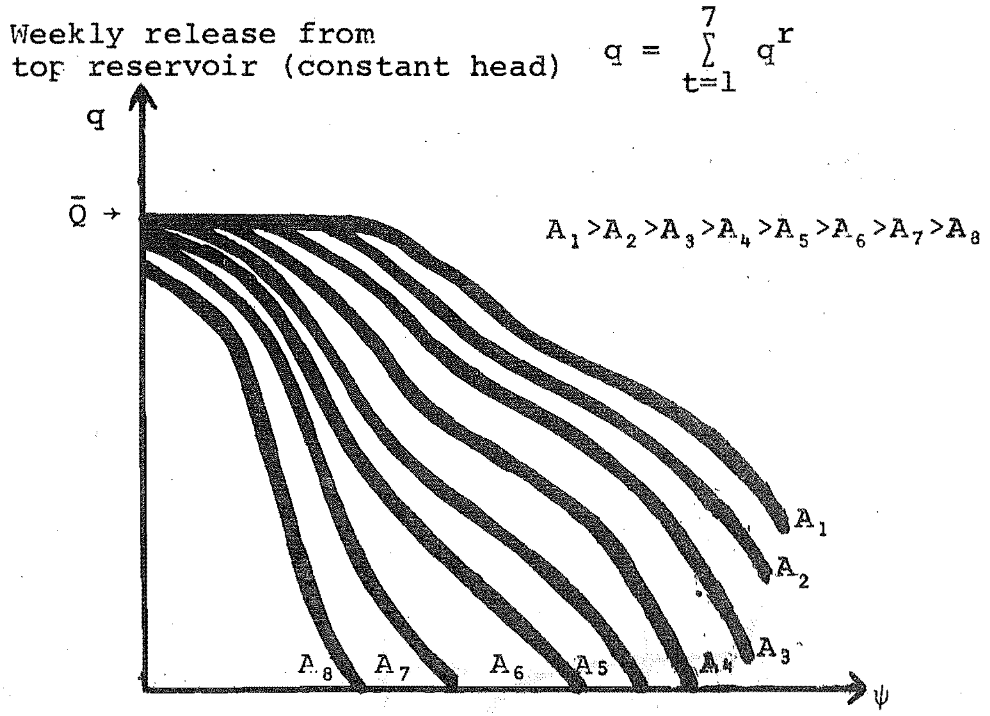


FIGURE (A-8): Typical curves for $q(\psi)$.

indices and on the variability of the streamflows during the week. Recall that the model assumes that the streamflows occur uniformly over the entire week. It is likely that these approximations are better for river systems that receive a major portion of their runoffs from snow-melt than for those mainly subject to rainfall runoffs. Systematic tests performed for a particular French river system indicate that under very unfavourable conditions the actual average production realized can deviate from the one estimated by the model by as much as 10% ([11]). The program is run for each river system modelled individually in the SGEP system. Its output is assumed valid as long as no major changes occur in the equipment or in the constraints of a river system. P1 can be used to advantage independently of SGEP for detailed analysis of an individual river system or parts of it.

In view of the detail required to obtain a valid representation of a river system, preparing each P1 run is a major investment in terms of man-hours. Each analysis of a river system requires a total of several man-months by operating people and mathematical analysts. For no other program in the SGEP system is it so important that a high degree of accuracy is achieved.

A.4 CONCLUSIONS

In this appendix we have outlined two systems proposed for application at EDF. A fuller description may be found in [17]. These systems share a similar basic philosophy which, for the reasons outlined in Section 1.3

we have chosen as the basis for our own model. Specifically, we have taken the "trajectory method" used in the GRAF system and used it to solve the long-term hydro sub-problems resulting from a SGEP type decomposition. We have further adapted this system by generalising its handling of the transmission network, short-term requirements and stochastic inflows. These adaptations are outlined in Section 1.5.

APPENDIX B

A SUMMARY OF NOTATION

We include here a summary of the notation used in this thesis. We list first the basic notation, introduced in Chapter 2. Then we list the extra notation used in each chapter. Unfortunately, the complexity of the model has required that some symbols be used to indicate different entities in different contexts. However it should always be clear from the context which meaning is intended. In general small letters indicate variables and capital letters constants or functions. Upper and lower bars indicate upper and lower limits, while ~ and * indicate optima. Superscripts indicate time, while subscripts indicate place (and, perhaps type). In general, where there is no possibility of confusion, we indicate vectors by omitting the appropriate subscripts (or superscripts). For example:

$$q_h = (q_j)_{j\varepsilon h}$$

$$= (q_j^t)_{j\varepsilon h}^{t=1,\ldots T}$$

$$= (q_j^r)_{j\varepsilon h}^{r\varepsilon t,t=1,\ldots T} \quad .$$

Equations have been labeled in accordance with the model or sub-model with which they are associated. The following table may be helpful.

LABEL	MODEL	SECTION WHERE INTRODUCED
(A-)	Aggregate Model	2.4
(C-)	Complete Model	2.2
(D-)	Dual Problem	6
(E-)	Exchange Problem	4
(F-)	EDF Model	Appendix A
(H-)	Hydro Sub-Model	5
(P-)	Economic Implications	10
(S-)	Stochastic Theory	8.2
(SA-)	Stochastic Aggregate Model	8.3
(SH-)	Stochastic Hydro Sub-Model	8.3.5
(T-)	Thermal Sub-Model	3

BASIC MATHEMATICAL NOTATION

∇	gradient
ε	is an element of
$\cup$	union
$\cap$	intersection
$\backslash$	set difference ($A\backslash B=\{x \mid x\varepsilon A$ and $x\not\varepsilon B\}$).
Δ	symmetric difference ($A\cup B\backslash A\cap B$)
X	cartesian product
$\sum$	summation
$<,>$	inner product
a.e	almost everywhere
a.s	almost surely
card	cardinality ($\equiv$ number of elements in)
$E\{\}$	expected value
inf	infimum
int	interior
sup	supremum
$\blacksquare$	denotes the end of the statement of a theorem

BASIC NOTATION

		Page
AA	(Assumption)	59
$b\varepsilon\mathcal{B}$	indexes thermal stations (subscript)	40
$C_i(g_T)$	cost of thermal generation	39
D_i^r	demand at i, in r	39
D^{tk}	demand in segment k, at n, in t	59
DA	aggregate dual problem	61
DC	complete dual problem	47
e_{ij}^r	energy sent from i to j in r	48
$\underline{E},\bar{E}$	limits on exchange	42
e_{nm}^{tk}	exchange between n and m in segment k of t	59
f_{ij}^r	energy received at i from j in r	42
F_h^r	inflows into valley h in r	40
g_{iT}^r	thermal generation at i in r	39
$\underline{G}_{iT},\bar{G}_{iT}$	limits on g_{iT}^r	39
G_{iF}^r	fixed generation at i in r	39
g_{iH}^r	hydro generation at i in r	40
g_n^{tk}	total generation in load segment k of t in region n	59
$h\varepsilon\mathcal{H}$	indexes hydro "valleys" (subscript)	40
$i\varepsilon\mathcal{I}$	indexes nodes (subscript)	38
$k=1,\ldots K$	indexes load segments (superscript)	57
L_{ij}^r	losses between i and j in r	42
L_{nm}^{tk}	losses between n and m in segment k of t	59
L_{nn}^{tk}	losses in n in segment k of t	59
$\mathcal{L}_A(z,\lambda)$	aggregate Lagrangian	61
$\mathcal{L}_C(z,\mu)$	complete Lagrangian	47

		Page
$n,m=1,...N$	index regions (subscript)	57
PA	aggregate primal problem	60
PA′	aggregate Lagrangian problem	61
PA″	aggregate modified Lagrangian problem	62
PAE	aggregate exchange problem	65
PAH	aggregate hydro problem	66
PAT	aggregate thermal problem	66
PC	complete primal problem	46
PC′	complete Lagrangian problem	47
PE	complete exchange problem	55
PG	complete generation problem	52
PH	complete hydro problem	55
PT	complete thermal problem	54
$P_A(\mu)$	aggregate dual objective	61
$P_C(\mu)$	complete dual objective	47
$q_h^r \varepsilon Q_h^r$	release pattern for h in r	41
$r=1,...R$	indexes instants (superscript)	38
s_h^r	storage pattern for h in r	41
$t=1,...T$	indexes periods (superscript)	57
$z \varepsilon Z$	feasible solutions for PC′	46
$\delta(\mu), \delta(\lambda)$	search direction in price adjustment	49,63
Δ	acceptable tolerance in PA	60
ε	predetermined tolerance in PC	49
λ	(aggregate) energy price	57
μ	(detailed) energy price	47
θ	step-size in DA, DC	49,63

EXTRA NOTATION

CHAPTER 3

		Page
$AC(g)$	average cost function	76
$\hat{g}$	minimum economic generation	76
$MC(g)$	marginal cost function	79
α, β, γ	coefficients for quadratic cost function	79
$\hat{\mu}$	minimum price for economic generation	78

CHAPTER 4

$h(\mu)$	net transfer function	90
$\hat{E}$	minimum economic transfer level	98
α, β, γ	coefficients for quadratic loss function	94
ρ	price ratio	90
$\hat{\rho}$	minimum price ratio for economic transfer	98

CHAPTER 5

A_j	reservoirs immediately above j	101
$\widehat{AJ}_j$	long-term reservoirs with short-term influence on j	109
$\widehat{AK}_j$	short-term reservoirs with short-term influence on j	109
d_i	total delay up to reservoir i	133
HAI, HAII	assumptions	108
$H_j^r(s_j^r)$	head at reservoir j in r	103
$j=1,\ldots J$	reservoirs in valley (subscript)	101
$j \varepsilon J$	long-term reservoirs in valley (subscript)	108
$k \varepsilon K$	short-term reservoirs in valley (subscript)	108

		Page
$\ell \varepsilon L$	long-term reservoirs in PASH (subscript)	129
$\mathcal{L}_H(g,\eta,\gamma,\sigma)$	(hydro) Lagrangian function	117
$n \varepsilon N$ $(n=1,\ldots N)$	short-term reservoirs in PASH (subscript)	129
$(\underline{P}^r_n,\bar{P}^r_n)$	range of feasible release instants for increment in PASH	135
PALH	long-term hydro problem	114
PASH	short-term hydro probelm	113
$\bar{Q},\underline{Q}$	upper and lower limits on release	102
$\hat{Q}$	maximum utilisable release	106
$\bar{S},\underline{S}$	upper and lower limits on storage	101
S^o,S^T	initial and final storage levels for reservoir	102
V^r_n	value of release of increment from n in r (in DP. solution of PASH)	134
$\hat{V}^r_n$	value of increment arriving at n in r (in D.P. solution of PASH)	143
w_k	delay time for water released from k	102
δ,γ,σ	multipliers on constraints in $\mathcal{L}_H$	117
Δ	release increment in PASH	117
ψ	marginal water value in PALH	117
π,π',π''	profit from optimal utilisation of releases	114,149

CHAPTER 6

$a = 1,\ldots A_h$	trajectory arcs for h (superscript)	180
g'^{r}_h	modified generation function	160
$H(\lambda)$	Hessian matrix for DA, DC	167
$H'(\lambda,\psi)$	alternative Hessian matrix	182
$\mathcal{L}'(z,\lambda,\psi)$	alternative Lagrangian	181
$P'(\lambda,\psi)$	alternative dual objective function	181
θ^*_{nG}	step size until corner of generation response curve	175
θ^*_{nmE}	step size until edge of exchange response curve	175

CHAPTERS 8 and 9

In these chapters we have attempted to generalise the model of the preceding chapters into the theoretical framework of [52] and [53]. Appendix C compares our notation with that of those papers. In general we have followed the notation of Chapters 2 to 7 with the following additions: (See notes at end of this list on the usage of the special symbols ^ ˇ and *).

		page
$\mathcal{D}(\xi)$	feasible region for response functions if ξ occurs	244
$d^t(\xi,\delta,\gamma)$		277
DS	stochastic dual problem	250
DSH	stochastic dual local hydro problem	276
DSM	modified stochastic dual problem	247
$E^t\{z\}$	conditional expectation given observations up to period t $(E\{z(\eta)\,\vert\,\hat{\eta}^t = \hat{\xi}^t\})$	252
F	σ field of Ξ	243
F^t	σ field of all sets of the form $((A\times[R^{\nu^{t+1}}\times\ldots\times R^{\nu^T})\cap\Xi)\Delta B$ where A is a borel set in $R^{\nu^1}\times\ldots\times R^{\nu^t}$, $B\varepsilon F$ and $\sigma(B) = 0$.	245
$f_o(\xi,u)$	objective function	243
$f_i(\xi,u)$	constraints on u	243
$f_i^t(\xi,u^t)$	constraints on u^t (in separable problem)	251
$f^*(\xi_j)$	(††)conjugate of $f(\xi_j)$	247
$g(\xi,\lambda,\rho)$		250
$g^t(\xi,\lambda)$		252
$h(\xi,u,\lambda,\rho)$	"Hamiltonian"	248
$h'(\xi,\lambda,q,\psi,\rho)$	variable part of Hamiltonian in hydro sub-problem	271

		page
$i=1,\ldots m$	(subscript) indexes constraints on u	242
$\ell=1,\ldots L$	(superscript) indexes inflow sequences	285
$\ell(\xi,z,\lambda)$		263
$\ell^t(\xi,u^t,\lambda)$		251
L_n^1	space of all integrable functions $\rho=\Xi \to R^n$	247
L_n^∞	space of all measurable essentially bounded recourse functions $u=\Xi \to R^n$	244
$(L_n^\infty)^*$	(††) space conjugate to L_n^∞	248
m	number of constraints, f_i	242
M	space for all "L^1 Martingales"	247
n^t	dimension of space from which $u^t(\xi)$ is chosen	242
n	$(= \sum_{t=1}^{T} (n^t))$	243
N_∞	space of all essentially non-anticipative recourse functions ($\equiv$ The set of all functions $u:\Xi \to R^n$ where: $u^t:\Xi \to R^{n^t}$ is F^t measurable for all $t=1,\ldots T$)	244
$\hat{N}_\infty^t$	(†) set of all nonanticipative decisions for the first t periods $(\hat{u}^t)$	246
PS	stochastic primal problem	244
PSA	stochastic aggregate primal problem	256
PSA′	stochastic aggregate Lagrangian problem	246
PSE	stochastic exchange problem	265
PSH	stochastic local hydro problem	267
$\hat{PSH}^t$	(†) projection of PSH	268
PSH^t	water value determination problem	269
PSM	modified stochastic primal problem	246

		page
$P\hat{S}M^t$	(†) projection of PSM	246
PST	stochastic thermal problem	264
$\mathcal{P}(\lambda,\rho)$	objective function for the dual problem	250
$\mathcal{P}_S(\lambda)$	"dual objective function" for practicable stochastic model	315
$\hat{Q}_h^t$	maximum utilisable release	
R^{n^t}	space from which $u^t(\xi)$ chosen	242
$R^n=R^{n^1}x\ldots R^{n^T}$	space from which $u(\xi)$ chosen	243
R^{ν^t}	space from which ξ^t drawn	242
$R^{\nu}=R^{\nu^1}x\ldots xR^{\nu^T}$	space from which ξ drawn	243
$\mathcal{R}(u,\lambda,\rho)$	Rockafellar-Wets Lagrangian	249
$\mathcal{R}^-(u,\rho)$	reduced Lagrangian	248
$\mathcal{R}'(u,\lambda)$	restricted Lagrangian	250
$\bar{S}'_h,\underline{S}'_h$	modified storage constraints	261
SDS	separable stochastic dual probelm	253
SPS	separable stochastic primal problem	251
SDSH	separable stochastic dual local hydro problem	277
SPSH	separable stochastic primal local hydro problem	266
$t=1,\ldots T$	(superscript) indexes time periods	242
$u^t(\xi)$	response at time t to ξ (= recourse function, decision rule, policy, control law)	242
$u(\xi)=(u^1(\xi),\ldots u^T(\xi))$		243
$\mathcal{U}^t(\xi^t)$	feasible set for $u^t(\xi)$ in separable problem	251
$\mathcal{U}(\xi)$	feasible set for $u(\xi)$	243
$v^t(\hat{\xi}^t,\hat{q}^t)$	the value function	246
$\hat{v}^t(\hat{\xi}^t,\hat{q}^t)$	(+) the past water value function (in hydro sub-model)	269

		page
$\overset{\vee}{V}^t(\hat{\xi}^t, \hat{q}^t)$	(†) the expected future water value function (in hydro sub-model)	269
w	dummy for u	246
$z(\xi) \varepsilon Z(\xi)$	$(g_T(\xi), q(\xi), e(\xi))$	256
η	dummy for ξ	246
$\lambda \varepsilon \Lambda$	vector of multipliers on f_i constraints	255
ν^t	dimension of space from which ξ^t drawn	242
ν	$(= \sum_{t=1}^{T} \nu^t)$	243
ξ^t	random vector observed at time t (inflows in hydro sub-model)	247
$\xi \varepsilon \Xi$	$(\xi^1, \ldots \xi^T)$	243
$\hat{\xi}^t$	(†) $(\xi^1, \ldots \xi^t)$	143
$\underline{\xi}_h(t,r), \bar{\xi}_h(t,r)$	maximum and minimum inflows between t and r in hydro sub-model	260
$\hat{\Xi}^t$	(†) projection of Ξ onto the first t components. i.e. $\{\hat{\xi}^t \mid \xi \varepsilon \Xi\}$	246
$\rho \varepsilon M$	multiplier on nonanticipativity restriction	247
σ	probability measure on Ξ	243
τ	computation time	311
$\bar{\psi}$	marginal expected water value	271
ψ_F^ℓ	marginal water value at end of first arc of simulated trajectory for inflow year ℓ	299

Notes: (†) Here the symbol ^ is used to indicate a projection rather than an optimum (e.g. $\hat{x}^t = (x^1, \ldots x^t)$. Also $^\vee$ is used to indicate a "tail projection". So $\overset{\vee}{x}^t = (x^{t+1}, \ldots x^T)$.

(††) Here the symbol * is used to indicate a conjugate rather than an optimum. This usage is restricted to the abstract recourse model of Section 8.2.

CHAPTER 10		page
$a=1,\ldots A$	indexes end uses for energy (subscript)	329
$B(d)$	social benefit function	329
B'^{r}_{ia}	(modified) social benefit function	346
d^{r}_{ia}	demand for electrical energy for use a, at i, in r.	330
$d^{r'}_{i}$	total demand at i, in r	330
$D^*(\mu)$	empirical demand function (for electricity)	338
DP	dual for optimal tariff problem	332
$\mathcal{H}_p(\mu)$	Hessian for DP	335
$\mathcal{L}_p(\mu)$	Lagrangian for optimal tariff problem	332
PD	demand sub-problem of PP′	333
PP	optimal tariff problem	330
PP′	Lagrangian problem for PP	332
$\mathcal{P}_P$	dual objective function (in DP)	332
$\mathcal{Z}_P$	feasible set for PP′	332
μ^{r}_{ia}	special tariff for use a	345
ρ	elasticity of demand curve	339
$\sigma(\mu)$	practicable tariff derived from μ	348

APPENDIX A

In general Appendix A follows the notational conventions of the remainder of this thesis with the following changes:

		page
A_h	tributary inflows to valley h	356
GRAF	Gestion du Réservoir Agrégé France	349
$i=1,\ldots I$	(subscript) indexes stations in river chain	366
$k=1,\ldots K$	(superscript) indexes load segments	356
$\ell=1,\ldots L$	(superscript) indexes inflow sequences	357
$\mathcal{L}_F^3$	Lagrangian for P3	358
$\mathcal{L}_F^2$	Lagrangian for P2	368
P1	short-term river scheduling program	353
P2	long-term local hydro program	353
P3	global optimisation program	353
P4	proposed thermal optimisation program	353
PL_o	thermal sub-problem	360
$PL_i, i=1,\ldots H$	local hydro sub-problem	360
R_i^t	"marginal rent" in P2	373
SGEP	Systeme de Gestion Energétique Prévisionelle	352
u^t	control variables for P2	367
$V(x,u)$	objective function for P2	368
x	desired trajectory in P2	357
X	ratio of off-peak to peak prices	376
σ^t	final water value	367
ϕ_i^t	multiplier on (F-30)	370
ω_i	"weights" in primal iterations of P2	370

APPENDIX C

NOTATIONAL COMPARISON OF STOCHASTIC MODELS

The purpose of this appendix is to facilitate comparison between our summary of the theoretical stochastic model (in Section 8.2) with the original expositions from which it was derived. Thus the following table sets out, for each of our notational conventions,the entity to which it refers and equivalent convention employed in references [52] and [53]. The table is arranged in the order in which the notation appears in Section 8.1.

OUR NOTATION	ENTITY	NOTATION OF [52]	NOTATION OF [53]
$t=1,\ldots T$ (superscript)	index for time periods	$k=1,\ldots N$ (subscript)	$k=1,\ldots N$
ξ^t	random vector	ξ_k	ξ_k
$R^{\nu t}$	space from which ξ^t drawn	$R^{\nu k}$	$R^{\nu k}$
$R^{\nu}=R^{\nu^1}x\ldots R^{\nu T}$	space from which ξ drawn	R^{ν}	R^{ν}
$u^t(\xi)$	response at time t to ξ (= recourse function, decision rule, policy, control law)	$x_k(\xi)$	$u_k(\xi)$
R^{n^t}	space from which $u^t(\xi)$ chosen	R^{n_k}	R^{n_k}
$R^n=R^{n^1}x\ldots xR^{n^T}$	space from which $u(\xi)$ chosen	R^n	R^n
$\mathcal{U}(\xi)$	basic feasible region in R^n from which $u(\xi)$ may be chosen	incorporated in $\mathcal{D}(\xi)$	$\mathcal{U}(\xi)$
$f_i(\xi,u)$ for $i=1,\ldots m$	further constraints on u		$f_i(\xi,u)$

OUR NOTATION	ENTITY	NOTATION OF [52]	NOTATION OF [53]
$f_o(\xi,u)$	objective function	$f(\xi,x)$	$f_o(\xi,u)$
$(\Xi,\mathcal{F},\sigma)$	probability space	$(\Xi,\mathcal{F},\sigma)$	$(\Xi,\mathcal{F},\sigma)$
$\hat{\xi}^r$	$=(\xi^1,\ldots\xi^r)$	ξ^k $[P_t(\xi)]$	ξ^k
$\mathcal{D}(\xi)$	the feasible region	$\mathcal{D}(\xi)$	$\mathcal{D}(\xi)$
$\alpha:\Xi\to\mathcal{R}^n$	summable function majorising f_o	$\mu:\Xi\to\mathcal{R}^n$	$\alpha:\Xi\to\mathcal{R}^n$
$\beta\varepsilon\mathcal{R}$	upper bound on f_i	-	$\beta\varepsilon\mathcal{R}$
$\mathcal{F}^r$	the σ field of all sets of the form: $((A\times[\mathcal{R}^{\nu^{t+1}}\times\ldots\times\mathcal{R}^{\nu^T}])\cap\Xi)\Delta B$ where A is a Borel set in $\mathcal{R}^{\nu^1}\times\ldots\times\mathcal{R}^{\nu^t}$, $B\varepsilon\mathcal{F}$ and $\sigma(B)=0$	$(\approx\mathcal{F}^k)$	$\mathcal{F}_k$
$\mathcal{N}_\infty$	the set of all essentially non-anticipative recourse functions ($\equiv$ the set of all functions $u:\Xi\to\mathcal{R}^n$ where $u^t:\Xi\to\mathcal{R}^{n^t}$ is $\mathcal{F}^t$ measurable for all t=1,...T.	$\mathcal{N}_\infty$	$\mathcal{N}_\infty$
$\mathcal{L}^\infty_n$	the set of all measurable essentially bounded recourse functions $u:\Xi\to\mathcal{R}^n$	$\mathcal{L}^\infty_n$	$\mathcal{L}^\infty_n$
$V^t(\hat{\xi}^t,\hat{q}^t)$	the value function	$q_k(\xi^k,x^k)$	-
$\hat{\mathcal{N}}^t_\infty$	the set of all non-anticipative decisions for the first t periods $(\hat{u}^t)$	$\mathcal{N}^k_\infty$	-
$\hat{\Xi}^t$	the projection of Ξ onto the first t components i.e., $\{\hat{\xi}^t\|\xi\varepsilon\Xi\}$	Ξ^k	-
		Definitions differ slightly	
$\mathcal{M}$	the set of '$\mathcal{L}^1$ martingales'	$\mathcal{M}_1$	$\mathcal{M}_1$
$\rho\varepsilon\mathcal{M}$	a particular 'multiplier on the nonanticipativity constraint'	ρ	ρ

OUR NOTATION	ENTITY	NOTATION OF [52]	NOTATION OF [53]
L^1_n	space of all integrable functions on R^n	$\mathcal{L}^1_n$	L^1_n
$f^*(\xi,\cdot)$	conjugate of $f(\xi,\cdot)$	$f^*(\xi,\cdot)$	-
$<u,\rho>$	inner product of u,ρ	$<u,\rho>$	$<u,\rho>$
$\mathcal{R}^-(u,\rho)$	$\left(=\begin{cases}E\{f_o(u,\xi)\}-<u,\rho> & \rho\varepsilon M\\ -\infty & \rho\not\varepsilon M\end{cases}\right)$ the reduced Lagrangian	$L(x,\rho)$ $\left(\begin{cases}I_f(x)-<x,\rho> & \rho\varepsilon M\\ -\infty & \rho\not\varepsilon M\end{cases}\right)$	$L(u,\rho)$
$(L^\infty_n)^*$	space conjugate to L^∞_n	$(\mathcal{L}^\infty_n)^*$	$(L^\infty_n)^*$
$\lambda_i\varepsilon\Lambda$	multiplier (measure) on i^{th} constant $f_i(\xi,u)$	-	$y_i\varepsilon Y$
$h(\xi,u,\lambda,\rho)$	"Hamiltonian"	-	$h(\xi,u,y,\rho)$
$\mathcal{R}(u,\lambda,\rho)$	Rockafellar-Wets Langrangian	-	$I_h(u,y,\rho)$
$E^t\{z\}$	conditional expectation given observations up to period t (information field F^t) $(E\{z(\eta)\,\vert\,\hat{\eta}^t=\hat{\xi}^t\})$	-	$E^t\{z\}$
λ	set of feasible λ's	-	$\mathcal{Y}$
$\mathcal{R}'(u,\lambda)$	the restricted Lagrangian	-	$I_\ell(u,y)$
$g(\xi,\lambda,\rho)$	-	-	$g(\xi,y,\rho)$
$P(\lambda,\rho)$	the 'objective functional' of dual problem	-	$I_g(y,\rho)$
$U^t(\xi^t)$	feasible set for $u^t(\xi)$	-	$U_k(\xi)$
$f^t_i(\xi,u^t)$	constraints on $u^t(\xi)$	-	$f_{ik}(\xi,u_k)$
$\ell^t(\xi,u^t,\lambda)$		-	$\ell_k(\xi,u_k,y)$
$g^t(\xi,\lambda)$		-	$g_k(\xi,y)$

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